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GENERALIZED SYMMETRIES OF THE ADS/CFT CORRESPONDENCE

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1 Introduction

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A year ago Maldacena[1] introduced a far-reaching correspondence involving SUSY Yang-Mills theory in d dimension and 10 (or 11) dimensional supergravity compactified to $d+1$ dimensions. Since its appearance this has been a topic of major interest. Many studies have been performed verifying the correspondence in various examples. Almost all the studies are limited to the boundary of the AdS space where the comparison of observables can be most easily performed[2, 3].

In ([4][5][6]) , we have adressed the question of an underlying symmetry principle behind this correspondence. In the most clear case of $d=4$ dimensional Yang-Mills theory the symmetry in question is given by the well known group of conformal transformations. In general for $d \neq 4$ the standard conformal group is not operational , the underlying symmetry is not obvious and one could have wondered about the general validity of the conjecture. In this case we have introduced a notion of a generalized conformal symmetry (GCS) . We then proceed to shown that this generalized symmetry holds on both sides of the correspondence for arbitrary d and arbitrary system of p-branes.

Among the intriguing features of the correspondence is the appearance in supergravity of an extra spatial dimension representing the radial direction of the generalized AdS space. Of major interest is the origin and nature of this bulk coordinate. The dependence on the extra coordinate on the gravity side is nontrivial. Consequently the (GCS) or any other symmetry is realized differently on the two sides of the correspondence. Notably in supergravity the extra radial coordinate appears nonlieарly. The extra spatial dimension corresponds to an expectation valuue associated with a Higgs field in Y-M theory. We show how the transmutation between the two descriptions happens and iluminate the origin of nonlinearities induced in the gravitational picture.

2 Yang-Mills$_4$

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The most clear case of Maldacena's conjecture is that of a 4-dimensional N=4 Yang-Mills theory [4]. Consider the corresponding action:

$$S_{Y-N} = \int d^4x \left\{ -\frac{1}{4g^2} F_{\mu\nu}F^{\mu\nu} + \frac{1}{4g^2}\left[X_m, X_n\right]^2 - \frac{1}{2g^2}\, D_\mu X \cdot D^\mu X + \text{fermions} \right\} \tag{1}$$

with $X_m \quad m = 1, 2, \cdots 6$ representing the scalar Higgs fields. Associated with this is a global $S0(6)$ R-symmetry . The space-time symmetry of 4d Yang-Mills theory is given by the well known conformal group

$$\begin{aligned} \delta^D X_m &= -\left(1 + x\cdot\partial\right) X_m \\ \delta^K X_m &= -2\epsilon \cdot x\, X_m - \delta_\epsilon x \cdot \partial X_m \end{aligned} \tag{2}$$

with

$$\delta_\epsilon x^\mu = 2\epsilon \cdot x\, x^\mu - \epsilon^\mu x^2 \tag{3}$$

Consider now the 5d (Super) gravity. It is defined by compactifying type Π_B SUGRA on $AdS_5 \times S_5$:

$$(ds)^2 = \alpha` \left\{ \frac{U^2}{R^2}\, d^2x_{(4)} + \frac{R^2}{U^2} + R^2 d^2\Omega_5 \right\} \tag{4}$$

with the radius of compatification specified by Maldacena:

$$R^2 = \left(2\, g^2_{YM} N\right)^{1/2} \tag{5}$$

The isommetry group of the S_5 sphere is SO(6) and is accounted for by the global R-symmetry of YM. The isometry of the Ads_5 space is the space-time group $S0(2,6)$ which will be our main concern . It is most easily exibited through an embedding in 6 dimension

$$-y^2_{-1} - y^2_0 + y^2_1 + y^2_2 + y^2_3 + y^2_4 = -R^2 \tag{6}$$

In terms of an independent set of coordinates (x_μ, U) one finds the AdS_5 metric and the form of the corresponding conformal transformations that read

$$\begin{aligned} \bar\delta x_\mu &= 2\epsilon \cdot x\, x^\mu - \epsilon_\mu\, x^2 - \epsilon_\mu\, \frac{R^4}{U^2} \\ \bar\delta U &= 2\epsilon \cdot x\, U \end{aligned} \tag{7}$$

Comparing with the conformal transformation of the 4d Yang-Mills theory, we see that there is agreement in the limit $U \to \infty$ when $\bar\delta x_\mu \to \delta_\epsilon x_\mu$. This is the well known agreement on the boundary of the AdS space.

We can ask the question, however, if the Yang-Mills theory provides the physics in the bulk and whether we can account for the full symmetry of the AdS space. The dependence on the radial coordinate U describing the interior of the AdS space is nonlinear and the question is if and how is this nonlinear dependence to be seen from Y-M theory. A positive answer to this was given in [4]: Through quantum Y-M effects the extra term was shown to appear in the correct nonlinear form. The effect provides a metamorphosis of a linear Yang-Mills symmetry transformation into a nonlinear gravitational symmetry.

A short description of the mechanism involved is as follows: The extra coordinate U appearing in supergravity is associated with the radius of the Higgs field

$$U \approx (X \cdot X)^{1/2} \,. \tag{8}$$

A transmutation occurs where this becomes a spatial coordinate on the gravity side. ˇo exibit this considers a background given by $B_m = (0, 0, \cdots 0, r_m)$ on the diagonal of the Higgs field. The dependence on r is then studied using the background field method, we write $X_m = B_m + Y_m$ with $withY_m$ representing fluctuations. The evaluation of quantum fluctuations is performed in the loop expansion which requires a gauge fixing in the form of a background gauge condition

$$\chi = \partial \cdot A - [B_m, Y_m] = 0 \tag{9}$$

It is through quantum effects that one finds a modifiation of the classical conformal symmetry due to the background. The origin of the modification comes from the fact that the (background) gauge condition is not invariant under the classical conformal symmetry. Its change under conformal transformation is given by

$$\delta\chi = \delta_\epsilon x \cdot \partial\, \chi - 2\epsilon \cdot x\, \chi + 4\epsilon \cdot A \tag{10}$$

and the next fact to notice is that this change can be compensated through an appropritely chosen gauge transformation. This restores our conformal symmetry but with an extra contribution appearing in the symmetry transformations

$$\tilde{\delta} X = \delta_\epsilon X + i[\Lambda, X] \tag{11}$$

The additional term and the gauge parameter Λ introduce explicite background dependence. Evaluating the expectation value of the second term results in

$$\langle \left[\Delta_B^{-1} \epsilon \cdot A, Y \right] \rangle \to \frac{g_{YM}^2 N}{r^2}\, \epsilon \cdot \partial B \tag{12}$$

leding to the quantum improved conformal transformation

$$\bar{\delta} B = -2\epsilon \cdot x B - \left(\delta_\epsilon x \cdot \partial + \frac{g_{YM}^2 N}{B^2}\, \epsilon \cdot \partial \right) B \tag{13}$$

With $\sqrt{B^2} \propto U$ this is seen to exactly reproduce the additional U-dependent term of the gravitational AdS isommetry.

The derivation of the full nonlinear symmetry transformation of the AdS_5 space involving the nontrivial dependence on the extra radial variable u is essentially equivalent to deriving the AdS metric itself (we have a case where the isometries fully specify the metric). We mention, however, that the extra nonlinear term was seen through a 1-loop effect in SUSY YM theory. The non-renormalization of this effect requires that no further contributions of this form appear in higher loops.

3 Generalized Symmetry of p-Branes

Yang-Mills theory in $d = p+1$ dimensions[5], representing a system of p-branes, does not have the conformal symmetry of the $d = 4$ dimensional theory. The obvious reason lies in the fact that the coupling constant in dimesion other than 4 is not dimensionless. Since the coupling constant carries explicite dimension it itself has to transform nontrivialy. This is what was used in [5,6] to introduce what we call a generalized conformal symmetry(GCS).

The dilatation and conformal transformations are now given by the transformations for X_m and A_μ supplemented by additional transformations[5].

$$\begin{aligned} \delta^D g^2 &= -\epsilon(3-p+x\cdot\partial)g^2 \\ \delta^K g^2 &= -2(3-p)\epsilon\cdot x g^2 - \delta_\epsilon x\cdot\partial g^2 \end{aligned} \tag{14}$$

This form of a transformation implies that we ought to consider the coupling constant g to be x dependent, in essence it is promoted to a dynamical variable and appears in the the action under the integral

$$S_{YM}\left(g(x), A_\mu, X_m\right) = \int d^{p+1}x \,\frac{1}{g^2(x)}\mathcal{L}(A,X) \tag{15}$$

The interpretation and clear meaning of this is given in an effective action framework which will play a central role in our computations.

Before describing the Yang-Mills calculations, consider the corresponding SUGRA theory. It is defined in $D = d+1$ dimension, with a background given by the near horizon metric of N coinciding p-branes which can be written in the form

$$(ds)^2_p = \frac{U^2}{\sqrt{\rho_p}}\,dx^2 + \frac{\sqrt{\rho_p}}{U^2}\,dU^2 + \sqrt{\rho_p}\,d\Omega^2_{8-p} \tag{16}$$

with

$$\rho_p = \frac{Q_p}{U^{3-p}} \quad , \quad Q_p = N\,g^2_{YM}d_p \tag{17}$$

Several features of this p-brane system are worth mentioning. The effective radius of this "AdS_{p+2}" metric is U-dependent. In addition, one has a nontrivial dilaton (and antisymmetric tensor) which are also u-dependent. The explicite coordinate dependence of the dilaton field is responsible for breaking of conformal symmetry and these theories were termed non-conformal.

Nevertheless, we observe that the system obeys a generalized conformal symmetry(GCS)

$$\begin{aligned}
\delta U &= -2\epsilon \cdot x U \\
\delta x_\mu &= 2\epsilon \cdot x\, x_\mu - \epsilon_\mu x^2 - \epsilon_\mu k \frac{Q_p}{U^{5-p}} \\
\delta Q_p &= -2(3-p)\epsilon \cdot x Q_p
\end{aligned} \tag{18}$$

Here the constant Q_p is seen to transform nontrivially under the GCS in agreement with $Q_p \propto g_{YM}^2 N$. One also has that the extra coordinate transforms with opposite conformal weight to the other coordinates x_μ. The correspondence with the GCS transformations of Y-M theory is seen to be exact except for the additional non-linear term dependent on U. This curvature term as we have seen in the $d=4$ case is to come from a quantum effects of Y-M theory. These we summarize in what follows.

The most complete calculation of this effect and interpretation of the dynamical coupling is in terms of effective action presented in [3]. Considering $g_{YM} = g(x)$ one defines through a path integral in a standard way the effective action

$$\Gamma\left(g(x), B_m(x), \cdots\right) \tag{19}$$

with B_m denoting a background and $g(x)$ having a meaning of a source in the Y-M action. For computing the effective action in a loop expansion one adds gauge fixing and ghost terms

$$\begin{aligned}
S_{gf} &= -\frac{1}{2}\int \frac{d^d x}{g^2} \mathrm{Tr}\, G^2 \,, & (20)\\
G &= -\partial_\mu A^\mu + i\left[B_m, Y_m\right] \,, & (21)\\
S_{gh} &= i\int d^d x \mathrm{Tr}\, \left(-\bar{C}\partial^\mu D_\mu C + \bar{C}\left[B_m, [X_m, C]\right]\right) . & (22)
\end{aligned}$$

The total action is invariant under the BRST symmetry transformations,

$$\begin{aligned}
\delta_B X_m &= -i\left[C, X_m\right]\lambda \,, \qquad \delta_B A_\mu = -\left[D_\mu, C\right]\lambda \,, & (23)\\
\delta_B C &= iC^2\lambda \,, \qquad \delta_B \bar{C} = \frac{i}{g^2} G\lambda \,. & (24)
\end{aligned}$$

Under the generalized conformal transformations the ghosts are regarded as scalar fields with dimension 0 and $d-2$ respectively. We find that, while the scale invariance is trivial, conformal invariance is not.The gauge fixing S_{gf} and the ghost term S_{gh} transform under under the special conformal transformations as :

$$\begin{aligned}
\delta_\epsilon S_{gf} &= 2(d-2)\int \frac{d^d x}{g^2} \mathrm{Tr}\, G A \cdot \epsilon \,, & (25)\\
\delta_\epsilon S_{gh} &= -2(d-2) i \int d^d x \mathrm{Tr}\, \bar{C}\epsilon^\mu D_\mu C \,. & (26)
\end{aligned}$$

A remarkable fact is that one can find a compensating field-dependent BRST transformation which removes these variations. The fermionic BRST parameter Λ which achieves a cancelation is found to be given by

$$\Lambda \;=\; -2(d-2)\int d^D y \mathrm{Tr}\left(\bar{C}(y)A(y)\cdot\epsilon\right), \tag{27}$$

Even though the BRST parameter is field dependent one still has the standard property of BRST transformations that they leave the classical action invariant. The field dependennce of the transformation leaves its mark on the functional integration measure which receives a a non-trivial contribution

$$\mathcal{D}(A+\Delta_\epsilon A)\mathcal{D}(\bar{C}+\Delta_\epsilon\bar{C}) \;=\; \mathcal{D}A\mathcal{D}\bar{C}\exp\left(i(-\delta_\epsilon S_{gf}-\delta_\epsilon S_{gh})\right), \tag{28}$$

This precisely cancels the troublesome conformal breaking terms found earlier .

To summarize, we have achieved symmetry through a combined effect of conformal and field dependent BRST transformations. This leads to a Ward identity

$$\int dx^d\left(\delta_\epsilon g^2\frac{\delta}{\delta g^2}+(\delta_\epsilon B+\delta_\Lambda B)\cdot\frac{\delta}{\delta B}\right)\Gamma(g,B)=0 \tag{29}$$

The compensating BRST transformation is now seen to be responsible for an extra effect and a modification of the YM GCS transformation. The evaluation of this effect can be done at the one loop level giving the total result

$$\bar{\delta}x_\mu=\delta_\epsilon x_\mu-\frac{p-1}{2}\,\epsilon_\mu k\frac{Q_p}{U^{5-p}} \tag{30}$$

The extra term is indeed found to be of the form needed for the generalized symmetry transformation in supergravity. The coefficient $\frac{p-1}{2}$ appearing in front of the Y-M symmetry transformation is hovewer different and to reconcile this with the value of the coefficient in SUGRA one observes that the B-I action on the gravity side does not have the $\partial X_\mu\cdot\partial Q_p$ coupling which can be seen to be present on the YM side. This term can be cancelled by a (space-time) reparametrization which once taken into account leads to an exact agreement with the effective action.

One can deduce the necessary reparametrization of supergravity without the explicite knowledge of the Yang-Mills result. We recall that the SCT for the worldvolume coordinate x^α in the supergravity solution takes the non-canonical form $\delta_\epsilon x^\alpha = 2\epsilon\cdot x x^\alpha-\epsilon^\alpha x^2-\epsilon^\alpha(kQ_p/U^{5-p})$. The corresponding coordinate in the Yang-Mills theory, in contrast, transforms canonically suggests that we should identify the latter as a new coordinate $y^\alpha(x)$ on supergravity side which transforms canonically. We set $y^\alpha=x^\alpha+\zeta^\alpha$ and find the condition on ζ^α

$$\delta_\epsilon\zeta^\alpha \;=\; 2\epsilon\cdot x\zeta^\alpha+2(\epsilon\cdot\zeta x^\alpha-x\cdot\zeta\epsilon^\alpha)+\epsilon^\alpha\frac{kQ_p}{U^{5-p}}-\epsilon^\alpha\zeta^2\,. \tag{31}$$

which leads us to

$$
\begin{aligned}
\zeta_\alpha &= \frac{1}{4}\partial_\alpha\left(\frac{kQ_p}{U^{5-p}}\right) = \frac{k}{4}\left(\frac{\partial_\alpha Q_p}{U^{5-p}} + (p-5)\frac{Q_p\partial_\alpha U}{U^{6-p}}\right) \\
&\simeq \frac{1}{4}\frac{k\partial_\alpha Q_p}{U^{5-p}}\,, \qquad (32)
\end{aligned}
$$

In leading order

$$
y^\alpha = x^\alpha + \frac{1}{4}\frac{k\partial_\alpha Q_p}{U^{5-p}}\,. \qquad (33)
$$

Performing a coordinate transformation gives

$$
\begin{aligned}
X'_m(x) &= X'_m(y-\zeta) = X'_m(y) - \zeta^\alpha\partial_\alpha X'_m(y) + \cdots \\
&\simeq X_m(y) - \frac{1}{4}\frac{k\partial_\alpha Q_p}{U^{5-p}}\partial_\alpha X_m(y)\,, \qquad (34)
\end{aligned}
$$

representing the redefinition needed to reconcile the gravitational SCT with the transformation law of the Higgs field in super Yang-Mills description.

As an application of the generalized conformal symmetry, let us consider the question of a p-brane moving in the N-fixed p-brane background. The backgrounds GLS can be used to determine the action of the probe (which was conjectured to have a born-infield form). We start from a most general scale and Lorentz invariant effective action for the radial coordinate of the p-brane $U(x)$

$$
S_p = -\int dx^{p+1}\, U(x)^{p+1} f(z, \rho_p) \qquad (35)
$$

where f is an arbitrary function of

$$
z \equiv \frac{\partial_\mu U \partial^\mu U}{U^4} \qquad (36)
$$

and $\rho_p(U)$ specifying the background. Requiring invariance under special conformal transformations of GLS we have

$$
\delta S_p = -2\int dx^{p+1}\, U^{p+1}\epsilon\cdot\partial U\frac{\rho_p}{U^3}\left(f - 2(z+\rho^{-1})\partial_z f\right) = 0 \qquad (37)
$$

which leads to

$$
f - 2\left(z + \rho^{-1}\right)\frac{\partial f}{\partial z} = -C(\rho) \qquad (38)
$$

an equation whose most general solution is

$$
f = a(\rho)\left(\sqrt{1+\rho z} - b(p)\right) \qquad (39)
$$

with $a(\rho)$ and $b(p)$ being two arbitrary functions. This is a general form compatible with the GLS transformations, the specification of a and b then relies on two nonrenormalization theorems that are expected to hold in this theory: first the requirement of the absence of a static potential (for BPS states) fixes b to be 1. Secondly the nonrenormalization from the level of the kinetic energy $(0(z))$ term fixes a leading us to the final result

$$S_p = -\int \frac{d_x^{p+1}}{(2\pi)^2} \frac{U^{7-p}}{g^2 Q_p} \left(\sqrt{1 + Q_p \frac{\partial U \cdot \partial U}{U^{7-p}}} - 1 \right) \tag{40}$$

Those found is of the form of a Born-Infield Lagrangian $S_p = S_{BI}$ which was conjecture to be the answer (previously this was checked at two loop level). This shows that the generalized conform symmetry has strong consequences on the dynamics of general p-brane systems.

4 Matrix Model

We finish this summary by discussing some features which concern the 0-brane[6] system .This theory as it appears is a special case $p = 0$ in the ladder of AdS correspondences. It is the candidate for a light-cone gauge description of the full 11 dimensional M-theory. The generalized conformal group found in the AdS framework is then likely to have further implication in relevant issues of M-theory. The action of matrix quantum mechanics:

$$S = \int dt\, Tr \left(\frac{1}{2g^2} \right) (D_t X_i)^2 + \frac{1}{4g^2} [X_i, X_j]^2 + \text{fermions} \tag{41}$$

can be easily seen to be invariant under infinitesimal scale transformations

$$\begin{aligned} t &\rightarrow \lambda^{-1} t \\ X_i &\rightarrow \lambda X_i \\ U &\rightarrow \lambda U \\ g_{YM}^2 &\rightarrow \lambda^3 g_{YM}^2 \end{aligned} \tag{42}$$

The opposite transformation properties of space (u) and time (t) imply an $IR - UV$ correspondence, they have a most natural interpretation in terms of the space-time uncertainty principle introduced by Yoneya[7].

The matrix operators representing the symmetry are written as

$$\begin{aligned} D &= \frac{1}{2} Tr\, (XP + PX) - 3i\, g_s \frac{\partial}{\partial g_s} \\ K &= Tr \left(X^2 \right) + c\, g^{2/3} \end{aligned} \tag{43}$$

Together with the Hamiltonian H they close an $SU(1,1)$ Lie algebra.

One topics of interest is the dependence on the dynamical coupling .One can study this at leding orther evaluating the effect linear in the quantity

$$\eta(\tau) \equiv \frac{\partial_\tau g_s}{g_s}. \tag{44}$$

The effective action follows from a the 1-loop calculation analogous to the one with constant g_s with modification due to the presence of $\eta(\tau)$. Expansion about the background field,leads the quadratic parts which are modified at $\mathcal{O}(\eta)$

$$\mathcal{L}_{YY} = \frac{1}{2g_s} Y_{m,ij}(-\partial_\tau^2 + \eta(\tau)\partial_\tau + B_{ij}^2) Y_{m,ji}\,, \tag{45}$$

$$\mathcal{L}_{\tilde{A}\tilde{A}} = \frac{1}{2g_s} \tilde{A}_{ij}(-\partial_\tau^2 + \eta(\tau)\partial_\tau + B_{ij}^2) \tilde{A}_{ji}\,, \tag{46}$$

$$\mathcal{L}_{Y\tilde{A}} = \frac{2i}{g_s}\left(\dot{B}_{m,ij} - \frac{1}{2}\eta(\tau) B_{m,ij}\right) Y_{m,ij}\tilde{A}_{ji}\,, \tag{47}$$

$Y\tilde{A}$-mixing can be analyzed in exactly the same way as for the constant g_s through the following replacement:

$$\dot{B}_{ij} \rightarrow V_{ij}\,, \tag{48}$$

$$\begin{aligned} V_{ij} &= \left[\sum_m \left(\dot{B}_{m,ij} - \frac{1}{2}\eta(\tau) B_{m,ij}\right)^2\right]^{1/2} \\ &= \dot{B}_{ij} - \frac{1}{2}\eta \frac{\sum_m \dot{B}_{m,ij} B_{m,ij}}{\dot{B}_{ij}} + \mathcal{O}(\eta^2)\,. \end{aligned} \tag{49}$$

Then the 1-loop effective action is then given as

$$\begin{aligned} e^{-(\Gamma_1+\Delta\Gamma_1)} = \prod_{i<j} &\det\left(-\partial_\tau^2 + \eta\partial_\tau + B_{ij}^2\right)^{-8} \\ &\times \det\left(-\partial_\tau^2 + \eta\partial_\tau + B_{ij}^2 + 2V_{ij}\right)^{-1} \\ &\times \det\left(-\partial_\tau^2 + \eta\partial_\tau + B_{ij}^2 - 2V_{ij}\right)^{-1} \\ &\times \det\left(-\partial_\tau^2 + B_{ij}^2 + \dot{B}_{ij}\right)^{4} \\ &\times \det\left(-\partial_\tau^2 + B_{ij}^2 - \dot{B}_{ij}\right)^{4} \\ &\times \det\left(-\partial_\tau^2 + B_{ij}^2\right)^{2}\,, \end{aligned} \tag{50}$$

One uses the eikonal approximation by seting

$$x \equiv B = b + \tilde{v}\tau = b + vt\,, \tag{51}$$

$$b \cdot \tilde{v} = 0\,, \tag{52}$$

$$r^2 \equiv B \cdot B = b^2 + \tilde{v}^2\tau^2 = b^2 + v^2 t^2\,, \tag{53}$$

Then, keeping terms linear in $\eta(\tau)$ and expanding up to $\mathcal{O}(\tilde{v}^4)$, gives $\Delta\Gamma_1$

$$\begin{aligned}\Delta\Gamma_1 &= 10\mathrm{Tr}\left\{\Delta\eta\partial_\tau - \tilde{v}^2\Delta\eta\partial_\tau\Delta\tau^2 + \tilde{v}^4\Delta\eta\partial_\tau\Delta\tau^2\Delta\tau^2\right\} \\ &\quad +4\mathrm{Tr}\left\{\tilde{v}^2(\Delta^2\eta\tau + 2\Delta^3\eta\partial_\tau) + \tilde{v}^4(-2\Delta\tau^2\Delta^2\eta\tau - 6\Delta^3\eta\partial_\tau\Delta\tau^2 + 4\Delta^4\eta\tau)\right\}, \end{aligned} \tag{54}$$

where $\Delta \equiv (-\partial_\tau^2 + b^2)^{-1}$ is the basic propagator. A longer calculation gives the result:

$$\Delta\Gamma_1 = N\int dt\frac{\dot{g}_s}{g_s}\left(-\frac{3}{2}\frac{v\cdot x}{r^3} - \frac{15}{8}\frac{v^2 v\cdot x}{r^7}\right). \tag{55}$$

Consider now the variation of this term under SCT's at fixed t which can be written as

$$\delta_K x = 2tx + t^2 v\,, \qquad \delta_K r = 2tr + t^2\frac{v\cdot x}{r}\,, \qquad \delta_K v = 2x + 4tv + t^2\dot{v}\,, \tag{56}$$

$$\delta_K\frac{\dot{g}_s}{g_s} = 6 + 2t\frac{\dot{g}_s}{g_s} + t^2\frac{d}{dt}\left(\frac{\dot{g}_s}{g_s}\right). \tag{57}$$

Droping $\dot{g}_s$ terms after the variation is made one finds:

$$\delta_K\Delta\Gamma_1 = \int dt\frac{-45N}{4}\frac{v^2 v\cdot x}{r^7}. \tag{58}$$

On the other hand, the usual 1-loop effective action Γ_1 is

$$\Gamma_1 = \frac{15N}{16}\frac{v^4}{r^7}, \tag{59}$$

and its variation takes the form

$$\delta_K\Gamma_1 = \int dt\frac{30N}{4}\frac{v^2 v\cdot x}{r^7}. \tag{60}$$

For the variation of the combined effective action we find

$$\delta_K(\Gamma_1 + \Delta\Gamma_1) = -\frac{1}{2}\delta_K\Gamma_1 = \left(\frac{p-1}{2}\right)_{p=0}\delta_K\Gamma_1\,. \tag{61}$$

This demonstrates that with the inclusion of a $\dot{g}_s$, term in the effective action one can reconcile the overall coeficient of the YM symmetry transformation with that of gravity.

Let us next describe a simple way to exibit the o-brane "AdS" metric

$$(ds)^2 = -\sqrt{\frac{U^7}{g^2}}dt^2 + \sqrt{\frac{g^2}{U^7}}du^2 \tag{62}$$

in terms of an effective matrix theory. Consider the 1-matrix theory inspired by the earlier effective action:

$$S = \frac{1}{g^2} \int dt \left(\frac{1}{2} \dot{U}^2 - \frac{1}{2} \frac{U^7}{g^2} \right) \cdots \tag{63}$$

It is such that it obeys the correct scaling transformations

$$U \to \lambda U, t \to \lambda^{-1} t, g^2 \to \lambda^3 g^2 \tag{64}$$

The connection between the two pictures requires a transition from a matrix description (where $\hat{U}$ is a matrix variable) to a field description. In general this phenomenon represents an integral part of the collective field technique. The matrix Hamiltonian reads

$$H = \frac{1}{2} \mathrm{Tr} \left(g^2 P^2 + \frac{U^7}{g^2} \right) \tag{65}$$

We now perform the transition to the collective field $\Phi(U,t), \Pi(U,t)$ or $\alpha_\pm(U,t) = \Pi \pm \Phi, u$ to have

$$H = \frac{1}{2} \int dU \left[\frac{g^2}{3} \left(\alpha_+^3 - \alpha_-^3 \right) + \frac{U^7}{g^2} \left(\alpha_+ - \alpha_- \right) \right] \tag{66}$$

The classical equation provides a stationary point

$$g^2 \alpha_0(U) = \sqrt{\frac{U^7}{g^2}} \tag{67}$$

and by linearizing

$$\alpha = \alpha_0 + \bar{\alpha} \tag{68}$$

we obtain the quadratic Hamiltonian

$$H_2 = \int du \, g^2 \alpha_0(U) \left(\Pi^2(u) + (\partial_U \varphi)^2 \right) \tag{69}$$

What we see is a scalar field in the background metric of the form:

$$(ds)^2 = -\frac{1}{g^2 \alpha_0(U)} dt^2 + g^2 \alpha_0(U) dU^2 \tag{70}$$

This metric is of the AdS(2) type and with the explicite U-dependence given by the stationary point we find precisely the correct 0-brane metric.

One other issue which we can illuminate in this case is our treatment of g (the coupling constant) as a dynamical variable. Recall the case of a relativistic particle

$$S = \int d_\tau e(\tau) \, \dot{X}_\mu \dot{X}^\mu \tag{71}$$

characterized by a reparametrization symmetry

$$\begin{array}{rcl} X_\mu(\tau) & \to & X_\mu\left(f(\tau)\right) \\ e(\tau) & \to & \dot{f}(\tau)^{-1}\, e\left(f(\tau)\right) \end{array} \tag{72}$$

The standard procedure involving a stationary point matrix quantum mechanics can be written in an analogous form, rescale

$$X_i \to g^\alpha X_i \tag{73}$$

with $\alpha = \frac{2}{3}$ and denoting $e = g^{2\alpha-2}$ the action. is written in the form

$$S_e = \int dt Tr \left(e(t)\dot{X}_i^2 + \frac{1}{e_{(t)}} [X_i, X_j]^2 \right) \tag{74}$$

where we have made e into a dynamical variable $e(t)$. One can check that this action still has the above reparametrization symmetry, the theory with $e = g^{2\alpha-2} = \text{const}$ corresponds to a gauge fixed version.This form implies the existence of a larger symmetry then the SO(2,1) group that we have described above.

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