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Yang-Mills 場とカイラルフェルミオン

Target continuum theory :

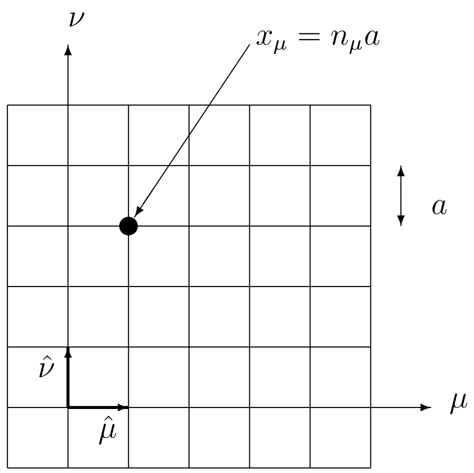
$$\mathcal{L}(x) = -\frac{1}{4} F^a_{\mu\nu}(x) F^{a\mu\nu}(x) + \sum_f \bar{\psi}(x)\, i\gamma_\mu(\partial_\mu + igA^a_\mu R_f[T^a])\psi(x)$$

$$F^a_{\mu\nu}(x) = \partial_\mu A^a_\nu(x) - \partial_\nu A^a_\mu(x) + g f_{abc} A^b_\mu(x) A^c_\nu(x)$$

格子理論

- 4-dim. Euclidean lattice $\mathbb{L}^4 = \{x_\mu = n_\mu a \mid n_\mu \in \mathbb{Z}^4\}$
  - 場は格子点, または, Link $(x, x+\hat{\mu})$ の上
  - Lorentz inv. $\rightarrow$ hypercubic rotational inv.
- momentum cut-off $p_\mu \in \left[-\frac{\pi}{a}, \frac{\pi}{a}\right]$
- Path-integral による量子化
  - Well-defined な Path-integral measure
  - 摂動論によらない regularization (cf. dimensional reg.)
  - Transfer-matrix $\Rightarrow$ 量子系の Hamiltonian
- 統計力学系 (スピン系) との対応
  - さまざまな (非摂動的) 解析方法
    Weak, Strong coupling expansion, RG, MC simulation, $\cdots$
  - 繰り込みの非摂動的な意味づけ

格子ゲージ理論



- Link variable: ベクトル場は Link $(x, x+\hat{\mu})$ 上に定義

$$U(x,\mu) = \mathrm{e}^{iB_\mu(x)} \ \in G \quad (G = U(1), SU(N), \cdots)$$
$$B_\mu(x) = ag\ T^a A^a_\mu(x)$$

  – ゲージ変換

$$U(x,\mu) \longrightarrow g(x)\,U(x,\mu)\,g(x+\hat{\mu})^{-1}, \qquad g(x) \in G$$

  – Field strength

$$P(x,\mu,\nu) = U(x,\mu)U(x+\hat{\mu},\nu)U(x+\hat{\nu},\mu)^{-1}U(x,\nu)^{-1} \simeq \exp[ia^2 gF_{\mu\nu}(x) + \cdots]$$

- Action:

$$S[U] = \beta \sum_{x,\mu,\nu} \frac{1}{2N} \mathrm{ReTr}(1 - P(x,\mu,\nu)) \qquad (\beta = 2N/g^2)$$

- Path-Integral measure: G 上の不変測度 $\quad d\mu(U) = d\mu(g_1 U) = d\mu(U g_2)$

$$Z = \int \prod_{x,\mu} dU(x,\mu)\ \exp\left(-S[U]\right)$$

  – SU(2) : $U = a_0 \mathbb{I} + i \sum_{k=1}^3 a_k \sigma_k$

$$dU = \frac{1}{2\pi^2}\, d^4 a\, \delta(a^2 - 1) = \frac{1}{2\pi^2}\ \sin^2\psi \sin\theta d\psi d\theta d\phi$$

$$\begin{aligned} a_0 &= \cos\psi \\ a_1 &= \sin\psi\cos\theta \\ a_2 &= \sin\psi\sin\theta\cos\phi \\ a_3 &= \sin\psi\sin\theta\sin\phi \end{aligned}$$