格子フェルミオン (1) Species doubling Problem, Wilson-Dirac Fermion

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Species Doubling Problem

$$S = a^4 \sum_x \bar{\psi}(x)\, \gamma_\mu \frac{1}{2} \left(\partial_\mu - \partial_\mu^\dagger\right)\, \psi(x) = \int_{-\pi/a}^{\pi/a} \frac{d^4k}{(2\pi)^4}\, \bar{\psi}(-k)\, \left\{ i\gamma_\mu \frac{1}{a} \sin k_\mu a \right\}\, \psi(k)$$

Massless poles : $k_\mu a = (0,0,0,0), (\pi,0,0,0), \cdots, (\pi,\pi,\pi,\pi)$

$$\left\langle \psi(k)\, \bar{\psi}(-k) \right\rangle = \frac{-i\gamma_\mu \frac{1}{a} \sin k_\mu a}{\sum_\nu \frac{1}{a^2} \sin^2 k_\nu a}$$

Nielsen-Ninomiya theorem

$$S = \int_{-\pi/a}^{\pi/a} \frac{d^4k}{(2\pi)^4}\, \bar{\psi}(-k)\, \tilde{D}(k)\, \psi(k)$$

1. $\tilde{D}(k)$ is a periodic and analytic function of momentum k_μ
2. $\tilde{D}(k) \propto i\gamma_\mu k_\mu \quad$ for $|k_\mu| a \ll \pi$
3. $\tilde{D}(k)$ is invertible for all k_μ except $k_\mu = 0$
4. $\gamma_5 \tilde{D}(k) + \tilde{D}(k)\gamma_5 = 0$

These four conditions cannot be satisfied simultaneously!

Note: analyticity and locality

$$\frac{\partial^l}{\partial k^l}\tilde{D}(k) = \sum_x e^{ikx}(ix)^l D(x) < \infty \qquad \Longleftarrow \qquad \|D(x)\| < Ce^{-\gamma|x|}$$

Wilson-Dirac fermion

$$S_{\mathrm{w}} = a^4 \sum_x \bar{\psi}(x) \left(\gamma_\mu \frac{1}{2}\left(\partial_\mu - \partial_\mu^\dagger\right) + \frac{a}{2}\left(\partial_\mu \partial_\mu^\dagger\right) + m \right) \psi(x)$$

For $k_\mu a = (\pi, 0, 0, 0), \cdots, (\pi, \pi, \pi, \pi)$

$$m + \sum_\mu \frac{a}{2}\left(\frac{2}{a}\sin\frac{k_\mu a}{2}\right)^2 \simeq m + \frac{2n}{a} \qquad (n : \text{number of } \pi)$$

- $SU(N_f)_L \times SU(N_f)_R$ flavor chiral symmetry is broken
- axial $U(1)$ anomaly due to the explicit breaking