# Yang-Mills as a holographic dual of string theory in flat space

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# Plan

- Motivation and goal
- Previous works, formulation of the problem, subtleties and obstacles
- Our results
- Discussion
- Conclusions

# What do I mean by "the holographic dual of string theory in flat space"?

- AdS/CFT :

**N=4 super Yang-Mills** **Type IIB strings on $AdS_5 \times S^5$**

- Yang-Mills as a non-perturbative definition of string theory in AdS.
- What about the textbook string theory defined in flat Minkowski space?

# Goal: flat space limit of AdS/CFT

- Can we take the flat space limit?
- This was discussed by Polchinski (1999) and Susskind (1999) soon after AdS/CFT was discovered.
- I will review other works.

# Other holographic formulations of string/M-theory in flat space

- M(atrix) theory (BFSS)
- Type IIB matrix model (IKKT)
- Matrix string theory

Common feature: Infinite N,<br>N is a regulator

Also true for the flat space limit of AdS/CFT

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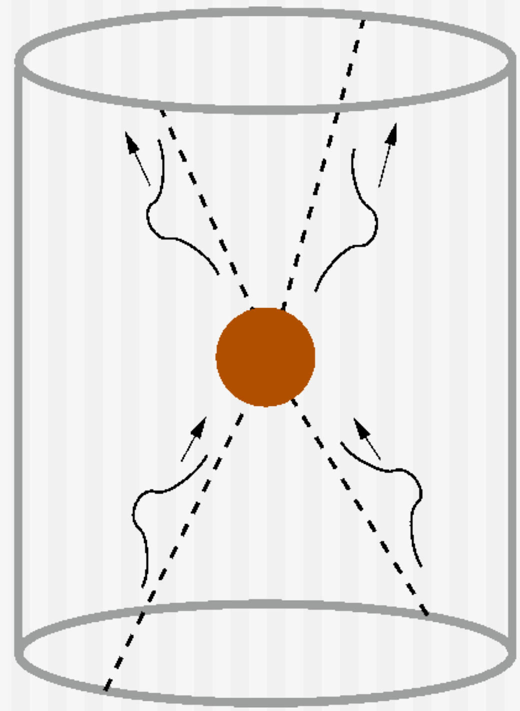
# Previous works on the flat space limit of AdS/CFT

- Polchinski (1999) and Susskind (1999)
    - Formulation of the problem

# Formulation of the problem: the flat space limit of AdS/CFT

The 4-point function is given by the bulk-to-boundary propagators and the amputated Green's function.



$$\langle \mathcal{O}_1(x_1) \ldots \mathcal{O}_4(x_4) \rangle$$

$$= \int \prod_{i=1}^{4} [dy_i] K_{B\partial}(x_i, y_i) G_{\text{amp}}(y_1, y_2, y_3, y_4)$$

# Previous works on the flat space limit of AdS/CFT

- Polchinski (1999) and Susskind (1999)
  - Formulation of the problem
  - Obtained a relation between the 4-point function and a scattering amplitude in flat space
  - Potential relevance of OPE singularity
  - Schematic
  - Could have gone further

# Previous works on the flat space limit of AdS/CFT

- Giddings, Balasubramanian and Lawrence (1999), Giddings (1999)
  - Potential obstacles (periodicity of geodesics, growth of amplitudes near boundary)
  - Need projection to avoid growing modes
- Jevicki and Nastase (2005)
  - pp-wave and flat space limit of AdS
  - Double limit AdS → pp-wave → flat

# Previous works on the flat space limit of AdS/CFT

- Gary, Giddings and Penedones (2009)
  - Construction of well-localized wave packets.
  - Explicit relation between singularity in a CFT correlator and scattering amplitudes for effective field theory in flat space.
  - Relation reproduces scattering amplitudes in effective field theory from known correlators
- Gary and Giddings (2009)
  - Cautionary remarks about power-law tails of the wave packets.

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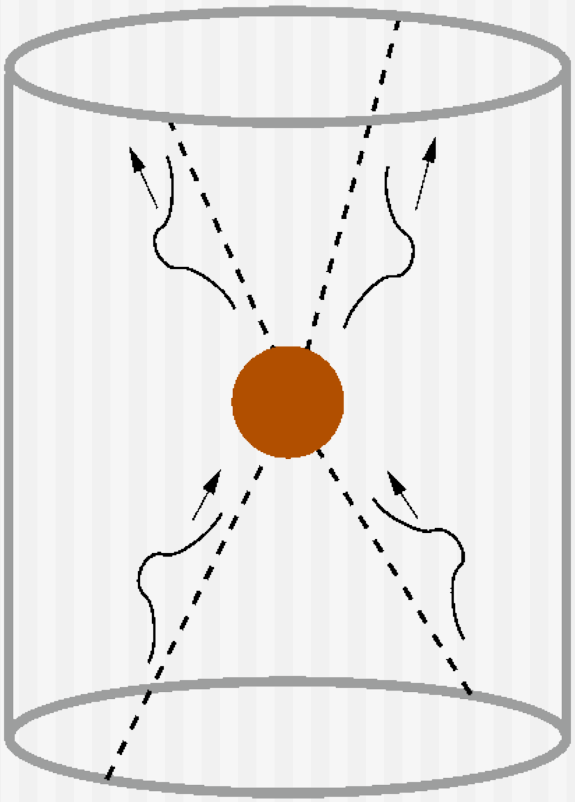
# Derivation of the relation
(Penedones and T.O.)

- Choose sources $J_i(x_i)$ with a compact support to produce wave packets

$$\Psi_i(y) = \int [dx] J_i(x) K_{B\partial}(x,y)$$

- In the flat limit the wave packets are well localized in AdS so that the interaction only occurs in a small flat region.



# Derivation of the relation

(Penedones and T.O.)

$$\int \prod_{i=1}^{4} [dx_i] J_i(x_i) \langle \mathcal{O}_1(x_1) \ldots \mathcal{O}_4(x_4) \rangle$$

$$= \int \prod_{i=1}^{4} [dy_i] \Psi_i(y_i) G_{\mathrm{amp}}(y_1, y_2, y_3, y_4)$$

In the limit and in the flat region, $\Psi_i(y_i)$ essentially becomes a plane wave, and $G_{\mathrm{amp}}$ gives the scattering amplitude by LSZ.

# Derivation of the relation
(Penedones and T.O.)

To account for all the momentum conserving delta functions on the gravity side, the four point function has to show a singular behavior at $\det x_{ij} = 0$ .
(similar but different from OPE)

A scaling variable $$\zeta^2 \sim \sqrt{g_{YM}^2 N} \frac{\det x_{ij}}{x_{12}^2 x_{34}^2 x_{13}^2 x_{24}^2}$$

Scaled correlator for dilaton scattering

$$\mathcal{F}(\zeta) \overset{\text{define}}{\sim} \lim_{\substack{N \to \infty \\ \zeta^2 : \text{fixed}}} (g_{YM}^2 N)^{-7/4} \langle \mathcal{O}(x_1) \ldots \mathcal{O}(x_4) \rangle$$

# Relation between the correlator and the scattering amplitude (Penedones and T.O.)

The rescaled corelator $\mathcal{F}(\zeta)$ and the string scattering amplitude $\mathcal{T}(\omega)$ are related, roughly, by the Laplace transform

$$i\mathcal{T}(\omega) \sim \int_{-i\infty}^{i\infty} d\zeta e^{\zeta l_s \omega} \mathcal{F}(\zeta)$$

$$\mathcal{F}(\zeta) \sim \int_0^{\infty} d\omega e^{-\zeta l_s \omega} i\mathcal{T}(\omega)$$

# Precise formulas

$$\frac{\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_3)\rangle}{\langle \mathcal{O}(x_1)\mathcal{O}(x_3)\rangle \langle \mathcal{O}(x_2)\mathcal{O}(x_4)\rangle} = \mathcal{A}(g_{\mathrm{YM}}, N, \sigma, \rho^2)$$

$$\sigma^2 = \frac{x_{13}^2 x_{24}^2}{x_{12}^2 x_{34}^2}, \quad \sinh^2 \rho = \frac{\det x_{ij}^2}{4x_{13}^2 x_{24}^2 x_{12}^2 x_{34}^2}$$

$$\mathcal{F}(g_{\mathrm{YM}}^2, \sigma, \zeta) = \frac{1}{\sigma^8} \lim_{N\to\infty} \frac{\mathcal{A}\left(g_{\mathrm{YM}}, N, \sigma, -\frac{(1-\sigma)\zeta^2}{\sigma g_{\mathrm{YM}}\sqrt{N}}\right)}{(g_{\mathrm{YM}}^2 N)^{7/4}}$$

$$iT\left(g_s, l_s, s, t/s\right) = \frac{2^{17}3^2\pi^3\sqrt{stu}}{l_s^5 s^7}$$

$$\times \int_{-i\infty}^{i\infty} \frac{d\zeta}{2\pi i} \zeta \mathcal{F}(4\pi g_s, -t/s, \zeta) e^{\zeta l_s \sqrt{s}}$$

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# Issues in the derivation

- Periodicity of geodesics in AdS might prohibit defining asymptotic states.
- Aren't the propagators modified by quantum corrections?
- String theory is defined on-shell. Does the LSZ reduction apply?

# Resolved issues in the derivation

- ✔ Periodicity of geodesics in AdS might prohibit defining asymptotic states.

  Use localized sources and wave packets.
- ✔ Aren’t the propagators modified by quantum corrections?

  Non-renormalization of 3-point functions.
- ✔ String theory is defined on-shell. Does the LSZ reduction apply?

  Local field theory only used outside the interaction region.

# Remaining issues in the derivation

- Are the wave packets sufficiently localized? Gary and Giddings (2009) found power-law tails.

- AdS and the Minkowski space have different asymptotic infinities. Aren't the theories defined on them completely unrelated?

# Translation of physical properties and questions

- Most interesting: gauge theory to string theory (e.g., study black hole physics).

  Difficult because of strong coupling.

- Easy: string theory to gauge theory.

  e.g., exponential fall-off at high energy

  ➔ high-order behavior in Taylor expansion of

  $\mathcal{F}(\zeta)$

# Conclusions

- Many but not all subtle issues in taking the flat space limit can be resolved. Made progress assuming that the remaining issues can be overcome.
- Identified a sub-sector of N=4 SYM dual to IIB strings in flat space.
- Scaled correlators and scattering amplitudes are related by the Laplace transform.

# Happy Birthday!