Unsolved Questions in String Theory

1 Historical overview: 45 years of string theory

2 Unsolved questions

3 Remarks on string theory in the Rindler coordinates and space-time uncertainty

4 Conclusion
To the memory of Bunji Sakita (1930-2002) and Keiji Kikkawa (1935-2013)
1 Historical (and personal) overview: 45 years of string theory

1969 ~ 1979 Initial developments

- Nambu-Goto action
- Light-cone quantization, no-ghost theorem, critical dimensions, ...
- Ultraviolet finiteness (modular invariance)
- Neveu-Schwarz-Ramond model
- Space-time supersymmetry

Developments related to field-theory/string connection

- Fishnet diagram interpretation, Nielsen-Olesen vortex
- Derivation of gauge theory, general relativity and supergravity from strings in the zero-slope limit
- Construction of various supersymmetric gauge and gravity theories
- String picture from strong-coupling lattice gauge theory
- 't Hooft's large $N$ expansion
- String field theories (light-cone)
1984~1989  First revolution

- Green-Schwarz anomaly cancellation
- Five consistent perturbative superstring vacua in 10D
- Compactifications, various connections to mathematics
- CFT technique, renormalization group interpretation

1990~1994  “Old” matrix models

- Double scaling limit
- c=1 strings, 2D gravity, ‘non-critical’ strings
- Topological field theories and strings

1995~1999  Second revolution

- Discovery of D-branes
- Statistical interpretation of black-hole entropy in the BPS or near-BPS limits
- Conjecture of M-theory
- New matrix models (BFSS, IKKT), supermembranes, M(atrix) theory conjecture, ....
- AdS/CFT correspondence, GKPW relation, ....
Current main stream:

general idea of gauge-gravity correspondence

unification of two old ideas on strings from the 70s

- hadronic strings for quark confinement from gauge theory
- string theory for ultimate unification as an extension of general relativity
Spiral stair case: review talks in JPS meetings

1 Dual model and field theory 1975.3
2 Condensation of monopoles and quark confinement 1979.3
3 Hamiltonian quantum gravity 1985.4
4 Strings and gravity 1986.10
5 Lower-dimensional quantum gravity and string theory 1993.9
6 Toward non-perturbative string theory 1997.3
7 General relativity and elementary-particle theory 2005.9
8 What is string theory? : reminiscences and outlook 2010.3
abstract of the JPS review talk in 1975

Dual model and field theory

1. 場の理論と dual model との関係に対するアプローチには,

   (1) local field theory の新しい解として dual model を導く,

   (2) field theory から dual model が導かれるか否かはかかわりなく,

   まず理論の模型としての dual model が通常の field theory と

   いかなる関係にあるかを追求する。

   という 2 つの視点があろうが, 以下では主に 2 つの視点で, 今後も得られる

   結果を議論する。 (1) の視点については, 純粋理論的困難をめぐってかかわらず,

   多くの興味深い suggestion がある。これについては以下でふれることがある。

2. 2 の方向について, 主に以下の 3 つの問題を議論する。

   (1) counting の問題

   (2) gauge 構造の関係

       (a) Yang-Mills fields  (← open string)

       (b) Gravity  (← closed string)

3. dual model と 場の理論 の "相互作用".

   最後に dual model と 場の理論の関係の研究を通じて, 双方に持たれた

   理論, および発表された事等を簡単に述べる。
Main message of that talk was:
Is it possible to resolve the following dichotomic duality relations?

1. Local field theories $\rightarrow$ strings
   - Non-perturbative and higher order effects
   - Fishnet diagrams, $1/N$-expansions
   - Lattice gauge theories, etc

2. Strings $\rightarrow$ local field theories
   - Zero-slope limits
   - Local gauge symmetry from world-sheet conformal invariance
   - Closed strings $\rightarrow$ general relativity
   - Open strings $\rightarrow$ gauge theory

“How could general relativity be contained in gauge theory?”
Gravity from strings: personal reminiscences of early developments

TAMIAKI YONEYA

Abstract

I discuss the early developments of string theory with respect to its connection with gauge theory and general relativity from my own perspective. The period covered is mainly from 1969 to 1974, during which I became involved in research on dual string models as a graduate student. My thinking towards the recognition of string theory as an extended quantum theory of gravity is described. Some retrospective remarks on my later works related to this subject are also given.

37.1 Prologue: an encounter with the dual string model

I entered graduate school at Hokkaido University, Sapporo, in April 1969. My advisor, Akira Kanazawa, who was an expert in the dispersion theoretic approach to strong interactions, proposed to have a series of seminars on Regge pole theory. However, the Regge pole theory was somewhat disappointing for me. I felt that it was too formal and phenomenological in its nature. Looking for some more favourable topics, I began studying the quantum field theory of composite particles, which, I thought, might be useful to explain the Regge behaviour from the dynamics of fundamental particles. I read many papers related to this problem, such as those on compositeness criteria, on the definition of the asymptotic field for a composite particle, the Bethe–Salpeter equation, and so on. Although I felt that these subjects themselves were not yet what I really wanted to pursue, I enjoyed learning various different facets of quantum field theory.

While still seeking subjects for my research, some senior students told me that a spectacular new development, triggered by a proposal made about a year before by Veneziano [Ven68], was springing up. After reading the paper of Veneziano and some others which extended the Veneziano amplitude to various directions, I gradually became convinced that this had to be the subject I should choose. In particular, when I was exposed to a short but remarkable preprint by Susskind [Sus69] giving a physical interpretation of the Veneziano formula in terms of vibrating strings (or ‘rubber band’ in Susskind’s terminology), I was
What we have achieved:

Gravity and and gauge forces emerge from quantum mechanics of relativistic strings, in loop expansions, in such a way that

- no ultraviolet divergence in loop corrections
- unitarity is preserved
- external degrees of freedom (or parameters) are not allowed
- deformations of backgrounds from flat space-time can be, at least infinitesimally, described using only the degrees of quantum strings (condensation of string fields)
- derivations of black hole entropy in some special cases
- new understanding about dual connections between gravity and gauge theory
All these suggest that string theory is quite promising as a unified theory of all natural interactions, the structure of matter and space-time.
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an artistic image of string unification?

Gunas
Isamu Noguchi (1904-1988)

Guṇa (Sanskrit: गुण) means 'string' or 'a single thread or strand of a cord or twine'. (from Wikipedia)
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But, leaving aside the ultimate question of phenomenological validity, there are many fundamental questions, which seem almost insurmountable at the present time, in both technical and conceptual aspects.
2 Unsolved questions

principles or any hidden (geometrical) symmetries behind
   Why strings and branes?
   Why matrices or gauge theories?

non-perturbative formulation
   Could gauge/gravity correspondence be the non-perturbative
definition of string theory?

exact formulation of strings and branes in curved space-time
   No string theory in nontrivial curved space-times
   which are understood at the same level as in flat space-time
background independent formulation

What are the primordial degrees of freedom of string theory?

Could holographic principle be formulated in the background-independent fashion?

Where are the degrees of freedom for deforming background space-time in “boundary” theory?

What is the non-perturbative formulation of open/closed string duality?
stringy description of space-time geometry

To what extent, space-time horizons and space-time singularities meaningful?

How to formulate information puzzle in stringy language? Is it possible to formulate “compactification” dynamically?

observables other than the S-matrix elements

What are the invariants or symmetries characterizing stringy observables? Is there any theory of measurement for quantum string theory?
Some of these questions motivated various new approaches appeared, especially in the period of second revolution (1995-1999)

M-theory conjecture, matrix models (BFSS, IKKT, ...), and general conjecture of gauge-gravity correspondence

Progress has been made. But it seems fair to say that we are still far and away in answering any of these fundamental questions.
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In the following, I would like to recall some of my past attempts motivated by these questions. Sole purpose is to stimulate you to think further.
An earliest suggestion
toward background independence:

conjecture of purely cubic action

\[ S = \frac{1}{6} \langle \psi^3 \rangle \]

\[ \psi_c^2 = 0 \quad \begin{array}{c} \downarrow \\ \psi = \psi_c + \tilde{\psi} \end{array} \]

\[ S = \langle \frac{1}{2} \tilde{\psi} \psi_c \tilde{\psi} + \frac{1}{6} \tilde{\psi}^3 \rangle \]

talk “Approaches to string field theory”
in ICOBAN’86 International conference on
grand unification (site near Kamiokande)

See also related works
T.Y. PRL55(1985)1828,
PLB197(1987)76
Q: P. Frampton (Univ. of North Carolina)
When such a string field theory has been successfully and completely formulated, will it answer the question of how many different string theories exist?

A: If the concept of field is fundamental to string theory, the final complete string field theory might answer all such questions. Different string models might be different solutions to a single fundamental field equation in a way as I suggested at the end of my talk.

Q: K. Kikkawa (Osaka Univ.)
I have a question about $\Psi^3$ action, which is supposed to be made background metric independent. In ordinary field theory we can make it if the metric tensor $g_{\mu\nu}$ is incorporated. Is there any geometrical possibility that $\Psi^3$ is made background independent without using the metric field which is now almost $\Psi$ itself?

A: My guess is that $\Psi^3$ might be itself something analogous to $\sqrt{\text{det}g}$ when $\Psi$ is properly interpreted as a basic geometrical entity.
After the conference, interesting attempts toward possible realization of this conjecture have been made in

Horowitz-Lykken-Rohm-Strominger, PRL 57, 283(1986)

In view of the present status of superstring field theory, this idea was a bit too naive. But in any case,

String fields, or (if any) other possible degrees of freedom for describing background independence, themselves should be regarded as a fundamental geometric entity of “string geometry”.

Classical geometry must be an emergent phenomena, or any property of space-time must be defined by physical processes of strings themselves.
Another *unfinished* project: *non-perturbative* understanding of *open-closed string duality*

motivation:

- **gravitational degrees of freedom** in gauge-theory description of D branes
  (and hence in gauge-gravity correspondence)
- are hidden in the whole quantum configuration space of gauge theories

For instance, the correct 3-point interactions of gravitons are obtained only as a loop effect in D0 susy gauge theory.

(Y. Okawa and T. Y., 1998)
an analogy: Mandelstam duality in 2D field theories

- massive Thirring model $\leftrightarrow$ D-particle field theory
- sine-Gordon model $\leftrightarrow$ closed string field theory

open-string field theories
- effective Yang-Mills theories
- first quantization or second quantization

D-brane field theories

closed-string field theories
- open-closed duality
- bosonization or Mandelstam duality

Fock space of D particle gauge theories with different $N$

Gauge symmetry
- = quantum statistics of D-particle Hilbert space

Gauge invariants
- = bi-linears of D-particle fields

Creation and annihilation of D particles and associated open strings

A Difficulty: necessity of non-associative structure
It is possible to rewrite the entire content of Yang-Mills quantum mechanics for D particles with all different $N$ as an extended quantum field theory.

Schrödinger equation

$$i \frac{\partial}{\partial t} \Psi[X] = - \text{Tr} \left( \frac{g_s \ell_s}{2} \frac{\partial}{\partial X^i} \frac{\partial}{\partial X^i} + \frac{1}{4g_s \ell_s^5} [X^i, X^j]^2 \right) \Psi[X]$$

$$= - \frac{1}{2} \text{Tr} \left[ g_s \ell_s \frac{\partial}{\partial X^i} \frac{\partial}{\partial X^i} + \frac{1}{g_s \ell_s^5} (X^i X^j X^i X^j - X^i X^i X^j X^j) \right] \Psi[X]$$

$$\Downarrow$$

$$\mathcal{H} | \Psi \rangle = 0$$

$$\mathcal{H} = i(4 \langle \phi^+, \phi^- \rangle + 1) \partial_t + 2g_s \ell_s \left( (\langle \phi^+, \phi^- \rangle + 1) \langle \phi^+, \partial_{\bar{z}} \phi^- \rangle + 3 \langle \phi^+, \partial_{\bar{z}} \phi^- \rangle \cdot \langle \phi^+, \partial_{z} \phi^- \rangle \right)$$

$$+ \frac{1}{2g_s \ell_s^5} (4 \langle \phi^+, \phi^- \rangle + 1) (\langle \phi^+, \phi^- \rangle + 1) \langle \phi^+, \left( (\bar{z}^i \cdot z^j)^2 - (\bar{z}^i \cdot \bar{z}^j)(\bar{z}^j \cdot z^i) \right) \phi^- \rangle$$

(with some minor constraints)
Remarks on string theory in the Rindler coordinates and space-time uncertainties

Another crucial unsolved problem is “information paradox” of black hole. But majority of recent works discussing this question ignore stringy nature of gravity, basing on low-energy effective (field) theory

(except perhaps for holographic arguments and “fuzz-ball” conjecture)

However, by definition, the black hole and space-time horizon involves arbitrarily high energy (short distance) physics, corresponding to infinite red shift associated with the horizon.
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It seems of vital importance, from the viewpoint of physics in the bulk space-time, to take into account the non-local nature of string theory.
What is the appropriate characterization of non-locality of strings?

This is also an unsolved question. In any theory of quantum gravity, if we set two of *classical* fundamental constants to unity,

\[
G = c = 1 \implies \ell_P = \sqrt{\hbar}
\]

Once we take into account gravity,

*quantization= introduction of fundamental length*

In string theory, the role of fundamental length is played by the string length \( \ell_s \)
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The space-time uncertainty relation was originally proposed with this in mind. In the absence of any definite mathematical formalism and axioms, only way was to adopt a qualitative approach.
DUALITY AND INDETERMINACY PRINCIPLE IN STRING THEORY

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ABSTRACT

We give an elementary explanation about how string theories overcome the ultraviolet difficulty of the local field theories. The indeterminacy principle is reinterpreted as a limitation on the smallness of the domain of observations.

Time-energy uncertainty relation is reinterpreted as the time-space relation.

\[ \Delta E \Delta t \gtrsim h \quad \rightarrow \quad \Delta X \Delta T \gtrsim \ell_s^2 \]

If we consider the high-energy regime,

\[ \Delta E \sim \Delta X \frac{h}{\ell_s^2} \quad \Delta t = \Delta T \]

This was derived by quantizing strings in the time-like gauge.
This relation can also be derived directly and in a more universal way using world-sheet conformal invariance.


For arbitrary quadrilaterals $\Omega$, on the world sheet, we have conformal invariants, called the extremal length which corresponds to proper time of particle trajectory

$$\lambda_{\Omega}(\Gamma) = \sup_{\rho} \frac{L(\Gamma, \rho)^2}{A(\Omega, \rho)}$$

arbitrarily chosen world-sheet metric

$$ds = \rho(z, \bar{z})|dz|$$

$$L(\Gamma, \rho) = \inf_{\gamma \in \Gamma} L(\gamma, \rho) \quad L(\gamma, \rho) = \int_{\gamma} \rho |dz|$$

$$A(\Omega, \rho) = \int_{\Omega} \rho^2 dz d\bar{z}$$
composition theorem:

\[ \Omega = \Omega_1 + \Omega_2, \quad \Omega_1 \cap \Omega_2 = \emptyset \]

1. If every \( \gamma \in \Gamma \) contains a \( \gamma_1 \in \Gamma_1 \) and \( \gamma_2 \in \Gamma_2 \), then
   \[ \lambda_{\Omega}(\Gamma) \geq \lambda_{\Omega_1}(\Gamma_1) + \lambda_{\Omega_2}(\Gamma_2). \]

2. If every \( \gamma_1 \in \Gamma_1 \) and \( \gamma_2 \in \Gamma_2 \) contains a \( \gamma \in \Gamma \), then
   \[ 1/\lambda_{\Omega}(\Gamma) \geq 1/\lambda_{\Omega_1}(\Gamma_1) + 1/\lambda_{\Omega_2}(\Gamma_2). \]

reciprocity theorem:

Let the two pairs of opposite sides of \( \Omega \) be \( \alpha, \alpha' \) and \( \beta, \beta' \).

\( \Gamma \) : the set of arcs joining \( \alpha \) and \( \alpha' \)
\( \Gamma^* \) : the set of arcs joining \( \beta \) and \( \beta' \)

\[ \lambda_{\Omega}(\Gamma)\lambda_{\Omega}(\Gamma^*) = 1 \]
The geometrical properties of target space-time are related through these conformal invariants:

\[
\lambda(\Gamma) = \frac{a}{b}
\]

\[
\lambda(\Gamma^*) = \frac{b}{a}
\]

Contribution to the amplitude:

\[
\Delta A = \sqrt{\langle A^2 \rangle} \sim \sqrt{\lambda(\Gamma)} \ell_s
\]

\[
\Delta B = \sqrt{\langle B^2 \rangle} \sim \sqrt{\lambda(\Gamma^*)} \ell_s
\]

\[
x^\mu(0, \xi_2) = x^\mu(a, \xi_2) = \delta^{\mu 2} B \xi_2 / b
\]

\[
x^\mu(\xi_1, 0) = x^\mu(\xi_1, b) = \delta^{\mu 1} A \xi_1 / a.
\]

\[
\exp \left[ -\frac{1}{\ell_s^2} \left( \frac{A^2}{\lambda(\Gamma)} + \frac{B^2}{\lambda(\Gamma^*)} \right) \right]
\]

\[
\Delta A \Delta B \sim \ell_s^2
\]
In black hole space-times;

For remote observers outside black holes, a finite length of time corresponds to an *infinitesimally short time on the horizon.*

Space-time uncertainty relation implies that the longitudinal extension of strings in the near horizon region is arbitrary large.

\[
\Delta X_1 \sim \frac{\ell_s^2}{\Delta T} \quad \rightarrow \quad \infty 
\]

Essentially the same relation is noticed later by Susskind (1994) who also emphasized its relevance in black hole physics.

Then there is almost no meaning in considering horizons using local field approximations. Information puzzle must be formulated by taking due account of *non-local nature of strings.*
Penrose diagram of Schwarzschild space-time

\[ ds^2 = -\left(1 - \frac{2GM}{r}\right)dt^2 + \frac{1}{1 - \frac{2GM}{r}}dr^2 + r^2d\Omega \]

\[ X_1 = 4GM\sqrt{1 - \frac{2GM}{r}} \]

\[ \tau = \frac{t}{4GM} \]

near horizon approximation

\[ ds^2 = -X_1^2(d\tau)^2 + (dX_1)^2 \]
String theory in the Rindler coordinates

\[ x^1 = X_1 \cosh \tau, \quad x^0 = X_1 \sinh \tau, \quad x_i = X_i \quad (i = 2, \ldots, D - 1) \]

\[ ds^2 = -X_1^2 (d\tau)^2 + (dX_1)^2 + \sum_{i=2}^{D-1} (dX_i)^2 \]

\[ S_{\text{string}} = -\frac{1}{4\pi \alpha'} \iint \sqrt{-\gamma(\xi)} \gamma^{ab}(\xi) \left[ -X_1^2 \frac{\partial \tau}{\partial \xi^a} \frac{\partial \tau}{\partial \xi^b} + \frac{\partial X_1}{\partial \xi^a} \frac{\partial X_1}{\partial \xi^b} \right. \]

\[ + \left. \frac{\partial X}{\partial \xi^a} \cdot \frac{\partial X}{\partial \xi^b} \right] \]

There are a large number of previous works studying string theory in the Rindler coordinates, but they are unsatisfactory in dealing with the Virasoro constraints or in using unjustifiable gauges. **It is in general impossible (or inconsistent) to satisfy the momentum constraint if one restricts oneself to incomplete space-like surfaces.**
Physically, the most natural gauge choice is the time-like and orthogonal gauge.

\[ \xi^0 = \tau \quad \gamma_{01} = 0 \]

residual freedom= time-independent re-parametrization of \( \sigma \)

canonical momenta

\[
P_{\tau} = -\kappa \sqrt{-\frac{\gamma_{11}}{\gamma_{00}}} X_1^2 \quad P_1 = \kappa \sqrt{-\frac{\gamma_{11}}{\gamma_{00}}} \frac{\partial X_1}{\partial \tau} \quad \kappa = \frac{1}{2\pi \alpha'}
\]

eqs. of motion

\[
\frac{\partial}{\partial \tau} \sqrt{-\frac{\gamma_{11}}{\gamma_{00}}} \frac{\partial X_1}{\partial \tau} - \frac{\partial}{\partial \sigma} \sqrt{-\frac{\gamma_{00}}{\gamma_{11}}} \frac{\partial X_1}{\partial \sigma} + \sqrt{-\frac{\gamma_{11}}{\gamma_{00}}} X_1 = 0
\]

\[
\frac{\partial}{\partial \tau} P_{\tau} = -\kappa \frac{\partial}{\partial \tau} \sqrt{-\frac{\gamma_{11}}{\gamma_{00}}} X_1^2 = 0 \quad \Rightarrow \quad \kappa \sqrt{-\frac{\gamma_{11}}{\gamma_{00}}} X_1^2 = e
\]

constant energy density

(closed string: \( \sigma \) with periodicity \( \sigma \to \sigma + 2\pi \) )
Virasoro conditions

\[ e^2 = X_1^2 \left[ P_1^2 + P^2 + \kappa^2 \left( \frac{\partial X_1}{\partial \sigma} \right)^2 + \kappa^2 \left( \frac{\partial X}{\partial \sigma} \right)^2 \right] \]

\[ P_1 \frac{\partial X_1}{\partial \sigma} + P \cdot \frac{\partial X}{\partial \sigma} = 0 \]

These complicated-looking (nonlinear) system of equations can in principle be exactly solvable, \textit{classically}.
But exact quantization is very difficult.

Let us take a \textit{qualitative approach and derive the space-time uncertainty relation} from this viewpoint.
In the regions of the world sheet where $X^2_1$ is small, velocity of longitudinal mode in the target space

$$\frac{\partial X_1}{\partial \tau} \sim X_1(\sigma, \tau)$$

velocity of propagating transverse oscillations along the world sheet

$$c(\sigma, \tau) = \frac{\kappa}{\epsilon} X^2_1$$

The Rindler horizon leads to **horizons in the world sheet**, which are only **dynamically determined self-consistently for each classical solution**.

$$X_1(\sigma_0, \tau) = 0, \quad \sigma_0 = F(\tau)$$

We cannot divide the Hilbert space of (1st quantized) strings into left and right wedges. (2nd quantized) string fields must be defined using both Rindler regions simultaneously. Hence, the Hilbert space of string fields cannot be decomposed either.
Let us denote the local horizon by a function 

\[ \nu \sim c/\lambda \sim nc = \frac{\kappa n}{e} X_1^2 \]

\( \mathcal{H} \): average excitation number of transverse oscillations along the world sheet 

\( \epsilon \): average energy density

Hamiltonian constraint requires \( e^2 \gtrsim n^2 \)

\[ \kappa X_1^2 \gtrsim \nu \quad \longrightarrow \quad \kappa |X_1| \Delta X_1 \gtrsim \Delta \nu \]

\[ \Delta \nu \Delta \tau \sim 1 \]

\[ |X_1| \Delta X_1 \Delta \tau \gtrsim 2\pi \alpha' \sim \ell_s^2 \]

Fluctuations of \( \epsilon \) and \( \mathcal{H} \) also contribute to uncertainties, but they are either of the same order or of non-leading order.
No thermalization!

This is as it should be:

There is no allowed invariants (observables) on the world-sheets which can be defined in restricted localized regions of $\sigma$.

We cannot localize the string fields in finite space-like regions if we assume the validity of the space-time uncertainty relation with respect to target space-time. Again no legitimate observables restricted space-like regions, other than S-matrices, and seems to support ‘black hole complementarity’.
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entangled pure states $\rightarrow$ thermal mixed state

cannot happen, at least in association with the existence of space-time horizons, in string theory.
This strongly indicates that the notion of space-time horizons is meaningless in string theory.

Of course, concrete resolution of the information puzzle is yet a big open question. Its final resolution would require the construction of genuinely stringy geometry of space-times. Note that, for instance, any non-linear sigma models for describing curved space-times intrinsically rely, still, upon classical geometry.

*Also, our claim does not mean that the idea of thermalization is completely devoid of meaning.*

*It must be useful as an approximate concept.*

But we have to keep in mind that there is a serious limit on this idea, as we can infer, for instance, also from the existence of limiting temperature (*Hagedorn temperature*) in string theory.
4 Concluding remark

No doubt, string theory is promising toward a final and complete unification. Unfortunately, however, there are many fundamental unsolved questions. They seem to be too difficult and almost insurmountable at this time.

But I believe that there must be different and entirely new ways of looking at these questions. Perhaps, and hopefully, steady efforts of pursuing what we can, such as various computer simulations and studies of simple models and so on, may somehow open up doors to new unexpected directions and angles of resolving these questions.
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*The next year 2015 is the centenary of General Relativity. Pray for the next (3rd) revolution of string theory in the not so distant future!*