格子フェルミオン (2) Exact chiral symmetry on the lattice

参考文献

[1] P. H. Ginsparg and K. G. Wilson, Phys. Rev. D **25**, 2649 (1982).

[2] M. Lüscher, Phys. Lett. B **428**, 342 (1998) [arXiv:hep-lat/9802011].

[3] D. B. Kaplan, Phys. Lett. B **288**, 342 (1992) [arXiv:hep-lat/9206013].

[4] Y. Shamir, Nucl. Phys. B **406**, 90 (1993) [arXiv:hep-lat/9303005].

[5] V. Furman and Y. Shamir, Nucl. Phys. B **439**, 54 (1995) [arXiv:hep-lat/9405004].

[6] H. Neuberger, Phys. Rev. D **57**, 5417 (1998) [arXiv:hep-lat/9710089].

[7] Y. Kikukawa and T. Noguchi, arXiv:hep-lat/9902022.

[8] H. Neuberger, Phys. Lett. B **417**, 141 (1998) [arXiv:hep-lat/9707022].

[9] H. Neuberger, Phys. Lett. B **427**, 353 (1998) [arXiv:hep-lat/9801031].

[10] Y. Kikukawa and H. Neuberger, Nucl. Phys. B **513**, 735 (1998) [arXiv:hep-lat/9707016].

[11] P. Hernandez, K. Jansen and M. Lüscher, Nucl. Phys. B **552**, 363 (1999) [arXiv:hep-lat/9808010].

[12] P. Hasenfratz, V. Laliena and F. Niedermayer, Phys. Lett. B **427**, 125 (1998) [arXiv:hep-lat/9801021].

[13] M. Lüscher, Phys. Lett. B **428**, 342 (1998) [arXiv:hep-lat/9802011].

[14] Y. Kikukawa and A. Yamada, Phys. Lett. B **448**, 265 (1999) [arXiv:hep-lat/9806013].

[15] D. H. Adams, Annals Phys. **296**, 131 (2002) [arXiv:hep-lat/9812003].

[16] K. Fujikawa, Nucl. Phys. B **546**, 480 (1999) [arXiv:hep-th/9811235].

[17] H. Suzuki, Prog. Theor. Phys. **102**, 141 (1999) [arXiv:hep-th/9812019].

[18] K. Fujikawa, Phys. Rev. Lett. **42**, 1195 (1979).

[19] K. Fujikawa, Phys. Rev. D **21**, 2848 (1980) [Erratum-ibid. D **22**, 1499 (1980)].

Ginsparg-Wilson relation

格子フェルミオンの作用 $S = a^4 \sum_x \bar{\psi}(x) D \psi(x)$ がカイラル極限にあることを表す条件

$$\gamma_5 D + D\gamma_5 = 2aD\gamma_5 D \tag{1}$$

Domain Wall Fermion と Ginsparg-Wilson relation

Block spin trans.	Domain wall fermion
≡Integration of high energy modes	≡Integration of heavy modes
$\Downarrow$	$\Downarrow$

Local low energy effective action

$$S = \sum_x \bar{\psi}(x) D \psi(x)$$

$$\Downarrow$$

Ginsparg-Wilson rel.

$$\gamma_5 D + D\gamma_5 = 2aD\gamma_5 D$$

$$S_{\mathrm{DWF}} = a_5 \sum_{t=1}^{N} \sum_x \bar{\Psi}(x,t)(D_{5w} - m_0)\Psi(x,t)|_{\mathrm{Dir.}} \qquad (0 < m_0 < 2, a = 1) \tag{2}$$

$$S'_{\mathrm{DWF}} = \sum_x \bar{\psi}(x) D_N \psi(x) + a_5 \sum_{t=1}^{N} \sum_x \bar{\Psi}'(x,t)(D_{5w} - m_0)\Psi'(x,t)|_{\mathrm{A.P.}} \tag{3}$$

$$Z_{\mathrm{DWF}} \equiv \int \mathcal{D}\Psi \mathcal{D}\bar{\Psi}\ \mathrm{e}^{-S_{\mathrm{DWF}}} = \int \mathcal{D}\Psi' \mathcal{D}\bar{\Psi}'\, \mathcal{D}\psi \mathcal{D}\bar{\psi}\ \mathrm{e}^{-S'_{\mathrm{DWF}}}$$

$$\det\left(D_{5\mathrm{w}} - m_0\right) = \det D_N \cdot \det\left(D_{5\mathrm{w}} - m_0\right)_{[AP]}$$

$$D_N = \frac{1}{2}\left(1 + \gamma_5 \frac{(1+H)^N - (1-H)^N}{(1+H)^N + (1-H)^N}\right) \xrightarrow{N\to\infty} \frac{1}{2}\left(1 + \gamma_5 \frac{H}{\sqrt{H^2}}\right) \tag{4}$$

$$H = a_5 H_w / [2 + a_5(D_w - m_0)] \qquad T = (1-H)/(1+H) \tag{5}$$

$$\langle q(x)\, \bar{q}(y)\rangle = \langle \psi(x)\, \bar{\psi}(y)\rangle - \delta_{x,y} = \langle (1 - D_N)\psi(x)\, \bar{\psi}(y)\rangle \tag{6}$$

Neuberger's Overlap Dirac operator

GW rel. を満足する ゲージ共変な Dirac op. の具体例

$$D = \frac{1}{2a}\left(1 + \gamma_5 \frac{H_w}{\sqrt{H_w^2}}\right) \tag{7}$$

$$H_w = \gamma_5 \left(D_w - m_0/a\right), \quad D_w = \sum_\mu \left\{\gamma_\mu \frac{1}{2}\left(\nabla_\mu + \nabla_\mu^*\right) - \frac{a}{2}\nabla_\mu \nabla_\mu^*\right\} \tag{8}$$

$$0 < m_0 < 2 \tag{9}$$

局所性

場同士の結合が、距離（格子スケール）とともに、指数関数的に小さくなる

$$\parallel D(x,y) \parallel \leq \kappa(1+ \parallel x-y \parallel^\nu) e^{-\|x-y\|/\varrho} \qquad \varrho/a \simeq \mathcal{O}(1) \tag{10}$$

D の局所性が成立するための十分条件 (admissibility condition)

$$\parallel 1 - U(x,\mu)U(x+\hat{\mu},\nu)U(x+\hat{\nu},\mu)^{-1}U(x,\nu)^{-1} \parallel \leq \ \epsilon \tag{11}$$

$$\epsilon < \frac{1}{30} \qquad \because \quad (aD_w - 1)(aD_w - 1)^\dagger \geq 1 - 30\epsilon \tag{12}$$

Locality of D $\Rightarrow$ GW rel. $\gamma_5 D + D\gamma_5 = 2D\gamma_5 D$ $\Rightarrow$ Index of D

$\Uparrow$ $\searrow$ $\Downarrow$

Local chiral anomaly $\sum_x -\mathrm{tr}\gamma_5 D(x,x)$

Index theorem on the lattice

$\searrow$ $\Uparrow$

Admissibility cond. $\|1 - P_{\mu\nu}(x)\| < \epsilon$ $\Rightarrow$ Topological structure of lattice gauge fields $\Rightarrow$ Topological charge $Q = -\frac{1}{2}\sum_x \mathrm{tr}\gamma_5 D(x,x)$

カイラル変換とカイラルアノマリー

$$\delta_\alpha \psi(x) = i\alpha\, \gamma_5 (1-2aD)\psi(x), \qquad \delta_\alpha \bar{\psi}(x) = i\alpha\, \bar{\psi}(x)\gamma_5 \tag{13}$$

D すなわち ゲージ場 $U(x,\mu)$ に依存する局所変換 $\Rightarrow$ $U(1)_A$ アノマリー

$$\delta_\alpha \Big[\prod_x d\psi(x) d\bar{\psi}(x)\Big] = \Big[\prod_x d\psi(x) d\bar{\psi}(x)\Big] \sum_x \alpha(x)(-2)\mathrm{tr}\left\{\gamma_5 (1-aD)(x,x)\right\} \tag{14}$$

Atiyah-Singer 指数定理 on the lattice

$$2\mathrm{Tr}\left\{\gamma_5(1-aD)\right\} = 2\,\mathrm{Index}(D) = 2(n_+ - n_-) \tag{15}$$

Topological charge:　整数値をとる, $U(x,\mu)$ の smooth な関数

$$Q = \mathrm{Tr}\left\{\gamma_5(1-aD)\right\} = -\frac{1}{2}\mathrm{Tr}\left\{\frac{H_w}{\sqrt{H_w^2}}\right\} \tag{16}$$

D の固有値　中心 $(1/2a, 0)$ 半径 $1/2a$ の円周上に分布

$$D + D^\dagger = 2aD^\dagger D = 2aDD^\dagger \ (\text{正規}), \quad D^\dagger = \gamma_5 D \gamma_5 \ (\gamma_5\text{–共役}) \tag{17}$$

$$\lambda + \lambda^* - 2a\lambda^*\lambda = (-2a)\left[(\lambda - 1/2a)(\lambda - 1/2a)^* - (1/2a)^2\right] = 0 \tag{18}$$

$$\lambda = 0: \qquad \gamma_5\psi_\lambda(x) = \pm\psi_\lambda(x) \tag{19}$$

$$\lambda = 1/a: \qquad \gamma_5\psi_\lambda(x) = \pm\psi_\lambda(x) \tag{20}$$

$$\lambda \neq 0, 1/a: \qquad \text{pair-wise} \begin{cases} \lambda \to \psi_\lambda \\ \lambda^* \to \gamma_5\psi_\lambda \end{cases} \tag{21}$$

cf. $D_E = \gamma_\mu D_\mu$ の固有値 $\longrightarrow$

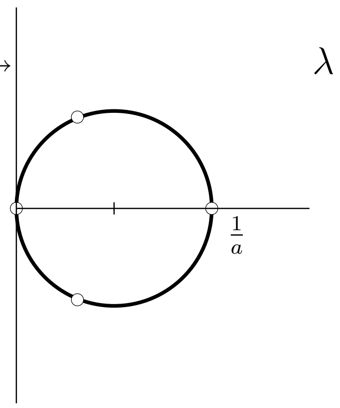

図 1: Overlap Dirac op. の固有値