AdS/CFT Correspondence and Greybody Factors in BTZ Black Holes[1]

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Abstract

It has been recently realized that 4D and 5D black hole solutions in string theories may be related through U-duality to 3D BTZ black holes. This suggests that various physical quantities may be understood in terms of BTZ solution in a unified manner and there may be some basic principle that governs the black hole physics. As an example, we discuss greybody factors for 3-dimensional BTZ and higher-dimensional black holes.

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[1] Talk given at the workshop Komaba '99.

1 Introduction

Understanding basic problems in quantum gravity is one of the most important and challenging subjects in current particle physics. Among others, we are faced with the difficult task of handling the strong coupling dynamics in the problems like

- How to find the true ground state among various topologies.
- How to understand the whole process of Hawking radiation from the black holes.

On the other hand, string theories are considered to provide a controllable theory of quantum gravity. Many works on the Hawking radiation and absorption cross sections have been done in 4D and 5D black holes by various techniques, including D-branes, semi-classical approximations and effective string models.

Here we would like to analyze these problems in the context of string theories. The black hole solutions in string theories are given by what are called the brane solutions. The fundamental branes are membranes, NS5-branes and D-branes, and other BPS solutions can be obtained as "intersecting" ones of these elementary solutions.

It has been known [1] that 4D and 5D black holes can be related through U-duality to BTZ black holes [2], and many of the important properties such as entropies and absorption cross sections can be understood using this relation. In this sense, **BTZ black holes contain essential informations of higher-dimensional black holes.**

What I would like to emphasize here is that this means that various physics could be understood through BTZ black holes in a unified manner, without examining various solutions separately.

On the other hand, it has recently realized that the large N limit or the near-horizon limit of the N brane system in d dimensions is equivalent to classical supergravity on $(d+1)$-dimensional AdS $\times M$. This is what is called **AdS/CFT correspondence** [3]. An example is the equivalence of $\mathcal{N}=4$ $SU(N)$ SYM in 4D with coupling g_{YM} to the type IIB superstring on $\mathrm{AdS}_5 \times S^5$ with $g_{st} \propto g_{YM}^2$. In the limit $N \to \infty$ with $g_{YM}^2 N$ fixed, weakly coupled superstring becomes relevant, which can be well described by low-energy effective theories – supergravities.

The basic idea in AdS/CFT correspondence which is important for us here is the following: In AdS background, the bulk field ϕ is completely determined by its field equation and given boundary values $\phi(\Omega)$ (modulo some subtlety in Lorentzian signature). This means that most of the on-shell (physical) informations can be obtained from the boundary theory. More explicitly, this relation can be used to evaluate correlation functions for the local operators on the boundary which couple to the boundary value of the fields.

The relevant relation is [4, 5]

$$e^{iS_{eff}(\phi_i)} = \langle T e^{i\int_B \phi_{b,i}\mathcal{O}^i} \rangle, \tag{1}$$

where $\mathcal{O}$ is an operator coupling to the field on the boundary.

Below we will review how these ideas are realized and then combined to give greybody factors for BTZ black holes (and hence those for higher-dimensional black holes) by using the AdS/CFT correspondence.

2 4D and 5D Black Holes from BTZ

The basic tool we will use is the following **shift transformation** [1]. Consider the wave solution in $(D+1)$ dimensions

$$\begin{aligned} ds^2 &= -K^{-1} f dt^2 + K \left[dy_1 + \left(K'^{-1} - 1 + \tanh\alpha \right) dt \right]^2 \\ &\qquad + f^{-1} dr^2 + r^2 d\Omega_{D-2}^2, \end{aligned} \tag{2}$$

with

$$\begin{aligned} K &= 1 + \frac{\mu^{D-3}\sinh^2\alpha}{r^{D-3}}, \\ K'^{-1} &= 1 - \frac{\mu^{D-3}\sinh\alpha\cosh\alpha}{r^{D-3}} K^{-1}, \\ f &= 1 - \frac{\mu^{D-3}}{r^{D-3}}, \qquad\qquad r^2 = x_1^2 + \cdots + x_{D-1}^2. \end{aligned} \tag{3}$$

We then apply an $SL(2,\mathbf{R})$ transformation

$$\begin{pmatrix} t \\ y_1 \end{pmatrix} = \begin{pmatrix} \cosh\alpha & -e^{-\alpha} \\ 0 & 1/\cosh\alpha \end{pmatrix} \begin{pmatrix} t' \\ y_1' \end{pmatrix}. \tag{4}$$

Then the solution is transformed into

$$ds^2 = -\tilde{K}^{-1} f dt^2 + \tilde{K} \left[dy_1 + \left(\tilde{K}^{-1} - 1\right) dt\right]^2 + f^{-1} dr^2 + r^2 d\Omega_{D-2}^2, \tag{5}$$

with

$$\tilde{K} = \frac{\mu^{D-3}}{r^{D-3}}. \tag{6}$$

It should be noted here that the asymptotic behavior is changed from flat to AdS $\times M$!!!

This shift transformation can be applied to wave solutions and can be combined with U-duality transformation to relate 4D and 5D black holes to BTZ solution.

As an example, take a 5D black hole and make the following transformations:

$$\begin{array}{ccccc}
\mathrm{F1} & \oplus & \mathrm{NS5} & \oplus & 0_w \\
(1) & & (12345) & & (1) \\
 & \mathrm{T}_1 & \downarrow & & \\
0_w & \oplus & \mathrm{NS5} & \oplus & \mathrm{F}_1 \\
(1) & & (12345) & & (1) \\
 & \mathrm{S} & \downarrow & & \\
0_w & \oplus & \mathrm{D5} & \oplus & \mathrm{D1} \\
(1) & & (12345) & & (1) \\
 & & & & \\
 & \text{Shift} & H_f \to \frac{\mu^2}{r^2} & & \\
 & & & & \\
 & \mathrm{T}_{1234} & \downarrow & & \\
\mathrm{F1} & \oplus & \mathrm{D1} & \oplus & \mathrm{D3} \\
(1) & & (5) & & (234) \\
 & \mathrm{S} & \downarrow & & \\
\mathrm{D1} & \oplus & \mathrm{F1} & \oplus & \mathrm{D3} \\
(1) & & (5) & & (234) \\
 & \mathrm{T}_5 & \downarrow & & \\
\mathrm{D2} & \oplus & 0_w & \oplus & \mathrm{D4} \\
(15) & & (5) & & (2345) \\
 & & & & \\
 & \text{Shift} & H_s \to \frac{\mu^2}{r^2} & &
\end{array}$$

Here the numbers in the brackets indicate the worldvolume of the brane solutions (except for trivial time direction 0). After this sequence of dualities, one finds

$$\begin{aligned} ds_{10}^2 &= -H_f^{-1}\left[-K^{-1}f dt^2 + K\left\{dy_1 - \left(K'^{-1}-1\right)dt\right\}^2\right] \\ &\quad + dy_1^2 + \cdots + dy_4^2 + H_s\left[f^{-1}dr^2 + r^2 d\Omega_3^2\right], \end{aligned} \tag{7}$$

with

$$\begin{aligned} e^{-2\phi} &= 1, \qquad B_{01} = \frac{1}{H_f} - 1, \\ H_{ijk} &= \frac{1}{2}\epsilon_{ijkl}\partial_l H_s, \quad i,j,k,l = x_1,\cdots,x_4, \\ H_f &= \frac{\mu^2}{r^2}, \qquad K = 1 + \frac{Q_K}{r^2}, \\ H_s &= \frac{\mu^2}{r^2}, \qquad f = 1 - \frac{\mu^2}{r^2}. \end{aligned} \tag{8}$$

This can be rewritten as

$$\begin{aligned} ds_{10}^2 &= -\frac{(\rho^2-\rho_-^2)(\rho^2-\rho_+^2)}{l^2\rho^2}dt^2 + \rho^2\left(d\varphi - \frac{\rho_+\rho_-}{l\rho^2}dt\right)^2 \\ &\quad + \frac{l^2\rho^2}{(\rho^2-\rho_+^2)(\rho^2-\rho_-^2)}d\rho^2 + l^2 d\Omega_3^2 + dy_1^2 + \cdots + dy_4^2. \end{aligned} \tag{9}$$

The first part is precisely the metric for BTZ black hole!!!

Thus entropies of 5D and 4D black holes can be related to those of BTZ black holes. Alternatively, instead of using the above transformations, one could use the large N approximation to justify the use of AdS $\times M$ (This is true for all physically interesting cases).

We note that **not only the entropies of BH but also absorption cross sections or greybody factors can be examined by going to AdS$_3$ and using the AdS/CFT correspondence.** This is the main subject we are going to discuss now.

3 AdS/CFT Correspondence and Greybody Factors for scalars

The purpose of this section is to evaluate greybody factors for BTZ black holes through two-point correlation functions by AdS/CFT correspondence, and relate them to those of

5D and 4D black holes [6]. This provides a check of AdS/CFT correspondence and some other issues.

Usually the greybody factors are considered for asymptotically flat solutions because only for this case we can have well-defined notion of particle modes in the asymptotic region. Since the BTZ black hole is not in this class, we should clarify the logic of our procedure, which may be summarized as follows.

The greybody factors for higher-dimensional, asymptotically-flat black holes can be related to two-point correlation functions. After these solutions are related to BTZ black holes, these correlation functions can in turn be computed by using the AdS/CFT correspondence and can be related to the greybody factors we want. This can be efficiently done if we derive **the greybody factors defined as the discontinuity of the correlation functions for the BTZ case**, which are the objects we are trying to calculate.

We now turn to the calculation. The two-point functions are usually calculated for AdS_3 metric in the Poincaré coordinate:

$$ds^2 = \frac{l^2}{y^2}(dy^2 + dw_+ dw_-). \tag{10}$$

In the following, the radius of AdS_3 space is set to $l = 1$.

The euclidean action of massive scalar field with mass m is given by

$$S(\phi) = \frac{1}{2}\int dy dw_+ dw_- \sqrt{g}\left(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + m^2\phi^2\right), \tag{11}$$

which has a solution with the behavior $\phi(y, w_+, w_-) \to y^{2h_-}\phi_0(w_+, w_-)$ for $y \to 0$ (boundary). This means that the dimension of the boundary value $\phi_0(w_+, w_-)$ is $2h_-$ which is defined by

$$h_\pm = \frac{1}{2}(1 \pm \sqrt{1 + m^2}). \tag{12}$$

The normalized bulk-to-boundary Green function in Poincaré coordinates is given by

$$K_P(y, w_+, w_-; w'_+, w'_-) = \frac{\Gamma(2h_+)}{\pi\Gamma(2h_+ - 1)}\left[\frac{y}{y^2 + (w_+ - w'_+)(w_- - w'_-)}\right]^{2h_+}. \tag{13}$$

The solution $\phi(y, w_+, w_-)$ to the field equation is expressed as

$$\phi(y, w_+, w_-) = \int dw'_+ dw'_- K_P(y, w_+, w_-; w'_+, w'_-)\phi_0(w'_+, w'_-), \tag{14}$$

where $\phi_0(w_+, w_-)$ is the boundary value of the bulk field $\phi(y, w_+, w_-)$.

On the other hand, the BTZ black holes have the metric

$$ds^2 = -\frac{(r^2 - r_+^2)(r^2 - r_-^2)}{r^2}dt^2 + \frac{r^2}{(r^2 - r_+^2)(r^2 - r_-^2)}dr^2 + r^2\left(d\phi - \frac{r_+ r_-}{r^2}dt\right)^2, \qquad (15)$$

with periodic identification $\phi \sim \phi + 2\pi$.

We define

$$\begin{aligned} w_\pm &= \left(\frac{r^2 - r_+^2}{r^2 - r_-^2}\right)^{\frac{1}{2}} e^{2\pi T_\pm u_\pm} \approx e^{2\pi T_\pm u_\pm}, \\ y &= \left(\frac{r_+^2 - r_-^2}{r^2 - r_-^2}\right)^{\frac{1}{2}} e^{\pi(T_+ u_+ + T_- u_-)} \approx \left(\frac{r_+^2 - r_-^2}{r^2}\right)^{\frac{1}{2}} e^{\pi(T_+ u_+ + T_- u_-)}, \end{aligned} \qquad (16)$$

($\approx$ for $r \gg r_\pm$) with

$$T_\pm = \frac{r_+ \mp r_-}{2\pi}, \quad u_\pm = \phi \pm t. \qquad (17)$$

It is easy to see that by this transformation, the metric for BTZ black hole is transformed into that of AdS_3 in the Poincaré coordinate.

We can now use **AdS_3/CFT correspondence to derive the relation between string theory in the bulk and field theory on the boundary:**

$$S_{eff} = \lim_{r\to\infty} \frac{1}{2}\int du_+ du_- \sqrt{g} g^{rr} \phi \partial_r \phi. \qquad (18)$$

getting

$$\begin{aligned} S_{eff} &= -\frac{h_+(2h_+ - 1)}{\pi}(r_+^2 - r_-^2)(2\pi T_+)^{-1}(2\pi T_-)^{-1} \\ &\times \int du_+ du_- du'_+ du'_- \phi_0(u_+, u_-)\left(\frac{\pi T_+}{\sinh \pi T_+(u_+ - u'_+)}\right)^{2h_+} \\ &\times \left(\frac{\pi T_-}{\sinh \pi T_-(u_- - u'_-)}\right)^{2h_+} \phi_0(u'_+, u'_-). \end{aligned} \qquad (19)$$

According to (1), one then obtains two-point correlation function

$$\begin{aligned} G(t, \phi) &= \langle \mathcal{O}(u_+, u_-)\mathcal{O}(0, 0)\rangle \\ &= \frac{2h_+(2h_+ - 1)}{\pi}\left(\frac{\pi T_+}{\sinh \pi T_+ u_+}\right)^{2h_+}\left(\frac{\pi T_-}{\sinh \pi T_- u_-}\right)^{2h_+}, \end{aligned} \qquad (20)$$

The greybody factors in BTZ black holes is defined by the discontinuity of the two-point correlation function:

$$\begin{aligned}\sigma_{abs} &= \frac{\pi}{\omega}\int dt \int_{-\infty}^{\infty} d\phi e^{ip\cdot x}[G(t-i\epsilon,\phi)-G(t+i\epsilon,\phi)] \\ &= \frac{2h_+(2h_+-1)(2\pi T_+ l)^{2h_+-1}(2\pi T_- l)^{2h_+-1}\sinh\left(\frac{\omega}{2T_H}\right)}{\omega\Gamma^2(2h_+)} \\ &\quad\times\left|\Gamma\left(h_+ + i\frac{\omega}{4\pi T_+}\right)\Gamma\left(h_+ + i\frac{\omega}{4\pi T_-}\right)\right|^2, \end{aligned} \tag{21}$$

where

$$\frac{2}{T_H} = \frac{1}{T_+} + \frac{1}{T_-}. \tag{22}$$

The decay rate for massless scalar field is now given by the product with thermal distribution:

$$\begin{aligned}\Gamma &= \frac{\sigma_{abs}(h_+ = 1)}{e^{\omega/T_H}-1} \\ &= \frac{\omega\pi^2 l^2}{(e^{\omega/2T_+}-1)(e^{\omega/2T_-}-1)}, \end{aligned} \tag{23}$$

which is consistent with semiclassical gravity calculations.

Let us compare the two-point correlation functions in Poincaré and BTZ coordinates:

$$\begin{aligned}\langle\mathcal{O}(w_+,w_-)\mathcal{O}(w'_+,w'_-)\rangle &\sim \frac{1}{(w_+-w'_+)^{2h_+}(w_--w'_-)^{2h_+}}, \\ \langle\mathcal{O}(u_+,u_-)\mathcal{O}(u'_+,u'_-)\rangle &\sim \frac{2h_+(2h_+-1)}{\pi}\left[\frac{\pi T_+}{\sinh\pi T_+(u_+-u'_+)}\right]^{2h_+} \\ &\qquad\times\left[\frac{\pi T_-}{\sinh\pi T_-(u_--u'_-)}\right]^{2h_+} \end{aligned} \tag{24}$$

We see that the Poincaré coordinates $w_\pm$ are related to BTZ coordinates $u_\pm$ by an exponential transformation in the region $r \gg r_\pm$. This means that the initial state of BTZ black holes can be described in 2D CFT by Poincaré vacuum state. The nonlinear coordinate transformation introduces a mapping of the original operator $\mathcal{O}(w_+,w_-)$ in Poincaré coordinates to the new one $\mathcal{O}(u_+,u_-)$ in BTZ coordinates, and this induces the Bogoliubov transformation on the operators. The operator $\mathcal{O}(u_+,u_-)$ sees Poincaré vacuum state as an excited mixed state.

We now relate the above result to the greybody factors in 5D and 4D black holes. Consider the near-horizon AdS_3 structure of 5D black holes ("boosted" D1/D5 configuration) in the large N limit:

$$\begin{aligned} ds^2 &= \frac{r^2}{R^2}(-dt^2+dx^2)+\frac{r_0^2}{R^2}(\cosh\sigma dt+\sinh\sigma dx)^2+\frac{R^2}{r^2-r_0^2}dr^2 \\ &\quad +R^2 d\Omega_3^2+\frac{r_1^2}{R^2}\sum_{i=1}^{4}dx_i^2, \end{aligned} \tag{25}$$

where r_0 is the extremality parameter, $r_1, r_5, r_0 \sinh\sigma$ are charges of D1-brane (Q_1), D5-brane (Q_5) and momentum in 5D black holes, respectively, and $R^2 = r_1 r_5$.

Using the connection discussed in sect. 2, **the two-point correlation functions for 5D BH** are then derived as

$$\langle \mathcal{O}(w_+, w_-)\mathcal{O}(w'_+, w'_-)\rangle = \eta_{5D}\frac{2h_+(2h_+-1)}{\pi}\frac{1}{(w_+-w'_+)^{2h_+}(w_--w'_-)^{2h_+}}, \tag{26}$$

where η_{5D} is a constant. Hence

$$\sigma_{abs}^{5D} = \eta_{5D}\sigma_{abs}^{BTZ} \tag{27}$$

We thus see that **the greybody factors in 5D and 4D black holes have their origin in BTZ black holes, and the dynamical informations of 5D and 4D black holes are encoded in BTZ black holes.**

4 Results for Fermions

A similar analysis can be repeated for spin $\frac{1}{2}$ fermions [7].

Most of the calculations of two-point correlation functions have been performed in the Poincaré coordinates, but to describe fermion emission from the BTZ black holes in the spirit of AdS/CFT correspondence, we need to calculate these in the BTZ coordinates. These coordinates are related by eq. (16).

A New feature here is that the bulk-boundary Green function in the BTZ coordinates (u_+, u_-) not only depends on Δu_+ and Δu_-, but also on $(u_+, u_-; u'_+, u'_-)$. The reason is that when we construct bulk-boundary Green functions in the Poincaré coordinates, we apply a local Lorentz transformation for spinor fields in deriving them, and this local

Lorentz transformation in the bulk breaks the manifest invariance under the translations in the boundary BTZ coordinates.

The free spinor action on AdS_3 is given by the sum of two terms

$$S_0 = \frac{1}{2}\int dy dw_+ dw_- \sqrt{g}\bar{\Psi}(\not{D} - m)\Psi. \tag{28}$$

$$S_1 = \lim_{y\to 0}\frac{1}{4}\int dw_+ dw_- \sqrt{g_0}\bar{\Psi}\Psi, \tag{29}$$

where g_0 is the determinant of the induced metric y^{-4}.

We denote the bulk fields by $\Psi(y, w_+, w_-)$ and $\bar{\Psi}(y, w_+, w_-)$ and the boundary values by $\psi(w_+, w_-)$ and $\bar{\psi}(w_+, w_-)$. We find

$$\begin{aligned}
\Psi(y, w_+, w_-) &= \frac{m+\frac{1}{2}}{2\pi}\int dw'_+ dw'_- \left[y\Gamma^y + (w_+ - w'_+)\Gamma^- + (w_- - w'_-)\Gamma^+\right] \\
&\times \ [y^2 + (w_+ - w'_+)(w_- - w'_-)]^{-3/2+m\Gamma^y} y^{1-m\Gamma^y}\psi(w'_+, w'_-), \\
\bar{\Psi}(y, w_+, w_-) &= \frac{m+\frac{1}{2}}{2\pi}\int dw'_+ dw'_- \bar{\psi}(w'_+, w'_-) y^{1+m\Gamma^y} \\
&\times \ [y^2 + (w_+ - w'_+)(w_- - w'_-)]^{-3/2-m\Gamma^y} \\
&\times \ \left[y\Gamma^y + (w_+ - w'_+)\Gamma^- + (w_- - w'_-)\Gamma^+\right].
\end{aligned} \tag{30}$$

The coupling between the operators and the boundary values of spinor fields is found to be

$$S(\bar{\psi}, \psi) = \frac{1}{2}\int dw_+ dw_- (\bar{\mathcal{O}}\psi + \bar{\psi}\mathcal{O})(w_+, w_-). \tag{31}$$

Using eq. (1), we thus find the two-point correlation function for spinor fields in the Poincaré coordinates:

$$\langle \mathcal{O}^\dagger(w_+, w_-)\mathcal{O}(w'_+, w'_-)\rangle = \frac{m+\frac{1}{2}}{\pi}\frac{1}{(w_+ - w'_+)^{2h_+}(w_- - w'_-)^{2h_-}}, \tag{32}$$

with

$$h_+ = h - \frac{1}{4}, \quad h_- = h + \frac{1}{4}, \quad h = \frac{m+1}{2}, \quad h_- - h_+ = \frac{1}{2}, \tag{33}$$

Repeating the calculations similar to those for scalars, one finds

$$\begin{aligned}
G(t, \phi) &= \langle \mathcal{O}^\dagger(u_+, u_-)\mathcal{O}(0,0)\rangle \\
&= \frac{h_+}{2\pi}\left(\frac{\pi T_+}{\sinh \pi T_+ u_+}\right)^{2h_+}\left(\frac{\pi T_-}{\sinh \pi T_- u_-}\right)^{2h_-},
\end{aligned} \tag{34}$$

In the derivation, there appear translationally non-invariant terms, but they disappear in the final result.

Again the operators $\mathcal{O}^{\dagger}(u_+, u_-)$ and $\mathcal{O}(u_+, u_-)$ see the Poincaré vacuum state as an excited mixed state.

We find **the greybody factors for spinor fields in the BTZ black holes**

$$\begin{aligned}
\sigma_{abs} &= \frac{2\pi}{\mathcal{F}} \int dt \int_{-\infty}^{\infty} d\phi e^{ip\cdot x} [G(t - i\epsilon, \phi) - G(t + i\epsilon, \phi)] \\
&= \frac{h_+}{\mathcal{F}} \frac{(2\pi T_+)^{2h_+ - 1} (2\pi T_-)^{2h_- - 1}}{\Gamma(2h_+)\Gamma(2h_-)} \cosh\left(\frac{\omega}{2T_H}\right) \\
&\quad \times \left| \Gamma\left(h_+ + i\frac{\omega}{4\pi T_+}\right) \Gamma\left(h_- + i\frac{\omega}{4\pi T_-}\right) \right|^2 ,
\end{aligned} \tag{35}$$

where $\mathcal{F}$ is the incident fermion flux.

For $(h_+, h_-) = (\frac{1}{2}, 1)$, $\mathcal{F} = 1$:

$$\sigma_{abs}^{(\frac{1}{2},1)} = \frac{\pi^2 \omega}{4} \frac{\cosh\left(\frac{\omega}{2T_H}\right)}{\cosh\left(\frac{\omega}{4T_+}\right) \sinh\left(\frac{\omega}{4T_-}\right)}. \tag{36}$$

The emission rate of the fermions is given by the product with thermal distribution:

$$\Gamma^{(\frac{1}{2},1)} = \frac{\pi^2 \omega}{2} \frac{d^2 k}{\left[\exp\left(\frac{\omega}{2T_+}\right) + 1\right] \left[\exp\left(\frac{\omega}{2T_-}\right) - 1\right]}, \tag{37}$$

in agreement with the semiclassical gravity calculations.

5 Outlook

Summary

1. We have discussed the relation of various physical quantities associated with 4D and 5D black holes to BTZ black holes.

2. We have also evaluated greybody factors for BTZ black holes by use of two-point correlation functions, which enable us to obtain those for higher-dimensional holes.

3. Through the analysis, we have confirmed the AdS/CFT correspondence.

Future problems

1. It is interesting to extend the analysis to other AdS cases. In particular, AdS_2 looks important in connection with D0-branes.

2. Our investigation suggests fundamental relations among all these black hole solutions and it is extremely interesting and important to use them to pin down the underlying mechanism of entropies and other properties of black holes.

3. It is also interesting to do quantum analysis of the classical solutions, in particular problems associated with black holes.

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References

[1] K. Sfetsos and K. Skenderis, Nucl. Phys. **B517** (1998) 179.

[2] M. Bañados, C. Teitelboim and J. Zanelli, Phys. Rev. Lett. **69** (1992) 1849, hep-th/9204099.

[3] J. Maldacena, Adv. Theor. Math. Phys. **2** (1998) 231, hep-th/9711200.

[4] S. Gubser, I. Klebanov and A. Polyakov, Phys. Lett. **B428** (1998) 105, hep-th/9802109.

[5] E. Witten, Adv. Theor. Math. Phys. **2** (1998) 253, hep-th/9802150.

[6] H.J.W. Müller-Kirsten, N. Ohta and J.-G. Zhou, Phys. Lett. **B445** (1999) 287, hep-th/9809193.

[7] N. Ohta and J.-G. Zhou, J. High Energy Phys. **12** (1998) 023, hep-th/9811057.