D-brane Superpotentials and Open Mirror Symmetry on Compact Calabi-Yau Manifolds

Hisao Suzuki

Hokkaido University

Work with H.Fuji, S.Nakayama and S.Shimizu


Happy birthday Kazama-san!

We enjoyed our works!

- I worked with Kazama san
- But
- I am not a student of Kazama-san….

Snow festival in Sapporo

How string would be related to physics?

- Compactification to four dimensions

N=2 world-sheet supersymmetry

Superstring in four dimensions

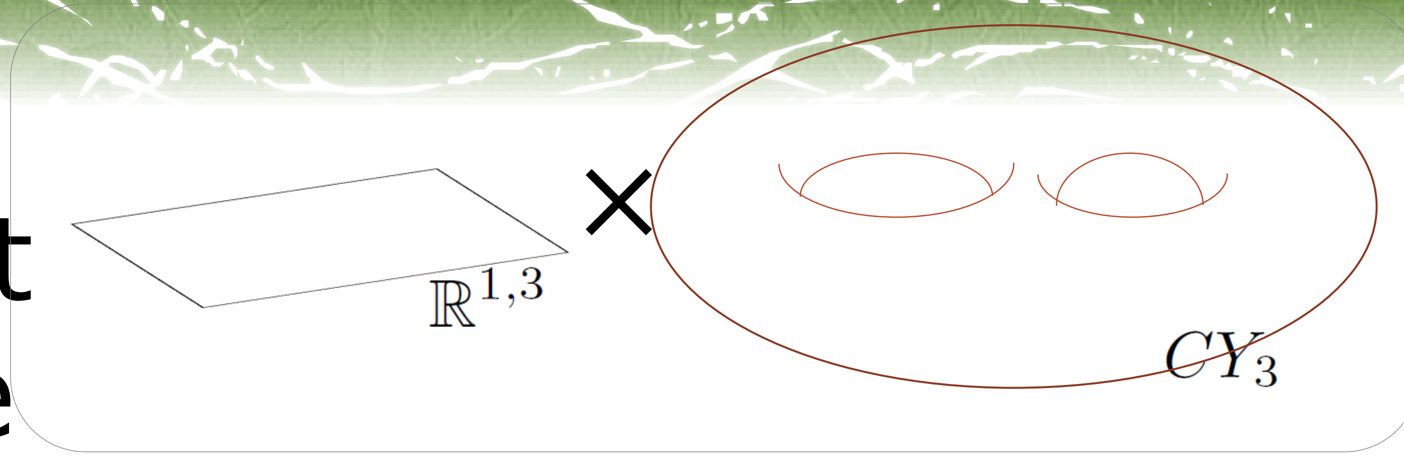
Supercoset models work with Kazama

Geometrical interpretations?

Mirror Symmetry

Ⅱ型 string theory / CY_3

Target space



◆4dim N=2

worldsheet

2次元 N=(2,2) SCFT

$S_{\mathrm{phys}}(t,z)$

A-twist B-twist

$S_{\mathrm{top.\ A}}(t)$ $S_{\mathrm{top.\ B}}(z)$

Some of the physical quantities can be obtained exactly

Witten

Topological A-model on CY_3

⟷

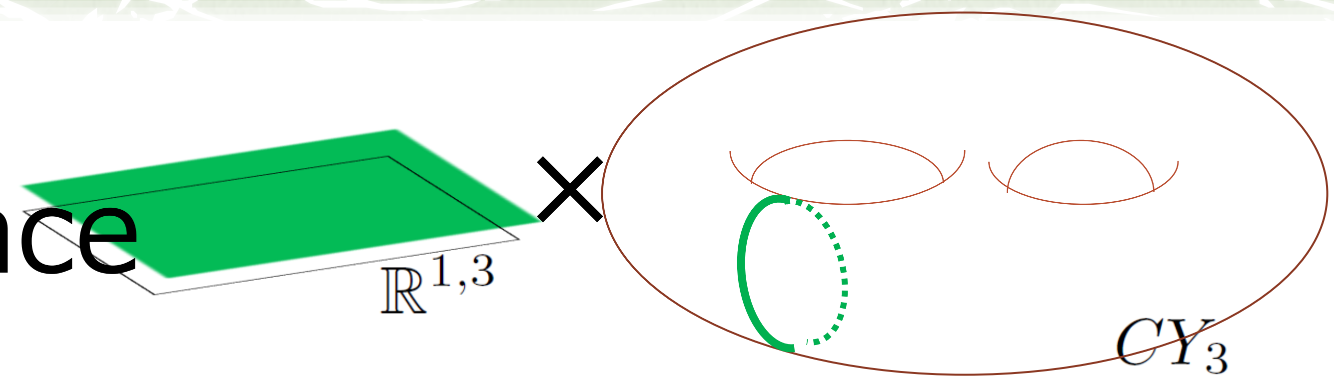
Topological B-model on "**mirror**" CY_3

Mirror symmetry

OPEN Mirror Symmetry

typeII string theory / CY_3 + supersymmetric D-brane

Target space



◆4dim N=1

ⅡA : special Lagrange submanifold (D6)

ⅡB : holomorphic (D3, D5, D7, D9)

Becker-Becker-Strominger
Ooguri-Oz-Yin

Worldsheet **2dim N=2 boundary SCFT**

$$S_{\mathrm{phys}}(t,z)$$

A-twist B-twist

$$S_{\mathrm{top.\ A}}(t) \qquad S_{\mathrm{top.\ B}}(z)$$

Topological A-model on CY_3 + **A-brane**

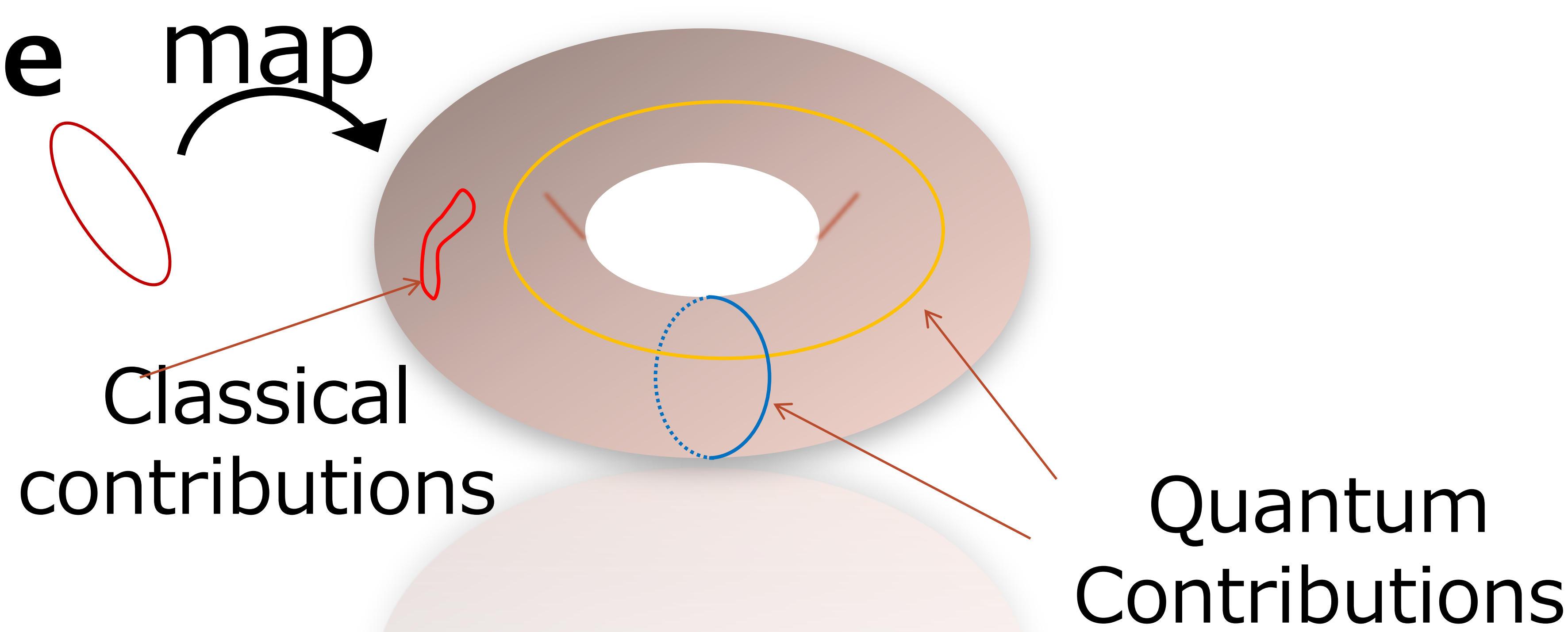
Open Mirror symmetry

Topological B-model on "**mirror**" CY_3 + "**mirror**" **B-brane**

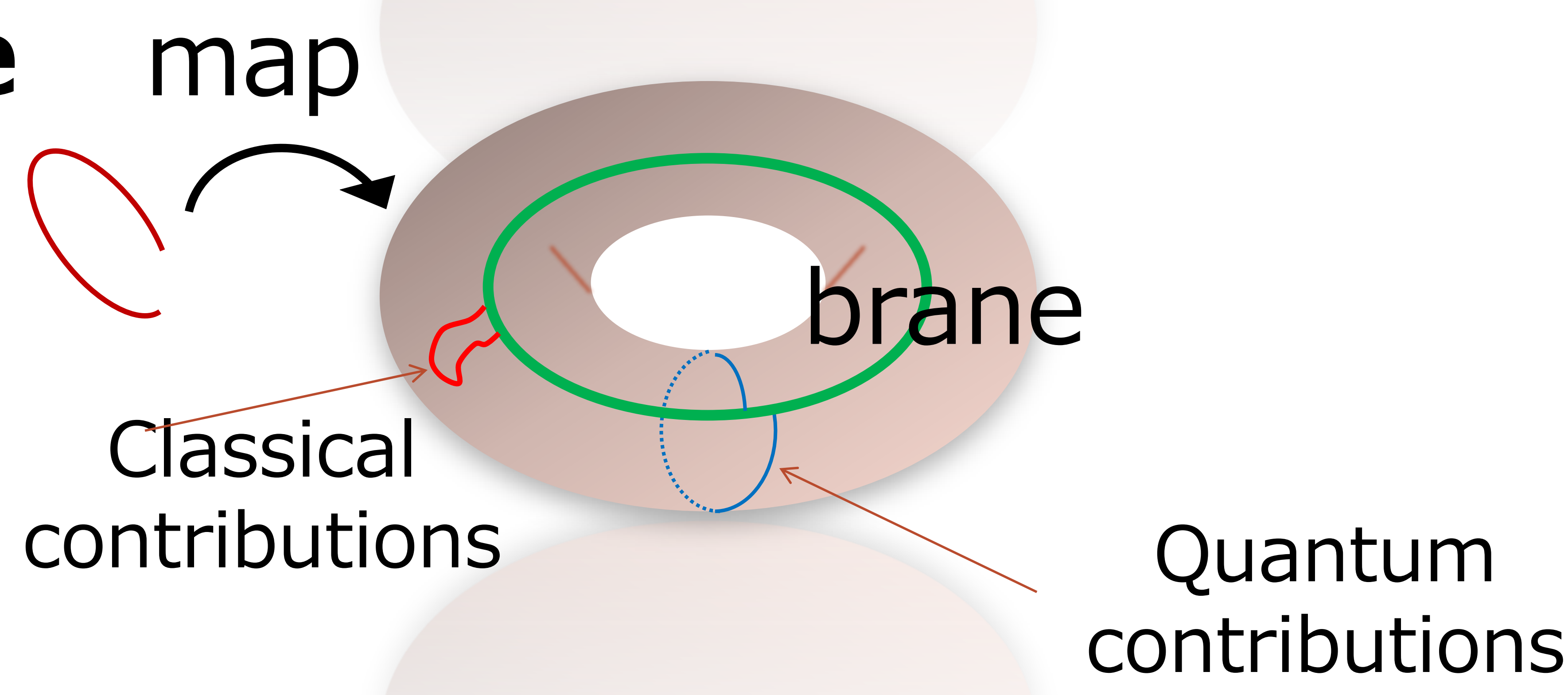
superpotentials

Worldsheet instanton effects (A-side)

Closed case



Open case



B-model side

Computation of superpotential

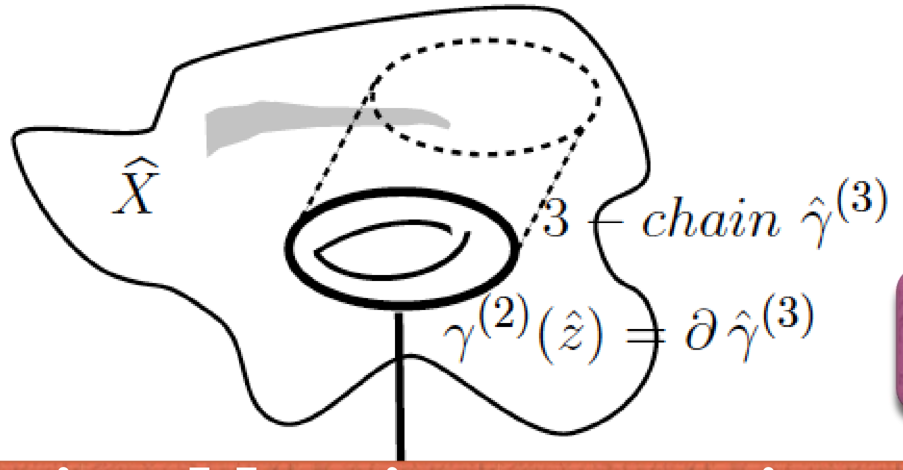
$$\mathcal{W}(z,\hat{z}) = \int_{\substack{\hat{\gamma}^{(3)}(\hat{z}) \\ \partial\hat{\gamma}^{(3)}=\gamma^{(2)}}} \Omega^{3,0}(z)$$

No instantons

Still hard to obtain

For non-compact CY successfull

For compact CY??

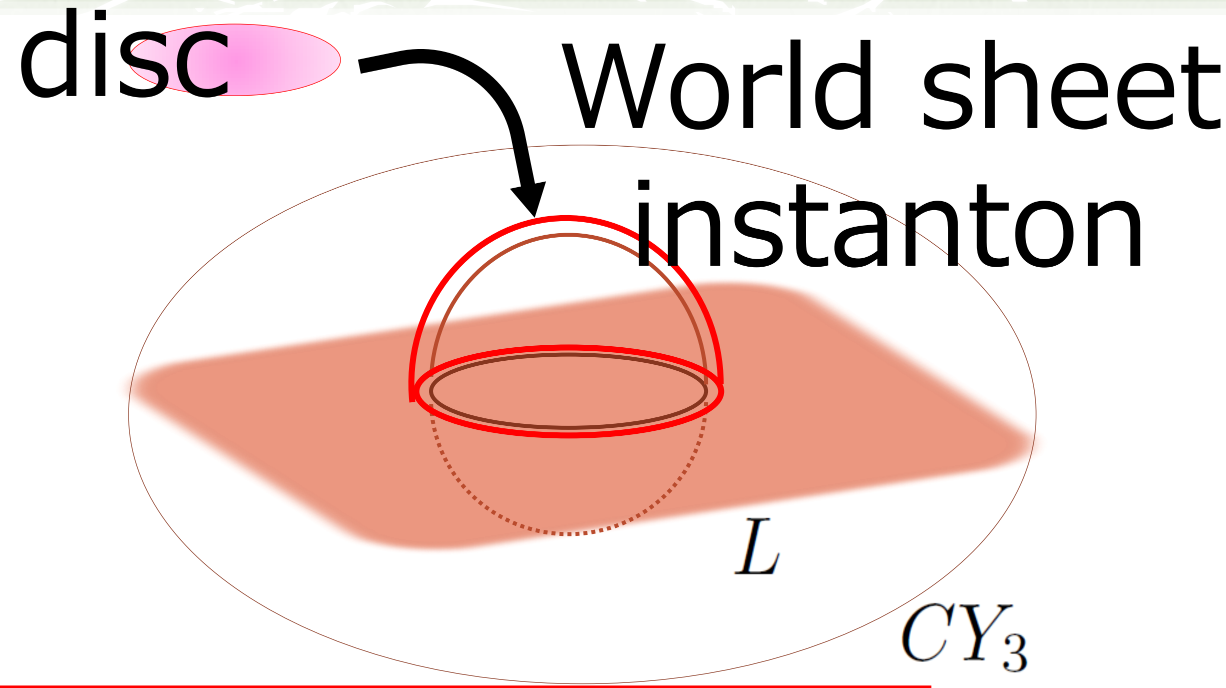


Lerche

Superpotentials for physically interesting models?
Cosmological applications

What is open string mirror symmetry?

- A-model

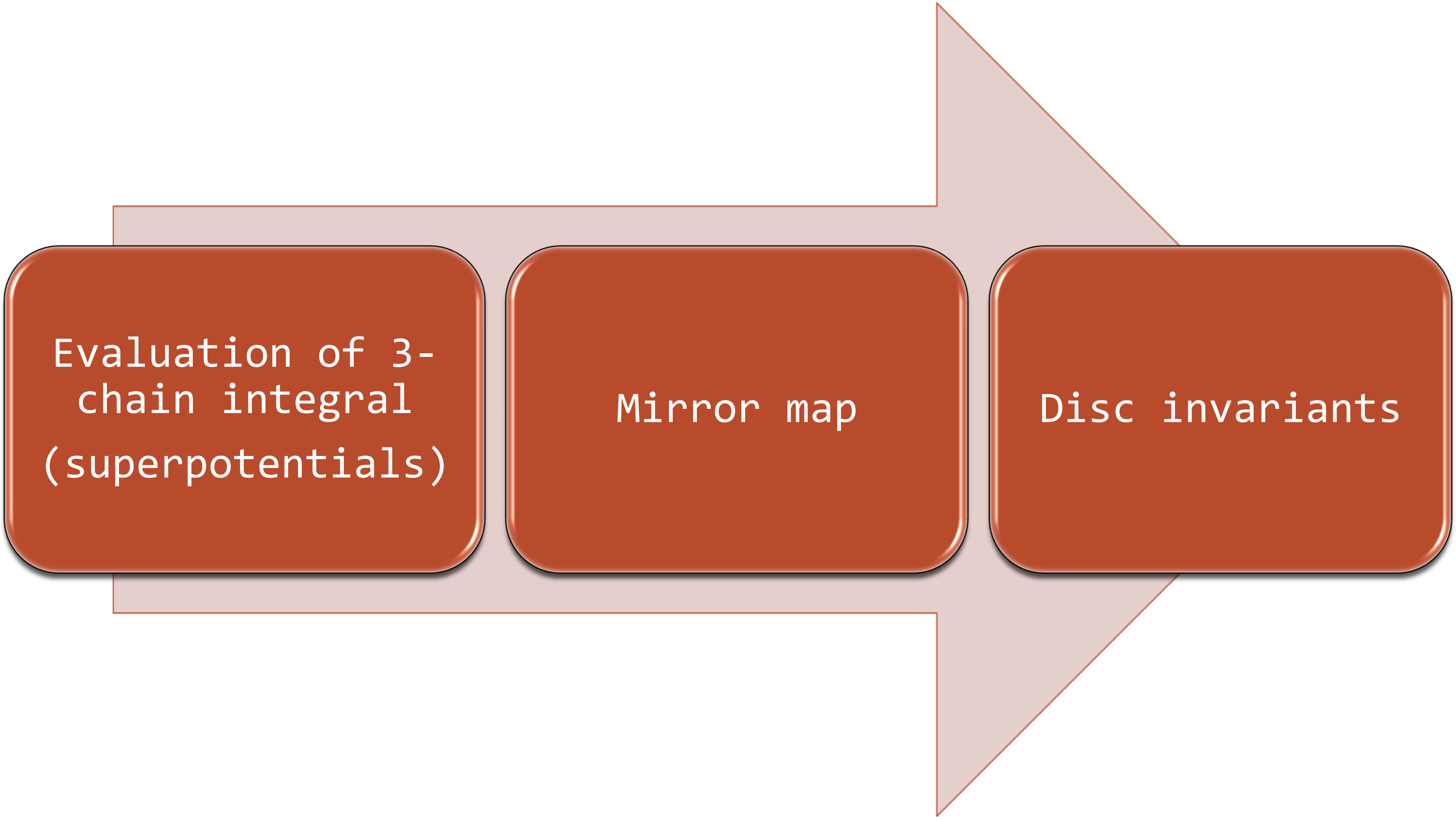


$$\boxed{\mathcal{T}_A(z) = \frac{1}{2}t + \frac{1}{2} + \frac{1}{2\pi^2}\sum_{k,d:\mathrm{odd}} \frac{n_d}{k^2} q^{kd/2}}$$

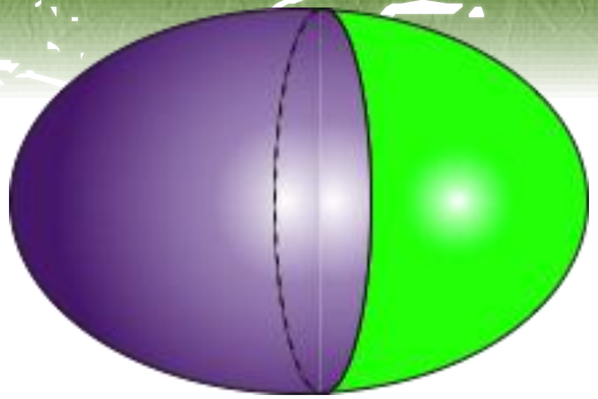
$$\boxed{q = e^{2\pi i t}}$$

$\boxed{n_d}$ Disc invariants

B-models to A-models



D-brane in Compact manifolds



- 3 chain integral

$$\frac{\Omega_{\Gamma_\pm}(z(q))}{\omega_0(z(q))} = \frac{t}{2} \pm \left(\frac{1}{4} + \frac{1}{2\pi^2} \sum_{k,d \text{ odd}} \frac{n_d}{k^2} q^{kd/2}\right),$$

Superpotential induced by D5-brane

Mirror symmetry on the quintic

Walcher 06

- Defining Polynomial

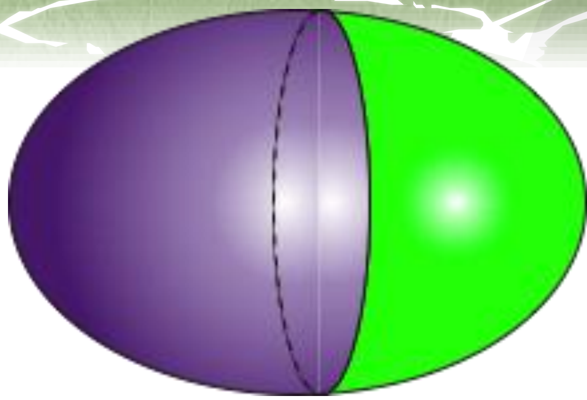
$$W = \frac{1}{5}(x_1^5 + x_2^5 + x_3^5 + x_4^5 + x_5^5) - \psi x_1 x_2 x_3 x_4 x_5 = 0$$

- Period integral

$$\Omega = \frac{5^3\psi}{(2\pi i)^4} \int \frac{dx_2 dx_3 dx_4 dx_5}{W}$$

3-chain integral

Picard-Fuchs equations



- Morrison Walcher arXiv:0709.4028

$$\tilde{\Omega}(z) := \frac{\omega}{W(z)}$$

$$\omega = \sum_i (-1)^{i-1} x_i dx_1 \wedge \ldots \wedge \widehat{dx_i} \wedge \ldots \wedge dx_5$$

$$\tilde{\mathcal{L}}\tilde{\Omega} := ((1-\psi^5)\partial_\psi^4 - 10\psi^4\partial_\psi^3 - 25\psi^3\partial_\psi^2 - 15\psi^2\partial_\psi - 1)\tilde{\Omega} = -d\tilde{\beta}$$

vanishes without boundaryes

$$
\begin{aligned}
\tilde{\beta} =& \frac{3!}{W^4}\left(x_2^4x_3^4x_4^4x_5^4\omega_1 + \psi x_2x_3^5x_4^5x_5^5\omega_2 + \psi^2 x_1x_2x_3^2x_4^6x_5^6\omega_3\right.\\
&\left.+\psi^3x_1^2x_2^2x_3^2x_4^3x_5^7\omega_4 + \psi^4x_1^3x_2^3x_3^3x_4^3x_5^4\omega_5\right)\\
&+\frac{2}{W^3}\left(\psi x_3x_4^5x_5^5\omega_3 + 3\psi^2x_1x_2x_3x_4^2x_5^6\omega_4 + 6\psi^3x_1^2x_2^2x_3^2x_4^2x_5^3\omega_5\right)\\
&+\frac{1}{W^2}\left(\psi x_4x_5^5\omega_4 + 7\psi^2x_1x_2x_3x_4x_5^2\omega_5\right)\\
&+\frac{1}{W}\left(\psi x_5\omega_5\right).
\end{aligned}
$$

- 3-chain integral

$$\boxed{\int_\Gamma \Omega(z)}$$

$$\boxed{\mathcal{LT}(z) = f(z)}$$

$$\boxed{\textit{inhomogeneous Picard–Fuchs equation}}$$

Where is the D-brane?

$$\boxed{W = \frac{1}{5}(x_1^5 + x_2^5 + x_3^5 + x_4^5 + x_5^5) - \psi x_1 x_2 x_3 x_4 x_5 = 0}$$

$$\boxed{P = \{x_1 + x_2 = x_3 + x_4 = 0\}}$$

- two singular points

$$\boxed{p_1 = \{x_1 = -x_2, x_3 = x_4 = x_5 = 0\}}$$

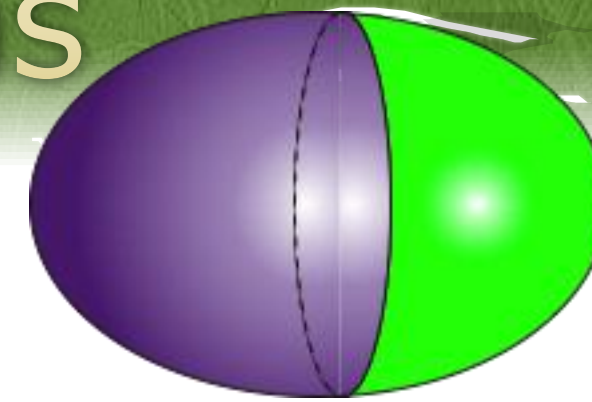
$$p_2 = \{x_1 = x_2 = x_5 = 0, x_3 = -x_4\}$$

main contributions

requires the resolution of singularities

- Morrison Walcher arXiv:0709.4028

boundary contributions



$$\mathcal{L}\big(\varpi_0(z)\mathcal{T}_A(z)\big) = \frac{15}{16\pi^2}\sqrt{z}.$$

terms added for consistency

- Mirror map

$$t = t(z) = \frac{1}{2\pi i}\frac{\varpi_1(z)}{\varpi_0(z)}, \qquad q(z) = \exp(2\pi i t(z))$$

$$\varpi_0(z)\mathcal{T}_A(z) = \frac{\varpi_1(z)}{4\pi i} + \frac{\varpi_0(z)}{4} + \frac{15}{\pi^2}\tau(z)$$

$$\tau(z) = \frac{\Gamma(3/2)^5}{\Gamma(7/2)}\sum_{m=0}^{\infty}\frac{\Gamma(5m+7/2)}{\Gamma(m+3/2)^5}\, z^{m+1/2} = \sqrt{z} + \frac{5005}{9}z^{3/2} + \cdots$$

$$\mathcal{T}_A(z) = \frac{1}{2}t + \frac{1}{2} + \frac{1}{2\pi^2}\sum_{k,d:\mathrm{odd}} \frac{n_d}{k^2} q^{kd/2}$$

Disc invariants

d	number of disks n_d
1	30
3	1530
5	1088250
7	975996780
9	1073087762700
11	1329027103924410
13	1781966623841748930
15	2528247216911976589500
17	3742056692258356444651980
19	5723452081398475208950800270

- Walcher 06

How to obtain disc invariants?

- Picard Fuchs analysis: straightforward

However…

Problems with Picard Fuchs analysis

- Rigorous but tedious
- boundary terms

Quintic

$$
\begin{aligned}
\tilde{\beta} = & \frac{3!}{W^4}\big(x_2^4 x_3^4 x_4^4 x_5^4 \omega_1 + \psi x_2 x_3^5 x_4^5 x_5^5 \omega_2 + \psi^2 x_1 x_2 x_3^2 x_4^6 x_5^6 \omega_3 \\
& \qquad + \psi^3 x_1^2 x_2^2 x_3^2 x_4^3 x_5^7 \omega_4 + \psi^4 x_1^3 x_2^3 x_3^3 x_4^3 x_5^4 \omega_5\big) \\
& + \frac{2}{W^3}\big(\psi x_3 x_4^5 x_5^5 \omega_3 + 3\psi^2 x_1 x_2 x_3 x_4^2 x_5^6 \omega_4 + 6\psi^3 x_1^2 x_2^2 x_3^2 x_4^2 x_5^3 \omega_5\big) \\
& + \frac{1}{W^2}\big(\psi x_4 x_5^5 \omega_4 + 7\psi^2 x_1 x_2 x_3 x_4 x_5^2 \omega_5\big) \\
& + \frac{1}{W}\big(\psi x_5 \omega_5\big).
\end{aligned}
$$

off-shell superpotentials

- Inclusions of boundary moduli

$$\int_{\Gamma_3} \frac{\Delta \log Q(x_i,\phi)}{\prod_a W_a(x_i,\psi)}.$$

A-model Interpretations?

H. Jockers and M. Soroush,

Fixing boundary moduli

Minimizing superpotentials → On-shell superpotentials

However

Problems on the normalization

another method we performed

- explicit integrations

$$\boxed{W=\frac{1}{5}(x_1^5+x_2^5+x_3^5+x_4^5+x_5^5)-\psi x_1x_2x_3x_4x_5=0}$$

$$\omega_0 \sim \int \frac{dx_1\wedge\cdots\wedge x_5}{W}$$

$$\omega_0 \sim \int \sum_{n=0}^{\infty} \frac{dx_1\wedge\cdots\wedge x_5}{x_1x_2x_3x_4x_5}\frac{(x_1^5+\cdots+x_5^5)^n}{(x_1x_2x_3x_4x_5)^n}\left(\frac{1}{\psi^5}\right)^n$$

$$\sim \sum_{n=0}^{\infty}\frac{(5m)!}{m!^5}\left(\frac{1}{\psi^5}\right)^n$$

resolution of the singularities

Fuji,Nakayama,Shimizu ,H.S. in preparation

mixture of the singularities

⇒ analytic continuations

Main claim of my talk

We can obtain superpotential induced by the presence of D-branes in compact CY by explicit integrations.

Simple integrals

2.Evaluation of 3-chain Integral

- Defining equation

$$\boxed{W = \frac{1}{5}(x_1^5 + x_2^5 + x_3^5 + x_4^5 + x_5^5) - \psi x_1 x_2 x_3 x_4 x_5 = 0.}$$

$$\boxed{x_2 \longrightarrow e^{i\pi/5} x_2,\, x_4 \longrightarrow e^{-i\pi/5} x_4}$$

$$\Rightarrow$$

$$\boxed{W = \frac{1}{5}(x_1^5 - x_2^5 + x_3^5 - x_4^5 + x_5^5) - \psi x_1 x_2 x_3 x_4 x_5 = 0}$$

”polar coordinates”

- $$X = \zeta w/(5\psi T)^{1/2},$$

  $$Z = \zeta^{-1} w/(5\psi T)^{1/2},$$

$$W = \frac{1}{5}[1 - T^5 + (5\psi)^{-5/2}(\zeta - \frac{1}{\zeta})w^5 T^{-5/2} Y^2 + Y(1 - w^2)]$$

Boundary $\zeta = 1, w = \pm 1, T = 1.$

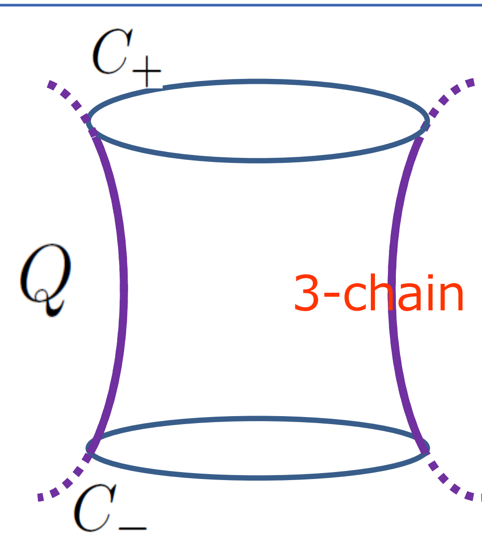
$$\Omega = \frac{10}{(2\pi i)^4}\int \frac{d\zeta}{\zeta}\frac{dT}{T} w\,dw\,dY$$

$$\times \frac{1}{1 - T^5 + (5\psi)^{-5/2}(\zeta - \zeta^{-1}) w^5 T^{-5/2} Y^2 + Y(1 - w^2)}$$

Integration over Y

Y no boundary even at the boundary

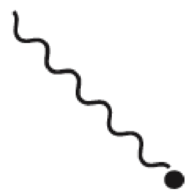
$$\Omega = 10\int \frac{d\zeta\, dT\, w\, dw}{(2\pi i)^3 \zeta \cdot T}\frac{1}{\sqrt{(1-w)^2 - 4w^5(\zeta - \zeta^{-1})(T^{5/2} - T^{-5/2}) z^{1/2}}}$$



$$z = 1/(5\psi)^5$$

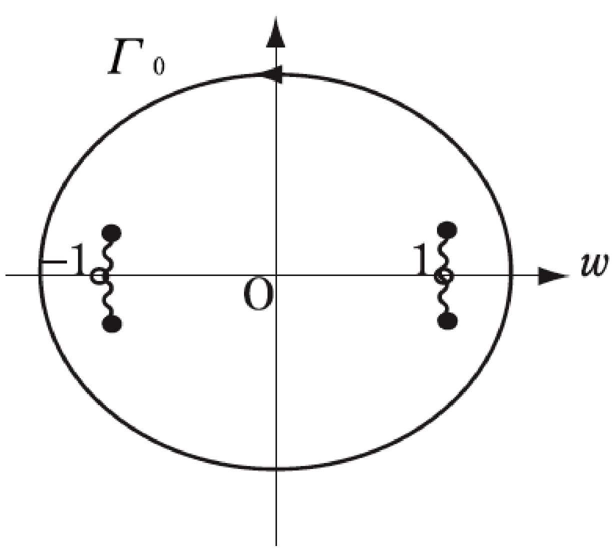
Singularities and the fundamental period

$$\Omega = 10 \int \frac{d\zeta dT w dw}{(2\pi i)^3 \zeta \cdot T} \frac{1}{\sqrt{(1-w)^2 - 4w^5(\zeta - \zeta^{-1})(T^{5/2} - T^{-5/2}) z^{1/2}}}$$



$$\zeta = e^{i\theta}, -\pi/2 < \theta < \pi/2$$

$$T = e^{i\phi}, 0 < \phi < 2\pi/5$$



Evaluation of the period

Factorized integrals

$$\Omega = 10 \sum_{n=0}^{\infty} \int \frac{dw}{(2\pi i)} \frac{(2n)!}{(n!)^2} \int \frac{dw}{2\pi i} \frac{w^{5n+1}}{(1-w^2)^{2n+1}}$$
$$\times \int \frac{d\zeta}{(2\pi i)\zeta} (\zeta - \zeta^{-1})^n \int \frac{dT}{(2\pi i)T} (T^{5/2} - T^{-5/2})^n z^{n/2}$$

Analytic continuations

$$\Omega = 40 \int \frac{ds}{2\pi i} \frac{\pi \cos(\pi s)}{\sin(\pi s)} \frac{\Gamma(2s+1)}{\Gamma(s+1)^2}$$
$$\times \int_1^0 \frac{dw}{2\pi i} \frac{w^{5s+1}}{(1-w^2)^{2s+1}} \int \frac{d\zeta}{2\pi i \zeta} (\zeta - \zeta^{-1})^s \int \frac{dT}{2\pi i T} (T^{5/2} - T^{-5/2})^s z^{s/2}$$

analytic continuation

Integral with boundary Line

$$\Omega = 40 \int \frac{ds}{2\pi i} \frac{\pi \cos(\pi s)}{\sin(\pi s)} \frac{\Gamma(2s+1)}{\Gamma(s+1)^2}$$
$$\times \int_1^0 \frac{dw}{2\pi i} \frac{w^{5s+1}}{(1-w^2)^{2s+1}} \int \frac{d\zeta}{2\pi i \zeta} (\zeta - \zeta^{-1})^s \int \frac{dT}{2\pi i T} (T^{5/2} - T^{-5/2})^s z^{s/2}$$

$$\frac{1}{2\pi i} \int \frac{d\zeta}{\zeta} (\zeta - \zeta^{-1})^s = \frac{1}{2} \cos \frac{\pi s}{2} \frac{\Gamma(s+1)}{\Gamma(\frac{s}{2}+1)^2}$$

$$\frac{1}{2\pi i} \int \frac{dT}{T} (T^{5/2} - T^{-5/2})^s = \frac{e^{\pi i s/2}}{5} \frac{\Gamma(s+1)}{\Gamma(\frac{s}{2}+1)^2}$$

$$\Omega_{\Gamma_+} = \frac{1}{2\pi i} \int \frac{ds}{2\pi i} \frac{\pi \cos(\pi s)}{\sin(\pi s)} e^{\pi i s/2} \frac{\Gamma(\frac{5}{2}s+1)}{\Gamma(\frac{s}{2}+1)^5} \left(\frac{2\pi \cos(\pi s/2)}{\sin(2\pi s)} \right) z^{s/2}$$

$$\Omega_{\Gamma_+} = \frac{1}{2\pi i}\int \frac{ds}{2\pi i}\frac{\pi\cos(\pi s)}{\sin(\pi s)} e^{\pi i s/2}\frac{\Gamma(\frac{5}{2}s+1)}{\Gamma(\frac{s}{2}+1)^5}(\frac{2\pi\cos(\pi s/2)}{\sin(2\pi s)}) z^{s/2}$$

- Singularities

s=2n+1: single poles

s=2n: double poles

logarithmic terms are derived

$$\Omega_{\Gamma_+} = \frac{1}{4\pi i}\omega_1 + \frac{1}{4}\omega_0 + \frac{1}{4}\sum_{n=0}^{\infty}\frac{\Gamma(n+7/2)}{\Gamma(n+3/2)^5} z^{n+1/2}$$

$$\omega_1(z) = \omega_0(z)\log z + \sum_{n=0}^{\infty}\frac{\Gamma(5n+1)}{\Gamma(n+1)^5}(5\Psi(5n+1) - 5\Psi(n+1))z^n$$

desired result

off-shell superpotentials

- Inclusions of boundary moduli

$$\int_{\Gamma_3} \frac{\Delta \log Q(x_i, \phi)}{\prod_a W_a(x_i, \psi)}.$$

A-model Interpretations?

H. Jockers and M. Soroush,

Fixing boundary moduli

F-theory Interpretations

- Calabi-Yau fourfold
- With intersection

Mina Aganagic, Christopher BeemarXiv:0909.2245

$$Q_4(\phi) = x_{n+2}x_{n+3} + Q(x_i,\phi) = 0,$$

$$\Pi_4 = \int_{\Gamma_4} \frac{\Delta_4}{\prod_a W_a(x_i,\psi) Q_4(\phi)},$$

$$\int_{\Gamma_3} \frac{\Delta \log Q(x_i,\phi)}{\prod_a W_a(x_i,\psi)}.$$

$$\partial_\phi \Pi_4 = \frac{-1}{\pi} \int_{\Gamma_3} \frac{\partial_\phi Q(x_i,\phi)\Delta}{\prod_a W_a(x_i,\psi) Q(x_i,\phi)}$$
$$= \frac{-1}{\pi} \partial_\phi \int_{\Gamma_3} \frac{\Delta \log Q(x_i,\phi)}{\prod_a W_a(x_i,\psi)}.$$

We can perform the integrals with open-string moduli

Hypergeometric series around on-shell points

6.Conclusions

- We have evaluated 3-chain integrals by explicit integrations (with open-string moduli)
- We could predict the results which has not been proved by A-models
- We need some generic formula for normalization, Case by Case?

Discussions

contributions from boundaries

Analytic continuations

?

Resolutions of the singular boundary

Off-shell extensions

Higher genus extensions for on-shell superpotentials

- Analysis for each models

Johannes Walcher
arXiv:0712.2775

Generic analysis?

Universal formula for genus one (one moduli models)

$$X_{d_1,d_2,,,d_k}(w_1,w_2,,,w_l).$$

Classical Yukawa coupling:
$$K_0=\frac{\prod_{i=1}^k d_i}{\prod_{i=1}^l w_i}.$$

Euler number
$$\chi=\frac{1}{3}\frac{\prod_{i=1}^k d_i}{\prod_{i=1}^l w_i}\left(-\sum_{i=1}^k d_i^3+\sum_{i=1}^l w_i^3\right)$$

The fundamental period

$$\omega_0=\sum_{n=0}^{\infty}\frac{\prod_{i=1}^k\Gamma(d_i n+1)}{\prod_{i=1}^l\Gamma(w_i n+1)}z^n.$$

the chain integral

$$\omega_\Gamma = \frac{\tau}{\omega_0} = c\frac{1}{2\pi^2}\sum_{n=0}^{\infty}\frac{\prod_{i=1}^{l}\Gamma(-w_i n - w_i/2)}{\prod_{i=1}^{k}\Gamma(-d_i n - d_i/2)} z^{n+1/2}$$

assumption

$c = N/|\mathcal{S}|$, N the number of branes

$|\mathcal{S}|$ the dimension of discrete subgroup which makes the brane invariant

$c = 1$

$$X_5(1^5), X_6(1^4,2), X_8(1^4,4), X_{10}(1^3,2,5), X_{3,3}(1^6)$$

$$n_2^{(1,\mathrm{real})} = 2^{-8}\pi^{-4}\frac{\prod_{i=1}^{l}\Gamma(-\omega_i/2)^2}{\prod_{i=1}^{k}\Gamma(-d_i/2)^2}\left[\frac{\prod_{\omega_i;\mathrm{odd}}\omega_i}{\prod_{d_i;\mathrm{odd}}d_i} - \frac{\prod_{i=1}^{l}\omega_i}{\prod_{i=1}^{k}d_i}c^2\right]$$

$$n_2^{(1,\mathrm{real})} = 0, 24, 72, 2^{12}, 0$$

for $X_5(1^5), X_6(1^4,2), X_8(1^4,4), X_{10}(1^3,2,5), X_{3,3}($

$$n_2^{(1,\mathrm{real})} = 2^{-8}\pi^{-4}\frac{\prod_{i=1}^{l}\Gamma(-\omega_i/2)^2}{\prod_{i=1}^{k}\Gamma(-d_i/2)^2}\left[\frac{\prod_{\omega_i;\mathrm{odd}}\omega_i}{\prod_{d_i;\mathrm{odd}}d_i} - \frac{\prod_{i=1}^{l}\omega_i}{\prod_{i=1}^{k}d_i}c^2\right]$$

$$n_2^{(1,\mathrm{real})} = 0, 24, 72, 2^{12}, 0$$

for $X_5(1^5), X_6(1^4,2), X_8(1^4,4), X_{10}(1^3,2,5), X_{3,3}(1$

$X_{4,4}(1^4,2^2)$ $\quad c = 4,$ $\rightarrow$ Negative integers

$c = 1$ $\rightarrow$ $n_2^{(1,\mathrm{real})} = 3/2$

We could not find any good generic formula for normalization

Generality of disc invariants?

I do not know

experiments

Shifting indices by 1/2

→ integer numbers?

very few examples

Strange phenomena

$X_8(1,1,2,2,2)$

We have claimed
the are no discs!

$$\omega_0(z_1,z_2) = \sum_{n_1,n_2=0}^{\infty} \frac{(4n_1+4n_2)!}{(n_1+n_2)!^2 n_1!^2 n_2!^2} z_1^{n_1}(z_1 z_2)^{n_2}$$

Shifting indices n_2 by 1/2?

$n_2 \to n_2 + 1/2$ ⇨

$X_4(1,1,1,1)$

d_2	d_1=1	3	5	7	9	11	13
1	16	960	75040	6173824	514892464	43219912640	3641042723936
3	0	-80	20352	11725184	2999936576	13711273908600672	1868277909330725376
5	0	0	720	6207360	5002228736	2147047861760	659882952731936
7	0	0	0	-8848	447544832	868822854144	679236361218048
9	0	0	0	0	126608	49265314880	159342867902112

Table 5: "Disc" instanton numbers for $X_8(1,1,2,2,2)$. A-model and B-model interpretations are both unknown.

$X_6(1,1,1,3)$

d_2	d_1=1	3	5	7	9	11	13
1	32	13184	6629952	3458928896	1824551060832	967408380640128	5144096626878
3	0	-1184	2866944	9187105536	14625343553664	17674937643907200	18256740403871
5	0	0	103840	4124378880	23328622659584	67719084870712320	138106880307719
7	0	0	0	-12504352	1942110616576	27809054323356672	150067275287732
9	0	0	0	0	1767088416	1397633136225408	33243587538451

Table 6: "Disc" instanton numbers for $X_{12}(1,1,2,2,6)$. A-model and B-model interpretations are both unknow. Otherwise it may be just an accident.

Elliptic fibered K3

$X_6(1,1,2,2)$
$x_1^3 x_2^3 + x_3^3 + x_4^3 - \psi_0 x_1 x_2 x_3 x_4 + \psi_1 (x_1^3 - x_2^3)^2$

$$\omega_0 = \frac{(3n_1 + 3n_2)!(2n_2)!}{n_1!(n_1+n_2)!^2 n_2!^3} z_1^{n_1+n_2} z_2^{n_2}$$

$$n_2 \to n_2 + 1/2$$

elliptic fibered K3

$X_6(1,1,2,2)$

$a[1,1] \to 12$	$a[1,3] \to 12$	$a[1,5] \to 36$	$a[1,7] \to 84$	$a[1,9] \to 144$
$a[3,1] \to 0$	$a[3,3] \to -48$	$a[3,5] \to -252$	$a[3,7] \to -1296$	$a[3,9] \to -5292$
$a[5,1] \to 0$	$a[5,3] \to 12$	$a[5,5] \to 492$	$a[5,7] \to 5184$	$a[5,9] \to 39552$
$a[7,1] \to 0$	$a[7,3] \to 0$	$a[7,5] \to -252$	$a[7,7] \to -15348$	$a[7,9] \to 227748$
$a[9,1] \to 0$	$a[9,3] \to 0$	$a[9,5] \to 36$	$a[9,7] \to -2916$	$a[9,9] \to -1007712$
$a[11,1] \to 0$	$a[11,3] \to 0$	$a[11,5] \to 0$	$a[11,7] \to 432$	$a[11,9] \to -1963752$

$X_8(1,1,2,4)$

$a[1,1] \to 16$	$a[1,3] \to 32$	$a[1,5] \to 144$	$a[1,7] \to 288$	$a[1,9] \to 800$
$a[3,1] \to 0$	$a[3,3] \to -144$	$a[3,5] \to -1536$	$a[3,7] \to -13056$	$a[3,9] \to -74016$
$a[5,1] \to 0$	$a[5,3] \to 32$	$a[5,5] \to 3504$	$a[5,7] \to 74016$	$a[5,9] \to 987712$
$a[7,1] \to 0$	$a[7,3] \to 0$	$a[7,5] \to -1536$	$a[7,7] \to -131344$	$a[7,9] \to -3889664$
$a[9,1] \to 0$	$a[9,3] \to 0$	$a[9,5] \to 144$	$a[9,7] \to 74016$	$a[9,9] \to 6076944$
$a[11,1] \to 0$	$a[11,3] \to 0$	$a[11,5] \to 0$	$a[11,7] \to -13056$	$a[11,9] \to -3889664$

$X_{24}(1,1,2,6,12)$

S,T,U model

Physically interesting

$a[1,1,1] \to 32$	$a[1,1,3] \to 320$	$a[1,1,5] \to 1824$	$a[1,1,7] \to 8000$
$a[1,3,1] \to 0$	$a[1,3,3] \to -1920$	$a[1,3,5] \to -76800$	$a[1,3,7] \to -1434240$
$a[1,5,1] \to 0$	$a[1,5,3] \to 320$	$a[1,5,5] \to 1674048$	$a[1,5,7] \to 180699840$
$a[1,7,1] \to 0$	$a[1,7,3] \to 0$	$a[1,7,5] \to -76800$	$a[1,7,7] \to -144582400$
$a[3,1,1] \to 0$	$a[3,1,3] \to 0$	$a[3,1,5] \to 0$	$a[3,1,7] \to 0$
$a[3,3,1] \to 0$	$a[3,3,3] \to -1824$	$a[3,3,5] \to -98304$	$a[3,3,7] \to -2211840$
$a[3,5,1] \to 0$	$a[3,5,3] \to 0$	$a[3,5,5] \to 327680$	$a[3,5,7] \to 27525120$
$a[3,7,1] \to 0$	$a[3,7,3] \to 0$	$a[3,7,5] \to -98304$	$a[3,7,7] \to -802258944$
$a[5,1,1] \to 0$	$a[5,1,3] \to 0$	$a[5,1,5] \to 0$	$a[5,1,7] \to 0$
$a[5,3,1] \to 0$	$a[5,3,3] \to 0$	$a[5,3,5] \to 0$	$a[5,3,7] \to 0$
$a[5,5,1] \to 0$	$a[5,5,3] \to 320$	$a[5,5,5] \to 296800$	$a[5,5,7] \to 31475520$
$a[5,7,1] \to 0$	$a[5,7,3] \to 0$	$a[5,7,5] \to -153600$	$a[5,7,7] \to -433073664$
$a[7,1,1] \to 0$	$a[7,1,3] \to 0$	$a[7,1,5] \to 0$	$a[7,1,7] \to 0$
$a[7,3,1] \to 0$	$a[7,3,3] \to 0$	$a[7,3,5] \to 0$	$a[7,3,7] \to 0$
$a[7,5,1] \to 0$	$a[7,5,3] \to 0$	$a[7,5,5] \to 0$	$a[7,5,7] \to 0$
$a[7,7,1] \to 0$	$a[7,7,3] \to 0$	$a[7,7,5] \to -98304$	$a[7,7,7] \to -71047712$

Higher dimensions?

$X_8(1,1,1,1,2,2)$

- mirror map produces integer numbers

$\{\{a[1] \to 192, a[3] \to 7727424, a[5] \to 1842113513280, a[7] \to 694520856180900672, a[9] \to$
$326365289149716415095552, a[11] \to 174638777858917520239006701888, a[13] \to$
$101821840062207643153782460776604992, a[15] \to 630963481647457778250071941714224691 40160, a[17]$
$409201246907152885132673321936499886614599 21088, a[19] \to 274917560393165740031457443028419 88$
$18998013695158230234850769075882328466073059534042130050752, a[23] \to 134342072796712939234 81$
$968359591473350848510101648105957792780868453206209804673741877595 1360, a[27] \to$
$70940512042198842699767102764597288076817720471986009705430103504257154 09152, a[29] \to$
$526956654463925506082084501088039589749172063496498384408821181695919512121545 2480, a[31] \to$
$3961606822180788923076174609511900919023575714462536019812732793267265299696693120454464, a$
$300973879966169030717787721624643378794317148388156908893924495629965992267446687529480540$
$230785417508587167685376689472117257863056793703930579055987573324659093141496032459343471$
$178427335977543175557200345226867742112500560130165708986972753115380495480868194654599825$
$138966576401846242801605245892677955237232019048567974058951712632790257609099634888626640$
$108951841021583212862521773449557076844108231246543945831848588323084942424329825769257041$
$859328699122086332997766978405569746584473136018401392702973261624280900786661480999784603$
$681471114784611288593427941293922866336111257632795743662701503743395313442353768507775933$
$543116344549344423876801890118655208661815426769540300711154371604336978161810748429438102$
$434825803051159897484539360361212795134413150469513558428860001119654745057036904277149499$
$349587729568688105153652967597858739554572412868995450536559690711098246808383610747324656$
$282146885426492838140923575585224193480431292110897244592905098582382999376107151150057081$
$2285324825109711497996679240317718251120931415756796061730788801058620815975919899988[illegible]7208$

- Relations to the geometries?
- should be continued