Results from lattice QCD with exact chiral symmetry

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Recent Developments in Strings and Fields
on the occasion of Y. Kazama's 60^{th} birthday

arXiv:0911.5555 (Fukaya et al.:JLQCD collaboration)

Outline

1. Chiral symmetry breaking in QCD
2. Banks-Casher relation
3. Partially quenched Chiral Pertubation Theory
4. Lattice Simulation
5. Conclusion

1. Chiral symmetry breaking in QCD

Lattice QCD

= A regularization of QCD with discrete spacetime

In recent years, lattice QCD with exact chiral symmetry has become feasible

Main topic of this talk

We can study the dynamics of QCD

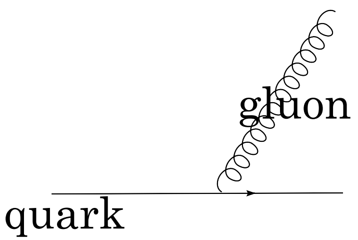
- Chiral symmetry breaking
- Determinations of parameters of QCD
- Hadron structure
- Weak matrix elements

quantitatively using numerical method.

One of the fundamental questions in QCD

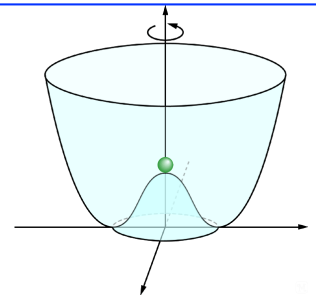
(Quantitative) study of chiral symmetry breaking (χSB) from the 1st principles of QCD

QCD



?

χSB and Chiral Lagrangian



NEWS@KEK

$$\langle \bar{\psi}\psi \rangle = -\Sigma \neq 0$$

$$L_{ChPT} = \frac{F^2}{4} \mathrm{Tr}\left[\partial_\mu U^\dagger \partial_\mu U\right] - \frac{\Sigma}{2} \mathrm{T}r\left[\mathcal{M}^\dagger U + U^\dagger \mathcal{M}\right] + \cdots$$

Chiral Pertubation Theory (ChPT)

Determine low energy const (LEC)

Important first step further application

However, life is not easy.

In order to show the χSB, we need to

1. Compute $\langle\bar{\psi}\psi\rangle$ with finite lattice spacing a, quark mass m, and volume $V \equiv L^4$
2. Take the infinite volume first,
3. Then take the chiral limit
4. Then take the continuum limit.

$$\langle\bar{\psi}\psi\rangle = \lim_{a\to 0}\lim_{m\to 0}\lim_{V\to 0}\langle\bar{\psi}\psi\rangle_{V,m,a}$$

Can we learn simulations with finite m, V ?
Yes, in principle, but it is not so trivial.

Divergence problem

Scalar operator mixes with divergent identity operator.

$$\left(\bar{\psi}\psi\right)^{\mathrm{lat}}(a) = \left[\cancel{c_1\frac{1}{a^3}} + c_2\frac{m}{a^2} + \cancel{c_3\frac{m^2}{a}} + c_4 m^3\right]\mathbf{1}$$
$$+ \quad Z\left(a\mu, g_0(a), am\right)\left(\bar{\psi}\psi\right)^{\overline{\mathrm{MS}}}(\mu) + O(a)$$

Absent due to chiral transformation property

The <u>divergence</u> is much larger than the <u>condensation</u> for typical values of a, m, V

$\longrightarrow$ Naïve method does not work

2. Banks-Casher relation

A method to avoid the divergence

Consider eigenvalues of the Dirac opeator

$$(D+m)\,\phi_n = (i\lambda_n + m)\phi_n$$

The chiral condensate can be written as

$$\left\langle \bar{\psi}(x)\psi(x) \right\rangle = -\frac{1}{V}\left\langle \mathrm{Tr}\left[\frac{1}{D+m}\right]\right\rangle = -\frac{2}{V}\left\langle \sum_k \frac{1}{m+i\lambda_k}\right\rangle$$

Defining the spectral density as $\rho(\lambda) \equiv \frac{1}{V}\langle \sum_k \delta(\lambda - \lambda_k) \rangle$,

$$\left\langle \bar{\psi}(x)\psi(x) \right\rangle = -2m \int_0^\infty d\lambda \frac{\rho(\lambda)}{m^2+\lambda^2}$$

→ The condensate can be described by the spectral density

$$\boxed{\begin{array}{l}\text{Banks-Casher relation}\\ \lim_{a\to 0}\lim_{m\to 0}\lim_{V\to 0}\langle\bar{\psi}\psi\rangle_{V,m,a} = -\pi\rho(0)\end{array}}$$

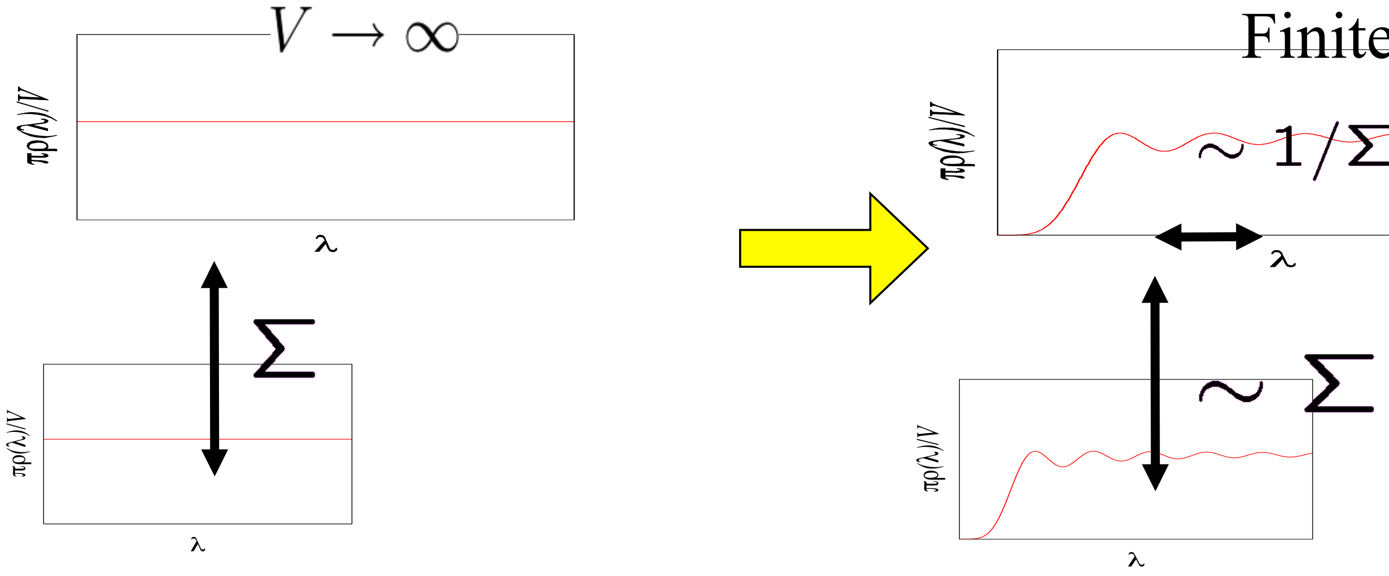
It is possible to study finite V, m effect, assuming equivalence with Chiral Random Matrix Theory (ChRMT). Damgaard Nishigaki (1998) : Leading Order (LO) in ε -regime

ε -regime: $1/\Lambda_{\rm QCD} \ll L \ll 1/m_\pi$

expansion parameter $\epsilon^2 \sim m_\pi/\Lambda_{\rm QCD} \sim p^2/\Lambda^2_{\rm QCD} \sim 1/(L\Lambda_{\rm QCD})^2$

cf. p-regime: $1/\Lambda_{\rm QCD} \ll 1/m_\pi \leq L$

expansion parameter $m^2_\pi/\Lambda^2_{\rm QCD} \sim p^2/\Lambda^2_{\rm QCD} \sim 1/(L^2\Lambda^2_{\rm QCD})$



Loop correction in ε-regime

$$\text{corr.} \sim \frac{1}{(4\pi F)^2}\sum_p \left(\frac{2\pi}{L}\right)^4 \frac{1}{p^2+m_\pi^2}$$

$$= \left(\frac{\frac{2\pi}{L}}{4\pi F}\right)^4 \sum_p \frac{1}{\left(\frac{p}{2\pi F}\right)^2 + \left(\frac{m_\pi}{2\pi F}\right)^2}$$

$$\sim \epsilon^4 [\underbrace{\frac{1}{\epsilon^4}}_{p=0 \text{ pion}} + \underbrace{\frac{1}{\epsilon^2+\epsilon^4}}_{p\neq 0 \text{ pion}}] = O(1) + O(\epsilon^2)$$

All order correction from zero mometum pion is needed.

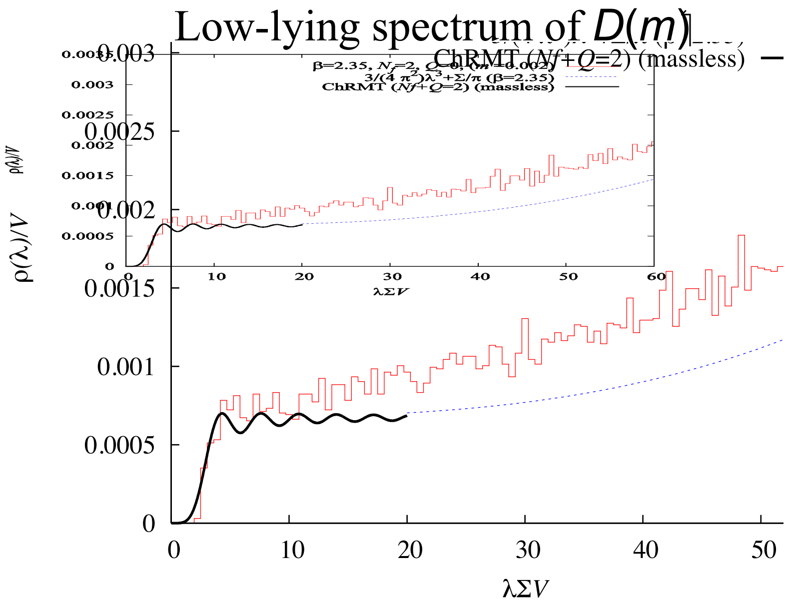
Special counting rule in ε-regime.

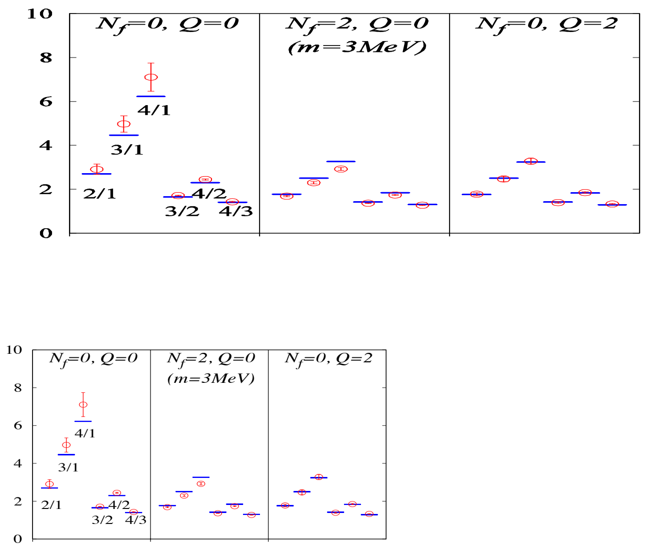
Elaborate ChPT to treat the zero-mode exactly using matrix theory technique. For microscopic eigenvalue spectrum , ChRMT (which is believed to be in the same universality class) is useful.

Previous study by JLQCD/TWQCD

(already presented at Komaba 2007) Fukaya, T.O. et al. (2007);

ε -regime: Nf=2, 16^3x32, a=0.11fm , m~3MeV





$$\Sigma^{\overline{MS}}(2\,\mathrm{GeV}) = (251 \pm 7(\mathrm{stat}) \pm 11(\mathrm{syst})\,\mathrm{MeV})^3$$

That was nice, but still unsatisfactory.

- We assumed that ChRMT is in the same universality class as ChPT. Direct comparison of ChPT and QCD has not been made.

- We see some deviation for larger values of λ. We hope the deviation can be explained by NLO effect but we do not know since ChRMT cannot be extended to higher order effects in 1/F.

3. Partially quenched Chiral Perturbation Theory

How to go beyond Leading Order?

Idea: extend QCD to partially quenched QCD (=pqQCD).

Smilga-Stern(93), Osborn-Toublan-Verbaarschot(99)

$$L = \sum_{i=1}^{N_f} \underbrace{\bar{q}_i(D + m_i)q_i}_{\text{dynamical quark}} + \underbrace{\sum_{j=1}^{N_v} (\underbrace{\bar{q}_v(D + m_v)q_v}_{\text{valence quark}} + \underbrace{\bar{Q}_v(D + m_v)Q_v}_{\text{ghost quark}})}_{\text{Determinant cancels}}$$

- Same dynamics as QCD (same partition fuunction)
- More observable if we include valence quark op.

$$\begin{tikzcd}[column sep=huge, row sep=huge] \boxed{\text{QCD}} \arrow[r, leftrightarrow, "\text{Same partition function}"'] \arrow[d] & \boxed{\text{pqQCD}} \arrow[d] \\ \boxed{SU(N_f)\ \text{ChPT}} \arrow[r, leftrightarrow, "\begin{array}{c}\text{Same partition function}\\ \text{Common LECs}\end{array}"'] & \boxed{SU(N_f+N_v|N_v)\ \text{ChPT}} \end{tikzcd}$$

Generalization of Banks Casher relation

Smilga-Stern (1993), Osborn-Toublan-Verbaarschot (1999)

$$\begin{aligned}
\rho^{QCD}_{m,V}(\lambda) &\equiv \frac{1}{V}\langle \sum_k \delta(\lambda+\lambda_k)\rangle^{QCD}_{m,V} \\
&= -\frac{1}{2\pi V}\sum_k \lim_{\epsilon\to 0}\left\langle \frac{1}{i(\lambda+\lambda_k)-\epsilon} - \frac{1}{i(\lambda+\lambda_k)+\epsilon}\right\rangle^{QCD}_{m,V} \\
&= \frac{1}{2\pi}\lim_{\epsilon\to 0}\left[\langle \bar{q}_v q_v\rangle^{pqQCD}_{m,m_v,V}\Big|_{m_v=i\lambda-\epsilon} - \langle \bar{q}_v q_v\rangle^{pqQCD}_{m,m_v,V}\Big|_{m_v=i\lambda+\epsilon}\right] \\
&= \frac{1}{2\pi}\lim_{\epsilon\to 0}\left[-\frac{d}{dm_v}\log(Z^{pqQCD}_{m,m_v,V})\Big|_{m_v=i\lambda-\epsilon} + \frac{d}{dm_v}\log(Z^{pqQCD}_{m,m_v,V})\Big|_{m_v=i\lambda+\epsilon}\right]
\end{aligned}$$

Important point

- Power divergence cancels in $\varepsilon \to 0$ limit.
- R.H.S is computable using pqChPT in ε -regime.
- L.H.S is obtained from ordinary lattice QCD.

NLO calculation in pQChPT

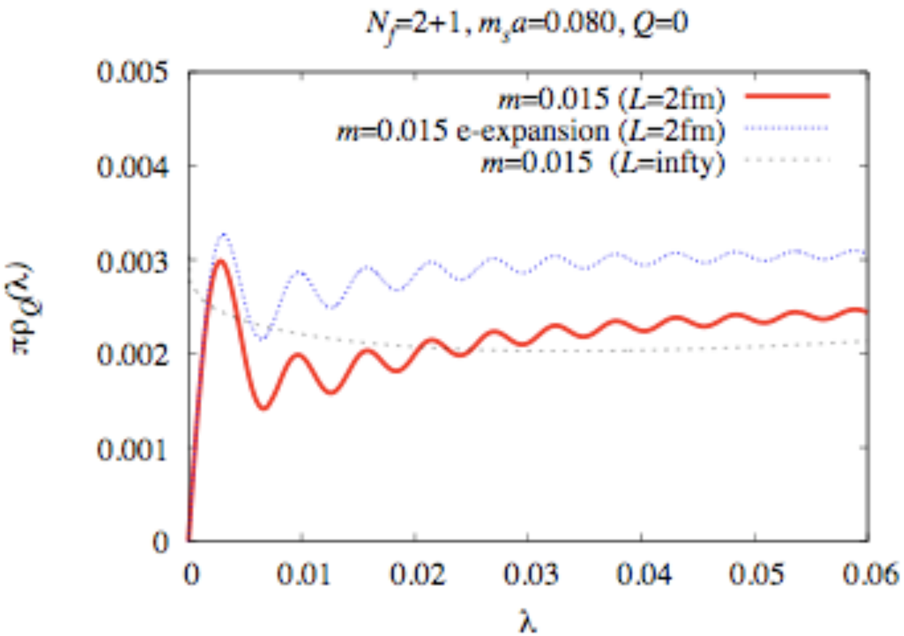
$$
\begin{aligned}
L_{\text{pqChPT}} \equiv & \frac{F^2}{4}\text{Tr}\left[\partial_\mu U^\dagger \partial_\mu U\right] - \frac{\Sigma}{2}\text{Tr}\left[\mathcal{M}^\dagger U + U^\dagger \mathcal{M}\right] \\
+ & L_4 \frac{2\Sigma}{F^2}\text{Tr}\left[\partial_\mu U^\dagger \partial_\mu U\right] \times \text{Tr}\left[\mathcal{M}^\dagger U + U^\dagger \mathcal{M}\right] \\
+ & L_5 \frac{2\Sigma}{F^2}\text{Tr}\left[\partial_\mu U^\dagger \partial_\mu U\left(\mathcal{M}^\dagger U + U^\dagger \mathcal{M}\right)\right] \\
- & L_6\left(\frac{2\Sigma}{F^2}\text{Tr}\left[\mathcal{M}^\dagger U + U^\dagger \mathcal{M}\right]\right)^2 - L_7\left(\frac{2\Sigma}{F^2}\text{Tr}\left[\mathcal{M}^\dagger U - U^\dagger \mathcal{M}\right]\right)^2 \\
- & L_8\left(\frac{2\Sigma}{F^2}\right)^2 \text{Tr}\left[\mathcal{M}^\dagger U \mathcal{M}^\dagger U + U^\dagger \mathcal{M} U^\dagger \mathcal{M}\right] - H_2\left(\frac{2\Sigma}{F^2}\right)^2 \text{Tr}\left[\mathcal{M}^\dagger \mathcal{M}\right]
\end{aligned}
$$

NLO Result: Damgaard-Fukaya

$$
\rho_Q(\lambda) = \Sigma_{\text{eff}}\hat{\rho}^{\epsilon}_Q(\lambda\Sigma_{\text{eff}}V, m\Sigma_{\text{eff}}V) + \rho^p(\lambda, m)
$$

Analytic function of $\Sigma_{\text{eff}}, F, L_6, m, m_v$

Σ_{eff} is a function of Σ, F, L_6, m, m_v



2. Lattice simulation

Chiral symmetry on the lattice

- Nielsen-Ninomiya's theorem

  Nielsen and Ninomiya, Nucl.Phys.B185(1981) 20

$$S_F = \bar{\psi} D \psi \qquad D\gamma_5 + \gamma_5 D = 0$$

Chiral symmetric fermion on the lattice with reasonable assumptions have doublers

- Wilson fermion :

  broken chiral symmetry, symmetry recovered in continuum.

- Staggered fermion: 4 spinors $\otimes$ 4 "tastes" (doublers)

  to apply QCD (u,d,s) one must take "the fourth root trick"

  Very dangerous compromise! Even locality is doubtful.

$$det(D) \sim det(D_{staggered})^{1/4}$$

Ginsparg-Wilson fermion

- Ginsparg-Wilson relation

Ginsparg and Wilson, Phys.Rev.D 25(1982) 2649.

$$D\gamma_5 + \gamma_5 D = aD\gamma_5 D$$

Exact chiral symmetry on the lattice (index theorem)

Hasenfratz, Laliena and Niedermayer, Phys.Lett. B427(1998) 125

Luscher, Phys.Lett.B428(1998)342.

$$\begin{aligned} \psi &\rightarrow \psi + i\theta\gamma_5(1 - aD)\psi = \psi + i\theta\widehat{\gamma_5}\psi \\ \bar{\psi} &\rightarrow \bar{\psi} + i\theta\bar{\psi}\gamma_5 \end{aligned}$$

- Overlap fermion (explicit construction)

$$D \equiv \frac{1}{a}\left[1 + \gamma_5 \epsilon(H_W)\right], \qquad H_W \equiv \gamma_5(D_W + M_0)$$

D_W : Wilson Dirac op., $\quad M_0$: negative mass

JLQCD+TWQCD collaborations

- JLQCD
  - SH, H. Ikeda, T. Kaneko, H. Matsufuru, J. Noaki, N. Yamada (KEK)
  - H. Fukaya (Nagoya)
  - T. Onogi, E. Shintani (Osaka)
  - H. Ohki (Kyoto)
  - S. Aoki, N. Ishizuka, K. Kanaya, Y. Kuramashi, K. Takeda, Y. Taniguchi, A. Ukawa, T. Yoshie (Tsukuba)
  - K. Ishikawa, M. Okawa (Hiroshima)

- TWQCD
  - T.W. Chiu, T.H. Hsieh, K. Ogawa (National Taiwan Univ)

- Machines at KEK (since 2006)
  - BlueGene/L (10 racks, 57.3 Tflops)

Parameters

- Nf=2+1 lattice QCD with overlap fermion
-
- β=2.30 (Iwasaki gauge action),

- a=0.11 fm, 16^3x48、24^3x48

- p-regime run: 5 ud quark masses in the range $m_s/6 \sim m_s$
- e-regime run: ud quark mass at m=0.002 ($m_q \sim 3$ MeV)

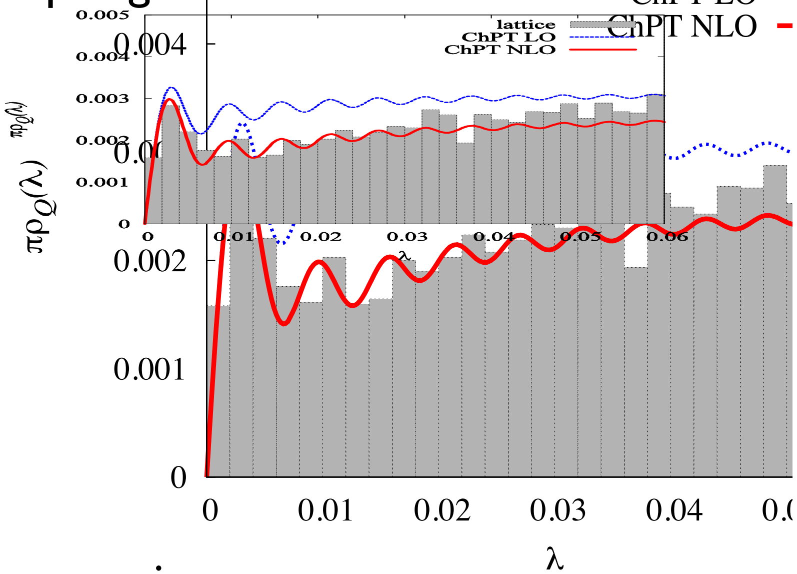
- s quark masses 2points

- Topological charge Q=0, 2

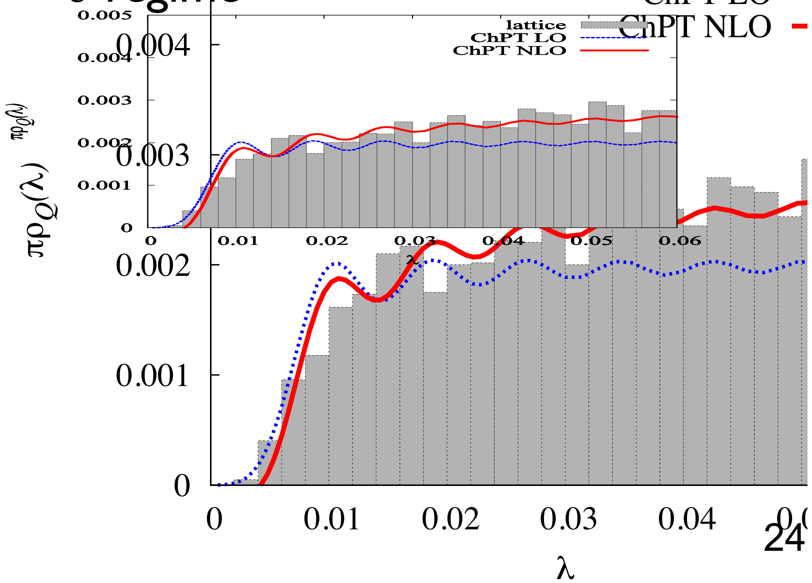
Our result (JLQCD 2009 arXiv:0911.5555)

- Comparison of the spectral funtion from lattice QCD, L=16^3 x48
- sea quark masses are in the the p-regime and e-regime.
- Fit of the shape with the pqChPT formula(Σ, F, L_6). is quite successful.

p-regime



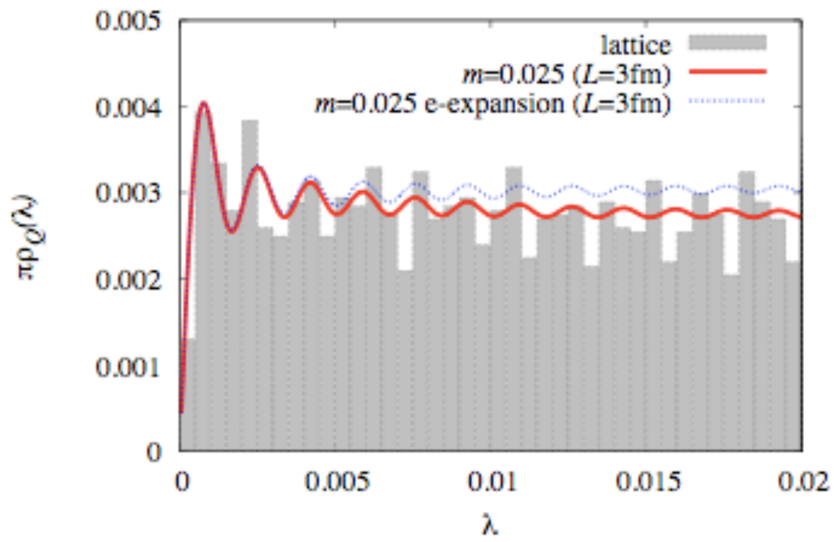
ε-regime



- 24^3x48 lattice (L ~ 2.6 fm) can be fitted with the same set of parameters as 16^3x48

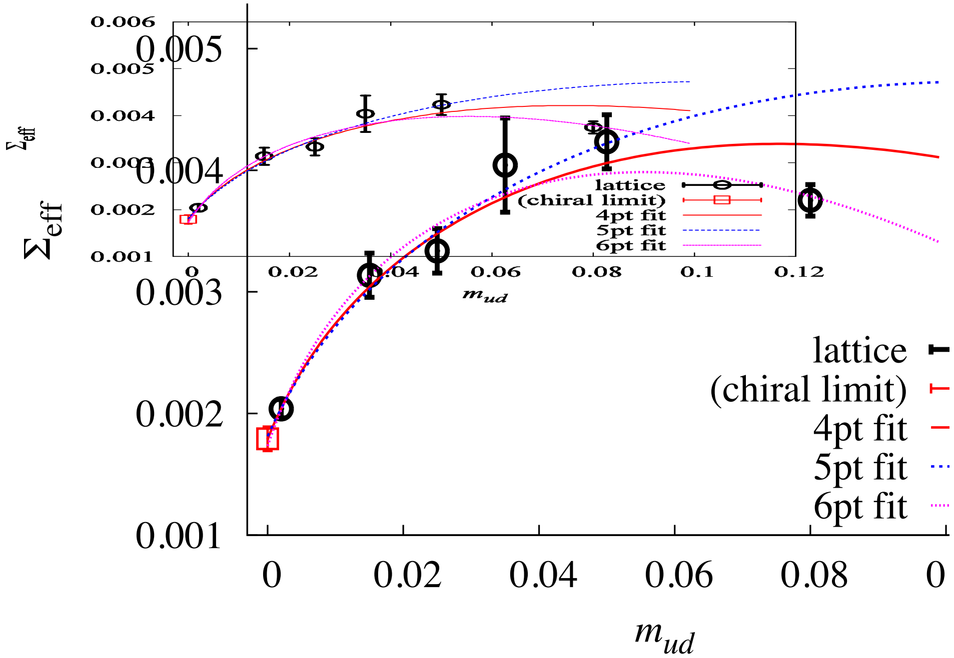
Finite volume effect well under control.

24^3x48



Chiral extrapolation

- Sea quark mass dependence of Σ



- Prediction from ChPT

$$\Sigma(m_{ud}, m_s) = \Sigma(0, m_s) \times \left[1 - \frac{3M_\pi^2}{32\pi^2 F^2} \ln \frac{M_\pi^2}{\mu^2} + \frac{32 L_6 M_\pi^2}{F^2} \right]$$

Σ, F, L_6 can be determined

$$\begin{aligned} \Sigma^{\overline{\mathrm{MS}}}(2(GeV)) &= \left[242(4)(^{+19}_{-18})\ \mathrm{MeV}\right], \\ F &= 74(1)(8)\ \mathrm{MeV}, \\ L_6^r(770\ \mathrm{MeV}) &= -0.00011(25)(11). \end{aligned}$$

scale input :

$$r_0 = 0.49\ \mathrm{fm},$$

Renormalization:

$$Z_S^{-1}(2\ \mathrm{GeV}) = 0.806(12)(^{+24}_{-26}).$$

Other topics from the project

1. Aoki et al., "*Topological susceptibility in two-flavor QCD…*," Phys. Lett. B665, 294 (2008).

2. Shintani et al. "*S-parameter and pseudo NG boson mass…*," Phys. Rev. Lett. 101, 242001 (2008); arXiv:0806.4222 [hep-lat].

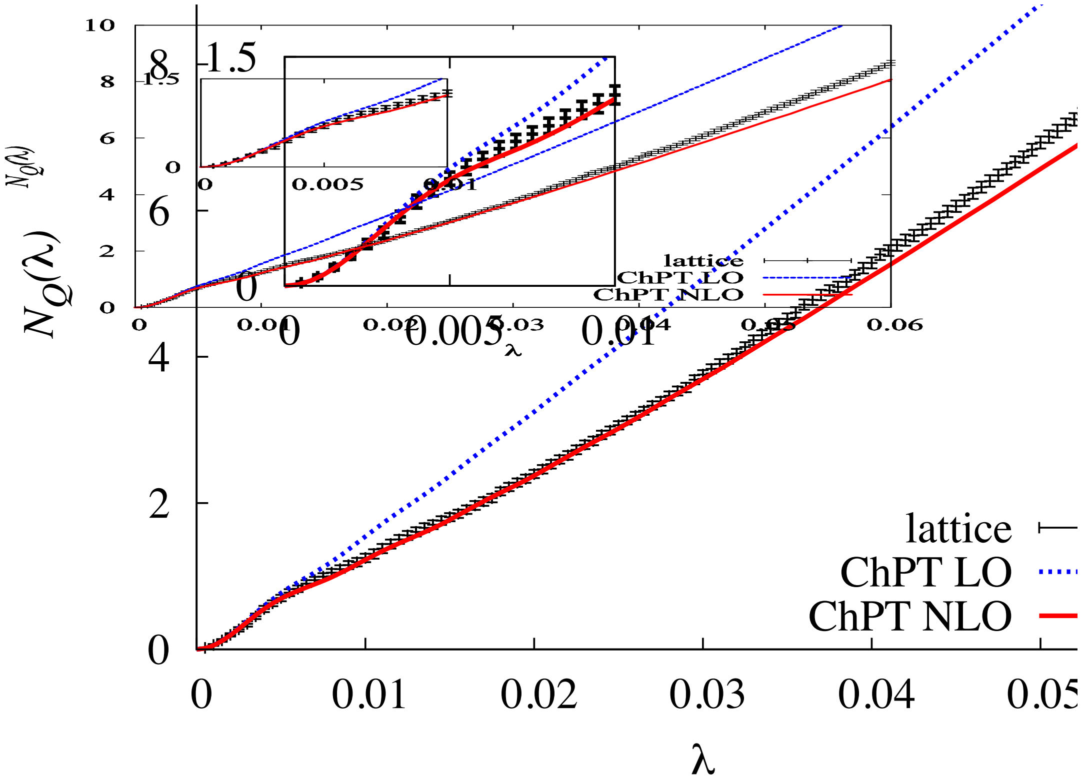
3. Ohki et al., "*Nucleon sigma term and strange quark content…*," Phys. Rev. D **78**, 054502 (2008); arXiv:0806.4744 [hep-lat] .

4. Shintani et al., "*Lattice study of the vacuum plarization functions and …*," Phys. Rev. D **79**, 074510 (2009); arXiv: 0807.0556 [hep-lat].

Conclusion

- Chiral symmetry breaking from the 1st principle QCD is now possible at the quantitative level with lattice QCD with exact chiral symmetry.
- With partially quenching we can make a direct link of QCD and ChPT and determine the low energy constant reliablly.
- With exact chiral symmetry on the lattice, we can now attack new problems in QCD.
  – Peskin-Takuchi S-parameter from vacuum polarization functions (VPF)
  – Strong coupling constant from VPF.
  – Nucleon sigma term (strange quark content)

Happy birthday, Kazama-san !



lattice
ChPT LO
ChPT NLO
$N_Q(\lambda)$
λ