Instability with Chern-Simons Terms

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based on

arXiv:0911.0679 with
Shin Nakamura and Chang-Soon Park

and work in progress with Chang-Soon Park.

A constant electric field is a solution to the vacuum Maxwell equations.

$$\partial^\mu F_{\mu\nu} = 0$$

A constant electric field is a solution to the vacuum Maxwell equations.

The Chern-Simons terms abhor the constant electric field.

3 dimensions

$$\mathcal{L} = -\frac{1}{4} F \ast F + \frac{\alpha}{2} A \wedge F$$

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$$\Rightarrow \quad d{*F} + \alpha F = 0$$

$$\mathcal{L} = -\frac{1}{4} F * F + \frac{\alpha}{2} A \wedge F$$

$$\Rightarrow \quad d * F + \alpha F = 0$$

A constant electric field is not a solution.

$$d^*F + \alpha F = 0$$

$$\begin{aligned} \Box F &= d^* d^* F \\ &= -\alpha\, d^* F \\ &= \alpha^2 F \end{aligned}$$

$$\mathcal{L} = -\frac{1}{4} F \, {}^*F + \frac{\alpha}{2} A \wedge F$$

$$\Rightarrow \quad (\Box - \alpha^2) F = 0$$

Gauge field becomes massive.

Deser, Jackiw and Templeton (1982)

5 dimensions

$$\mathcal{L} = -\frac{1}{4} F * F + \frac{1}{3!} \alpha A F F$$

$$\mathcal{L} = -\frac{1}{4} F * F + \frac{1}{3!} \alpha \, A F F$$

Gauge field in the vacuum is massless.

A constant electric field is a solution.

$$\mathcal{L} = -\frac{1}{4} F * F + \frac{1}{3!} \alpha A F F$$

A constant electric field is a solution.

But, it is **unstable** (as I will show).

$$\mathcal{L} = -\frac{1}{4} F {}^{*} F + \frac{1}{3!} \alpha A F F$$

$$\Rightarrow \quad d {}^{*} F + \frac{1}{2} \alpha F \wedge F = 0$$

$$d^*F + \frac{1}{2}\alpha\, F \wedge F = 0$$

$$F = F^{(0)} + f$$

Linearize.

$$d^*f + \alpha\, F^{(0)} \wedge f = O(f^2)$$

Linearized equation with the Chern-Simons term:

$$d^* f + \alpha F^{(0)} \wedge f = 0$$

Take: $$F^{(0)}_{01} = E$$

Linearized equation with the Chern-Simons term:

$$d * f + \alpha F^{(0)} \wedge f = 0$$

Take: $F^{(0)}_{01} = E$

$$(\partial^\mu \partial_\mu + \partial^j \partial_j) f_i - 4 \alpha E \epsilon_{ijk} \partial_j f_k = 0$$

$$\left(\begin{array}{c} \mu, \nu = 0, 1 \quad , \quad i, j, k = 2, 3, 4 \\ f_i = \epsilon_{ijk} f_{jk} \end{array} \right)$$

$$(\partial^\mu \partial_\mu + \partial^j \partial_j) f_i - 4 \alpha E \epsilon_{ijk} \partial_j f_k = 0$$

momentum eigenstate $\quad p_{0,1} \quad k_{2,3,4}$

circular polarization

$$(\partial^\mu \partial_\mu + \partial^j \partial_j) f_i - 4\alpha E\, \epsilon_{ijk} \partial_j f_k = 0$$

momentum eigenstate $\quad p_{0,1} \quad k_{2,3,4}$

circular polarization

$$(p_0)^2 - (p_1)^2 = k^2 \pm 4\alpha E\, k$$

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$$(p_0)^2 - (p_1)^2 = k^2 \pm 4 \alpha E k$$

$$= (k \pm 2 \alpha E)^2 - 4 \alpha^2 E^2$$

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$$(p_0)^2 - (p_1)^2 = k^2 \pm 4 \alpha E k$$

$$= (k \pm 2 \alpha E)^2 - 4 \alpha^2 E^2$$

The fluctuarion is tachyonic for

$$0 < k < 4 \alpha E$$

$$\mathcal{L} = -\frac{1}{4} F * F + \frac{1}{3!} \alpha A F F$$

A constant electric field is unstable for

$$0 < k < 4 \alpha E$$

In contrast, a constant **magnetic** field is stable.

Gauge field fluctuations around it is massive.

The Chern-Simons terms abhor constant electric field.

In odd dimensions, the Chern-Simons term is induced by a massive electron in one-loop.

It is exact in the limit of large mass.

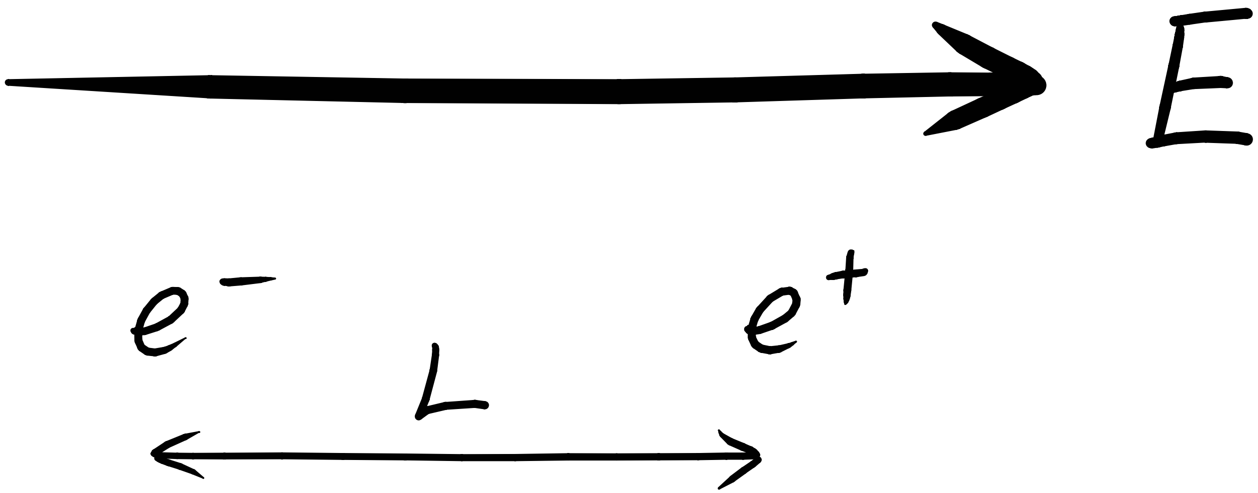
$$\log \det (i \not{\partial} + \not{A} + m)$$

$$= \frac{m}{|m|} \int_{\mathbb{R}^{2d,1}} A \wedge F^d + O\left(\frac{1}{m^2}\right)$$

Redlich (1984)

Schwinger mechanism:

A constant electric field can be screened by electron-positron pair creation.



$$qEL > 2mc^2$$

Schwinger mechanism:

A constant electric field can be screened by electron-positron pair creation.

The effect is stronger in lower dimensions.

AdS/CFT Correspondence

A charged black hole in AdS_5

is dual to

a conformal field theory in 4 dimensions

at non-zero temperature and chemical potential.

Condensed Matter Physics
Meets High Energy Physics

February 8 - 12 at IPMU

In the extremal limit (temperature = 0),

the near-horizon geometry of
the charged black hole in AdS_5

is

$AdS_2 \times R^3$

with electric field ~ volume form of AdS_2.

Chern-Simons terms abound in supergravity theories in AdS.

Instability of black holes in AdS

corresponds to

phase transition in dual CFT.

Things to be careful about:

(1) Stability conditions in AdS are different.

(2) Mixing of photons and graviton.

(1) Stability conditions in AdS are different.

Breitenlohner-Freedman bound

(1) Stability conditions in AdS are different.

instability range:

$$0 < k < 4\alpha E \quad \text{for } \mathbb{R}^{4,1}$$

$$\Downarrow$$

$$4\alpha E \gamma < k < 4\alpha E (1-\gamma)$$

for $AdS_2 \times \mathbb{R}^3$

$$2\gamma = 1 - \sqrt{1 - \frac{1}{16\alpha^2 E^2 R^2}}$$

AdS_2 radius

(1) Stability conditions in AdS are different.

Instability happens at non-zero momenta.

$$4\alpha E\gamma < k < 4\alpha E(1-\gamma)$$

for $AdS_2 \times \mathbb{R}^3$

$$2\gamma = 1 - \sqrt{1 - \frac{1}{16\alpha^2 E^2 R^2}}$$

AdS_2 radius

(1) Stability conditions in AdS are different.

In the near horizon geometry, $E\mathcal{R} = \sqrt{2}$.

Instability requires $\gamma < \frac{1}{2} \iff \alpha > \frac{1}{4\sqrt{2}}$

$$4\alpha E\gamma < k < 4\alpha E(1-\gamma)$$

for $AdS_2 \times \mathbb{R}^3$

$$2\gamma = 1 - \sqrt{1 - \frac{1}{16\alpha^2 E^2 \mathcal{R}^2}}$$

AdS_2 radius

(1) Stability conditions in AdS are different.

The Maxwell + Chern-Simons system in the near horizon geometry of the extremal charged black hole is unstable

$$\text{if} \quad \alpha > \frac{1}{4\sqrt{2}} .$$

c.f., for the minimal gauged supergravity in 5 dimensions,

$$\alpha = \frac{1}{2\sqrt{3}} > \frac{1}{4\sqrt{2}}$$

Things to be careful about:

(1) Stability conditions in AdS are different.

(2) Mixing of photons and graviton.

With the background electric field,

the gauge kinetic term causes the mixing.

$$g^{IJ} \, g^{KL} \, F_{IK} \, F_{JL}$$

The mixing raises the critical value
of the Chern-Simons coupling:

$$\alpha > 0.2896 \cdots$$

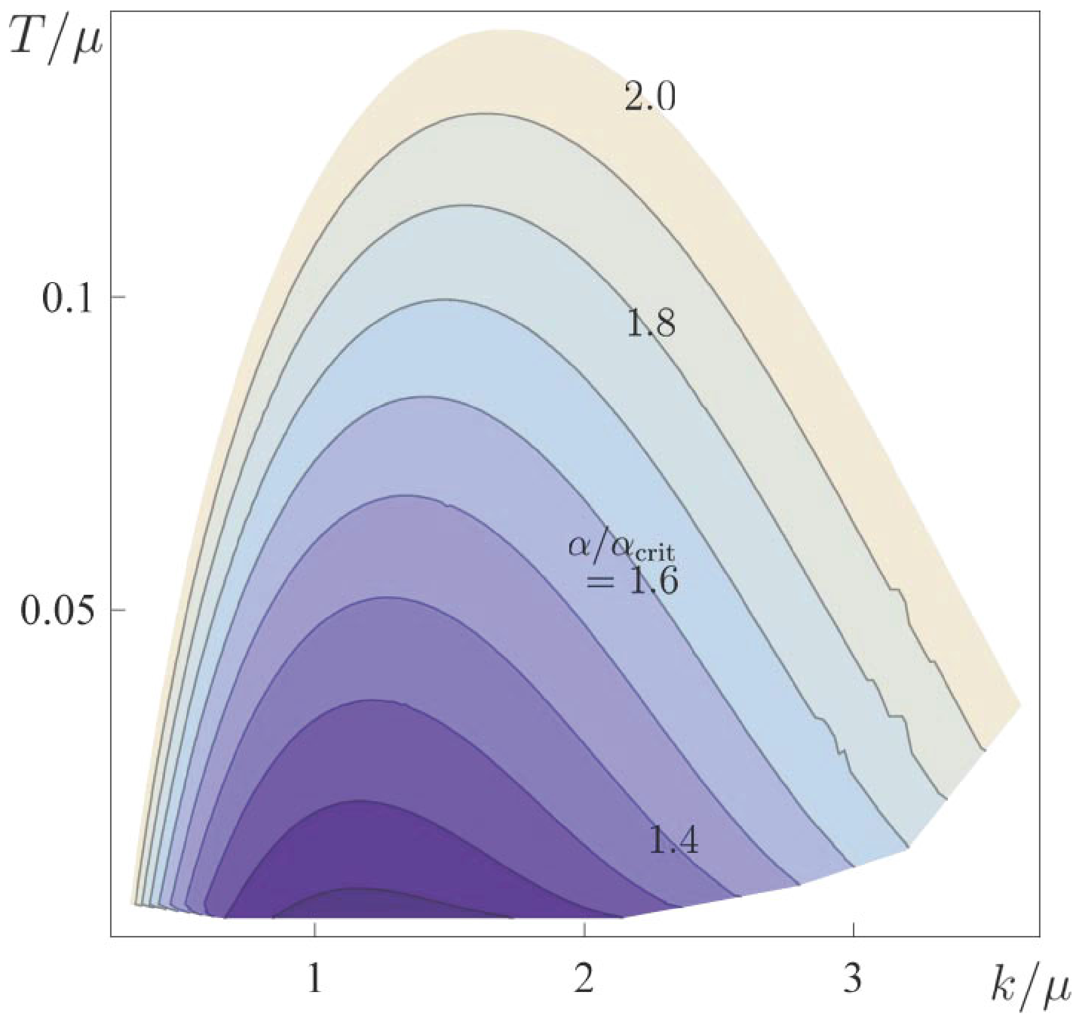
$$\text{c.f.} \quad \frac{1}{2\sqrt{3}} = 0.2887 \cdots$$

AdS_2 x R^3 is the near horizon geometry

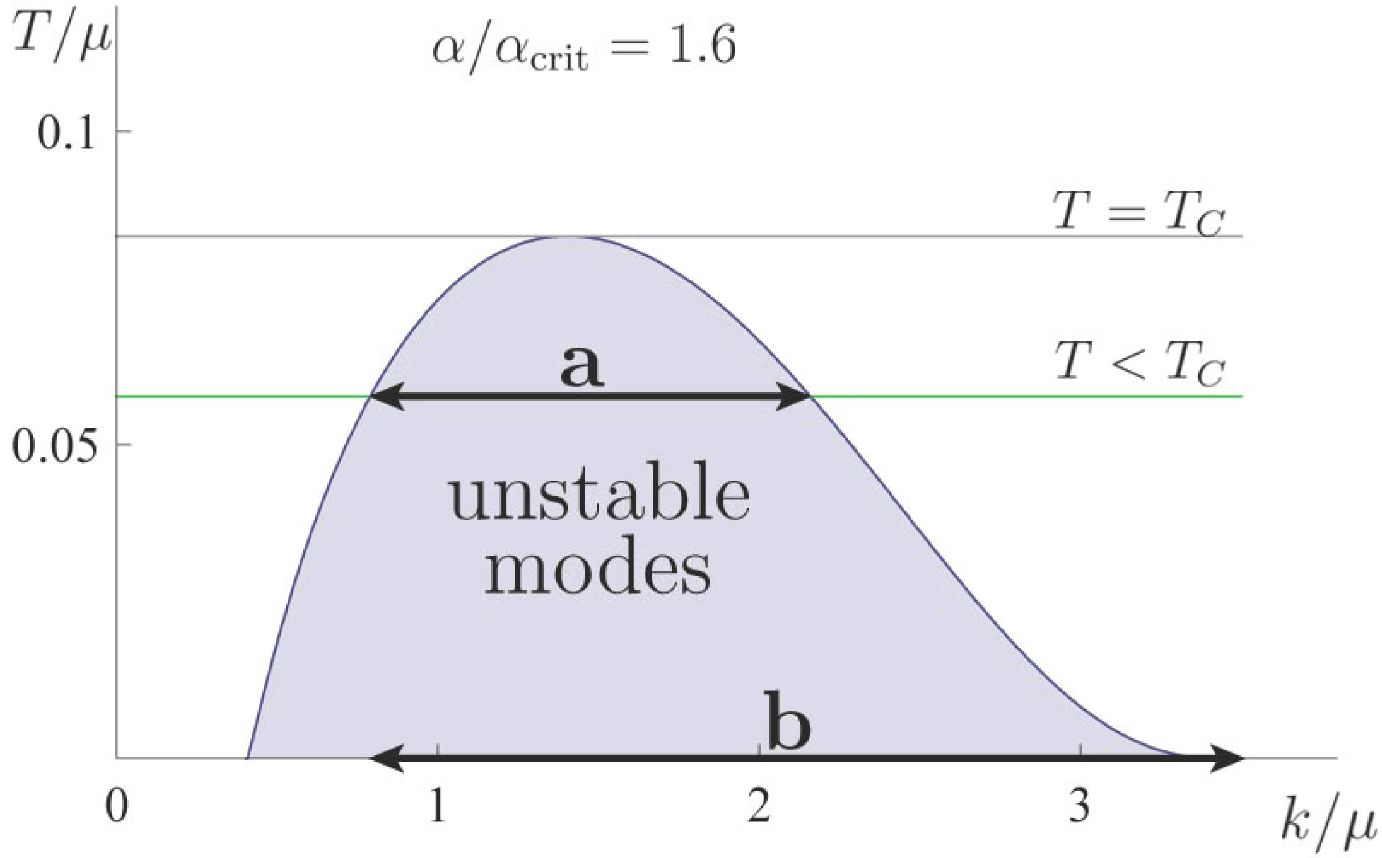
of the extremal black hole (T=0).

We also analyzed stability of

charged black holes with T > 0.

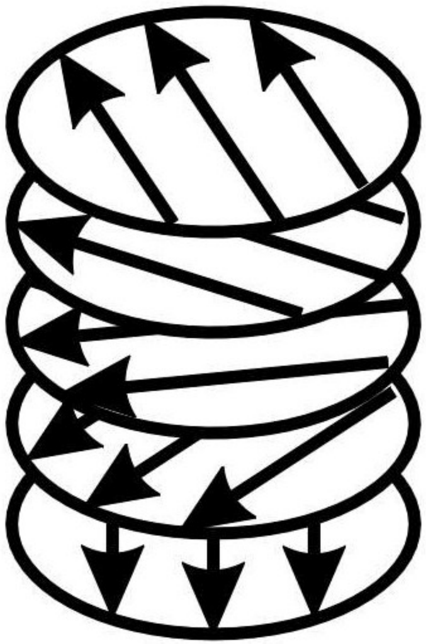


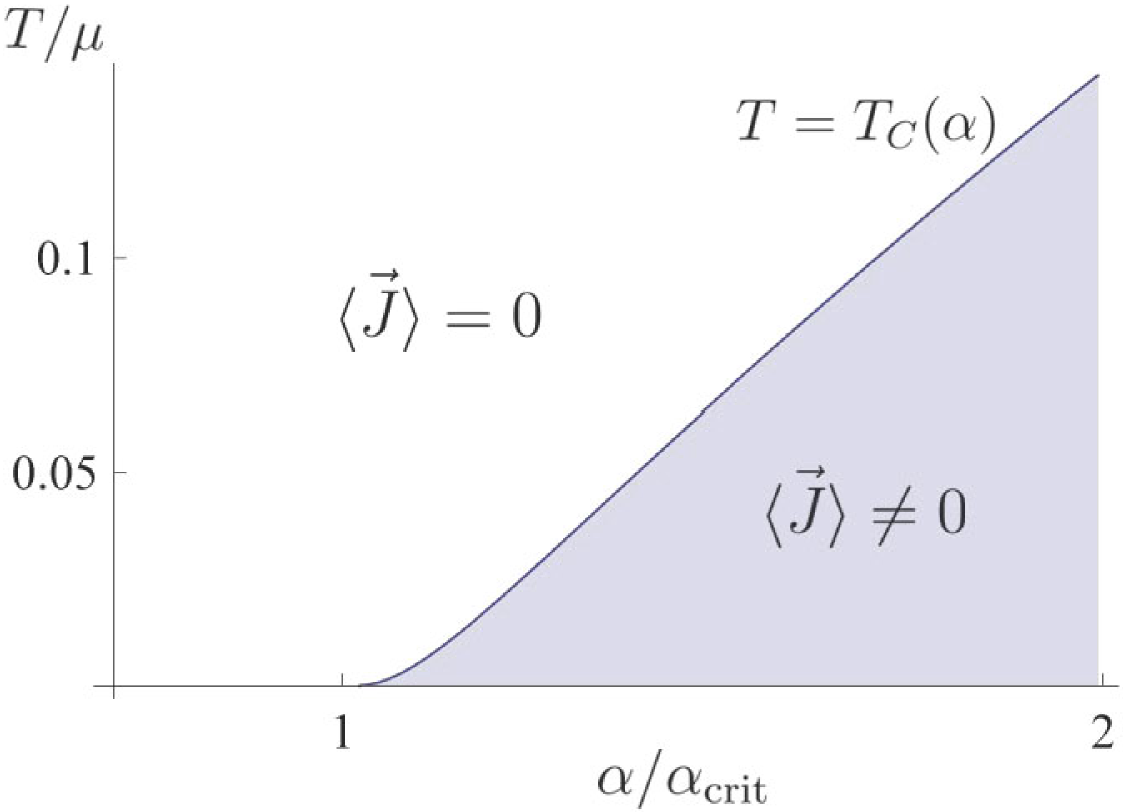
Unstable momenta at various values of the Chern-Simons coupling and temperature



Unstable momenta at \alpha = 1.6 \alpha_crit.

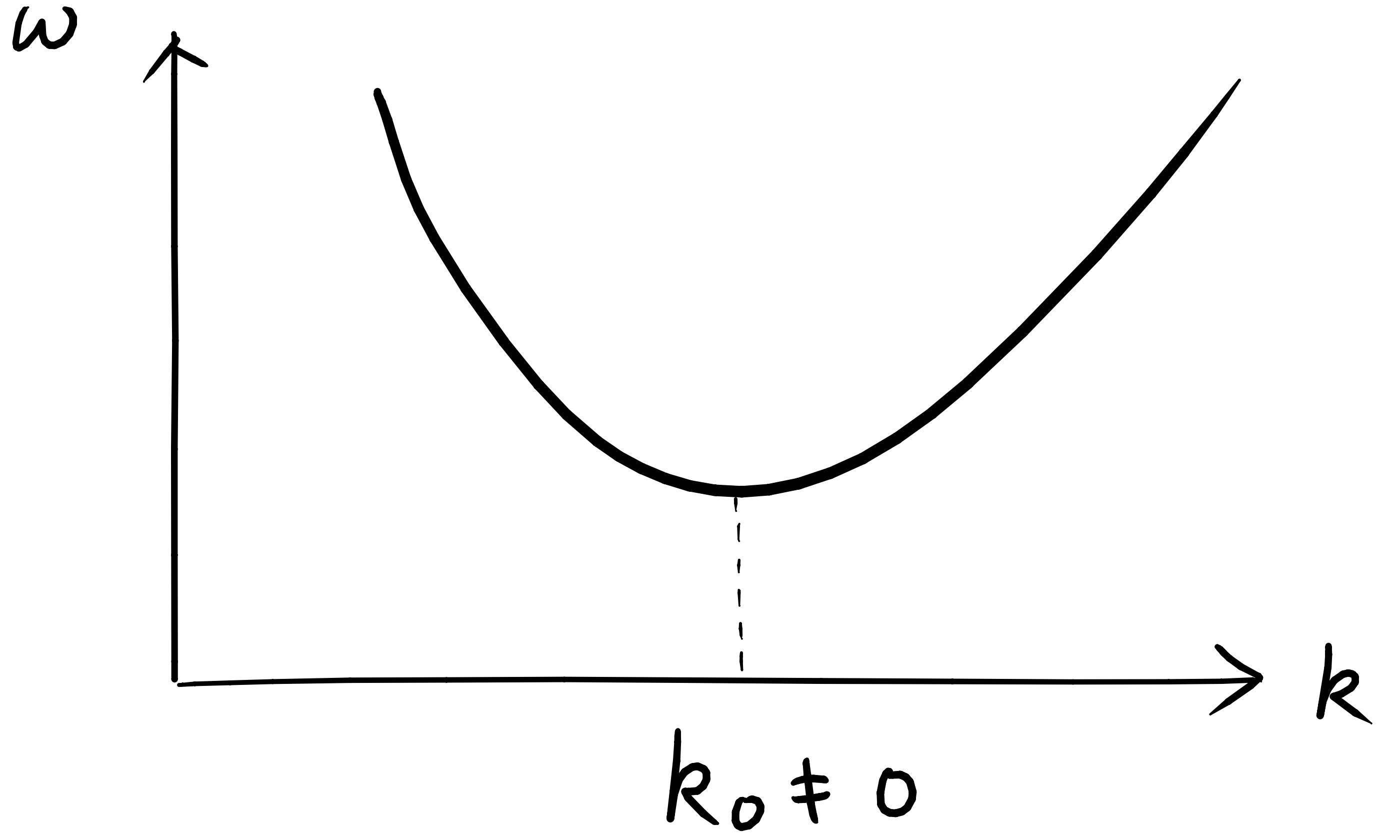
The range b is by the near horizon analysis at T=0.





The instability of the gauge field means that the corresponding current in the dual CFT acquires a vacuum expectation value.

Even below the critical Chern-Simons coupling, effects of the spatially modulated phase can be seen in the dispersion relation.



Since our analysis has been at the linearized level, what we have observed is an onset of phase transition.

To understand the nature of the new phase, it would be helpful to know more about non-linear solutions.

Work in progress:

We have found non-linear solutions to the Maxwell + Chern-Simons equation.

$$d^{*}F + \frac{1}{2}\alpha\, F \wedge F = 0$$

Some of them are stable.

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$$d^*F + \frac{1}{2}\alpha F \wedge F = 0$$

Some of them are stable.

We also need to consider coupling to the gravity.

Spatially modulated phases are known
in condensed matter physics and in QCD.

e.g., Fulde-Ferrell-Larkin-Ovchinnikov

involving Cooper pair of two species of
fermions with different Fermi momenta.

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e.g., Cholestric phase in liquid crystal

The Chern-Simons coupling in the bulk

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the chiral anomaly in the dual CFT.

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This correspondence has turned out
to have important implications on
the hydrodynamical regime of the CFT.

The Chern-Simons coupling in the bulk

corresponds to

the chiral anomaly in the dual CFT.

How does the chiral anomaly causes the spatially modulated phase transition?

風間様

還暦

おめでとう
ございます。

益々
活躍ください。