E_{10}, Cosmobilliards and Quantum Gravity

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Based on joint work with
T. Damour, M. Henneaux and A. Kleinschmidt
(in various combinations)

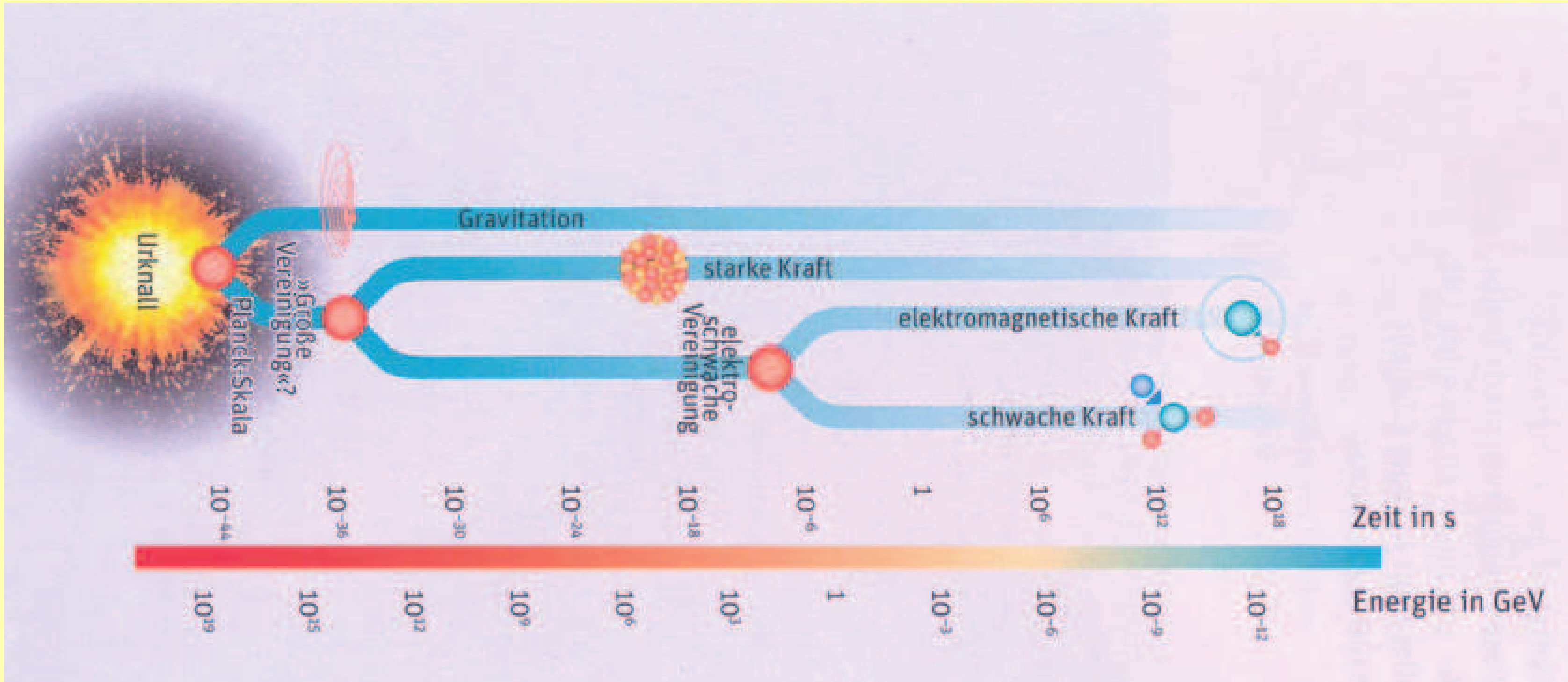
[and: A.Kleinschmidt, M.Koehn, HN: arXiv:0907.3048[hep-th]]

Main theme: Symmetry

... arguably the most successful principle of physics!

- **Space-time symmetries**
  - **Rotations and translations in Newtonian physics**
  - **Special relativity and the Poincaré group**
  - **General relativity and general covariance**
- **Internal symmetries**
  - **Isospin $SU(2)$ symmetry:** $m_{\text{neutron}} = 1.00135 m_{\text{proton}}$
  - **Flavor symmetry $SU(3)$ and the strong interactions**
  - **Standard model and $SU(3)_c \times SU(2)_w \times U(1)_Y$**
- **The two fundamental theories of modern physics, General Relativity and the Standard Model of Particle Physics, are based on and largely determined by symmetry principles!**

Symmetry and Unification



Like a ferromagnet: symmetry is broken more and more with decreasing temperature as universe expands.

But where do we go from here?

Idea: symmetry enhancement as a guiding principle!

- **Grand Unification:**

$$SU(3)_c \times SU(2)_w \times U(1)_Y \subset SU(5) \subset SO(10) \subset E_6 \subset \ldots ?$$

$\rightarrow$ **quark lepton unification, proton decay, ...**

- **'Fusion' of space-time and internal symmetries?**
- **Supersymmetry: relates Bosons $\leftrightarrow$ Fermions, or:**
Forces (vector bosons) $\leftrightarrow$ Matter (quarks & leptons)?
- **Duality symmetries, e.g. electromagnetic duality**

$$\mathbf{E} + i\mathbf{B} \rightarrow e^{i\alpha}(\mathbf{E} + i\mathbf{B}) \ , \quad q + ig \rightarrow e^{i\alpha}(q + ig)$$

- **Quantum symmetry and quantum space-time?**

Habitat of Quantum Gravity?

- **Cosmological evolution as 'geodesic motion' in the moduli space of 3-geometries** [Wheeler, DeWitt,...]

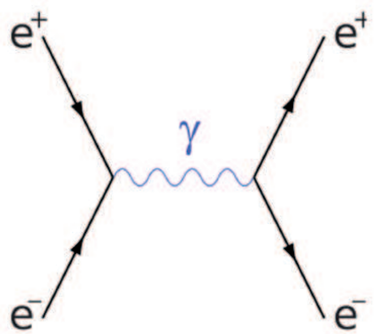
$$\mathcal{M} \equiv \mathcal{G}^{(3)} = \frac{\{\text{spatial metrics } g_{ij}(\mathbf{x})\}}{\{\text{spatial diffeomorphisms}\}}$$

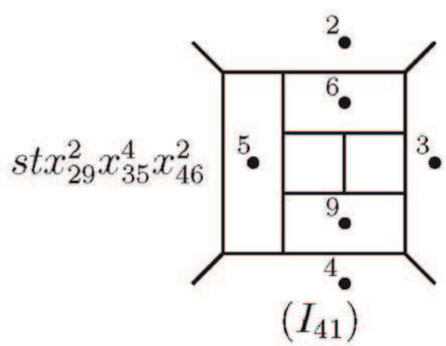
- **Formal canonical quantization leads to WDW equation ("Schrödinger equation of quantum gravity")**
  - **Functional differential equation: mathematically ill-defined**
  - **Unsolved conceptual and interpretational problems**
- **Unification of space-time, matter and gravitation: configuration space $\mathcal{M}$ for quantum gravity should consistently incorporate matter degrees of freedom**
- **Can we understand and 'simplify' $\mathcal{M}$ by means of embedding into a group theoretical coset $G/K(G)$?**

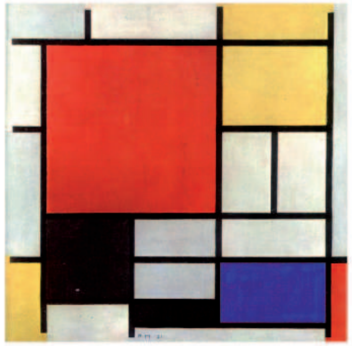
Four-loop finiteness of $N = 8$ supergravity

[Bern,Carrasco,Dixon,Johansson,Roiban, PRL103,081301(2009); Z.Bern: talk at Strings'09]

- **Use unitarity based arguments to reduce all amplitudes to integrals over products of tree amplitudes.**
- **All particles are on-shell $\rightarrow$ only 3-point vertices.**
- **Instead of $\mathcal{O}(10^{20})$ Feynman diagrams need only calculate $\mathcal{O}(50)$ 'Mondrian-like' diagrams!**







Thus, $N = 8$ supergravity could be *UV finite to all orders* $\rightarrow$ WHY? $\rightarrow$ Unknown Symmetry?

Exceptional Symmetries and Supergravity

- $N = 8$ **supergravity in** $D = 4$ **has more symmetry than meets the eye:** $E_{7(7)}$**!** [Cremmer,Julia,1979]

  (Recall: G_2, F_4, E_6, E_7, E_8 **are the five *exceptional Lie groups*.)**

- **Duality symmetries for maximal supergravities in** $D \neq 4$**:** $E_{n(n)}$ **for maximal supergravity in** $D = 11 - n$**.**

- **Idem for other non-compact real forms, e.g.**

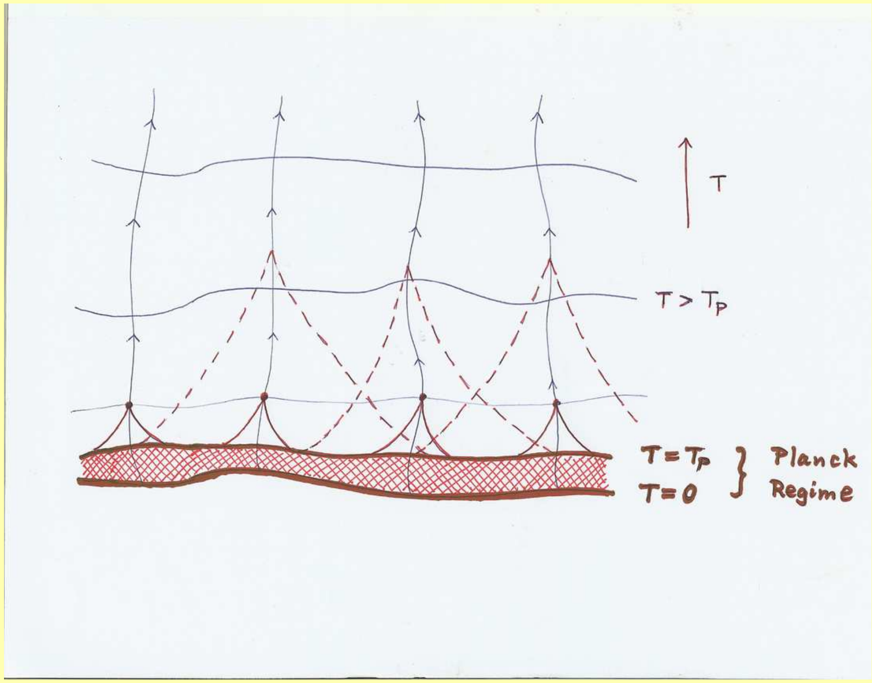
$$\begin{array}{ll} E_{7(-25)} & \textbf{for 'magic' } N = 2 \textbf{ in } D = 4 \\ E_{7(-5)} & \textbf{for } N = 12 \textbf{ in } D = 3 \\ E_{8(-24)} & \textbf{for 'magic' } N = 4 \textbf{ in } D = 3 \end{array}$$

- $E_{9(9)} \equiv E_8^{(1)}$ **for maximal supergravity in** $D = 2$**.**

- **... suggests** $E_{10(10)}$ **for** $D = 1$ **....**

- **... or even** $E_{11(11)}$ **for** $D = 0$**??** [→ P. West]

BKL and Spacelike Singularities (I)



Hypothesis: for $T \to 0$ spatial points decouple and the system is effectively described by a continuous ***superposition of one-dimensional systems*** $\to$ **effective dimensional reduction to $D = 1$!** [Belinski,Khalatnikov,Lifshitz (1972)]

BKL and Spacelike Singularities (II)

Near cosmological singularity parametrize metric as

$$ds^2 = -N^2 dt^2 + g_{mn} dx^m dx^n \quad , \qquad g_{mn} = e_m{}^a e_{n\,a}$$

Iwasawa decomposition of spatial zehnbein $e_m{}^a \equiv e_m{}^a(t, \mathbf{x})$

$$e_m{}^a = e^{-\beta^a} \theta_m{}^a \quad , \qquad \det \theta_m{}^a = 1$$

From classical BKL analysis we know that:

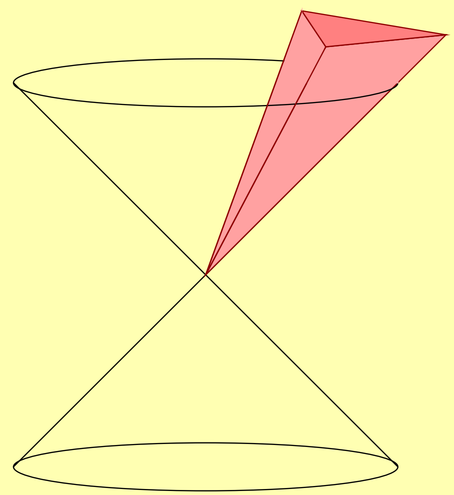
[Belinski,Khalatnikov,Lifshitz (1972); Misner (1969); Chitre (1972); DHN (2003)]

- **Dynamics near singularity is dominated by logarithmic scale factors** $\beta^a \to \infty$ **and leading 'wall forms' which result from 'integrating out' non-diagonal metric and matter degrees of freedom.**
- $\Rightarrow$ **off-diagonal metric components** $\theta_m{}^a$ **and matter degrees of freedom 'freeze' as** $T \to 0$**.**

Walls and Roots

- ‘Integrating out’ remaining degrees off freedom leads an effective description in terms of ***cosmological billiards*** taking place in β-space of scale factors.
- The ***Lie algebra connection*** : identify space of logarithmic scale factors $\{\beta^a\}$ with the Cartan subalgebra of some indefinite Kac Moody algebra.
- Walls of billiard table are defined by ***spacelike normal vectors*** which can be identified with ***real roots*** of some indefinite Kac Moody algebra.
- For maximal supergravity this Kac Moody algebra is the **maximally extended hyperbolic algebra E_{10}**.

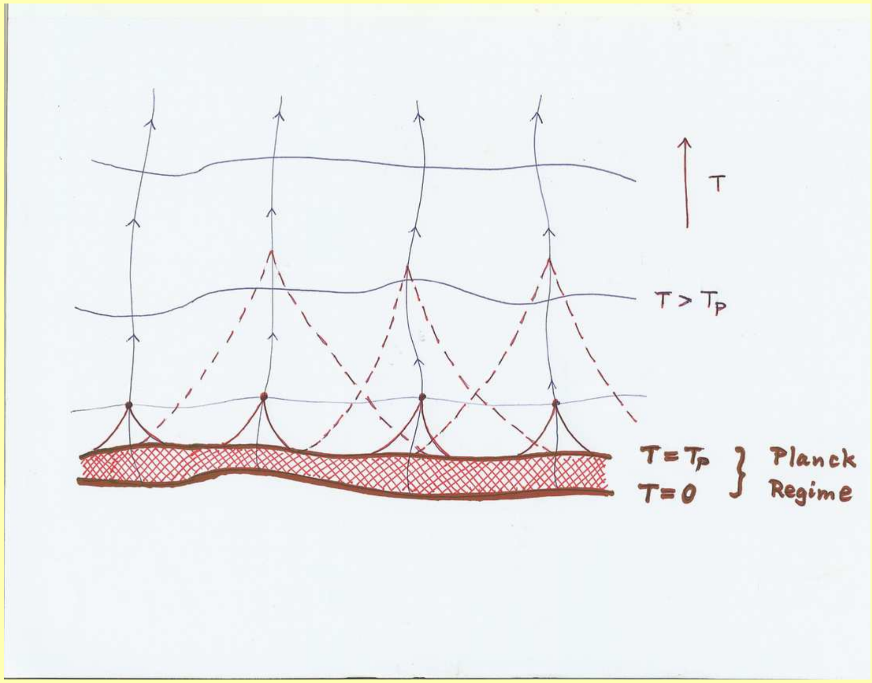
Cosmobilliards in 'β-spacetime'



The 'Kasner billiard ball' moves in the 'billiard chamber' on lightlike straight lines ('free Kasner flights'), bouncing off the walls of the chamber ('Kasner bounces').

Chaotic oscillations of metric if chamber is contained within forward lightcone, otherwise 'AVD' behavior.

E_{10}: The Basic Picture



Conjecture: **for** $0 < T < T_P$ **space-time 'de-emerges', and space-time based (quantum) field theory is replaced by (quantized)** $E_{10}/K(E_{10})$ **σ-model** [Cf. DN, 0705.2643]

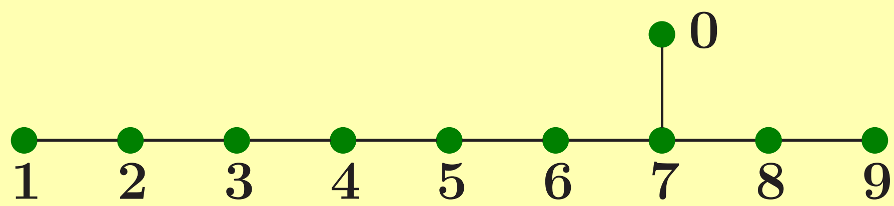
Why E_{10}?

- E_{10} occupies a **uniquely distinguished place among all infinite-dimensional Lie algebras** (much like E_8 among the finite-dimensional Lie algebras)
- **In BKL approximation, classical dynamics of SUGRA$_{11}$ near the initial singularity is well approximated by cosmological billiards in Weyl chamber of E_{10}**
- E_{10} **'knows all' about maximal supersymmetry:**
  - Different 'slicings' of the E_{10} algebra yield correct supermultiplets for maximal supergravities (SUGRA$_{11}$, mIIA, IIB, ...)
  - $E_{10}/K(E_{10})$ σ-model dynamics at low levels matches with respective equations of motion when truncated to first order spatial gradients
- E_{10} may provide ***Lie-algebraic mechanism*** **for the 'de-emergence' of space and** (upon quantization) **time** near the singularity (that is, for $0 < T < T_P$)

What is E_{10}?

(No one knows, really....)

E_{10} is the 'group' associated with the Kac-Moody Lie algebra $\mathfrak{g} \equiv \mathfrak{e}_{10}$ defined via the Dynkin diagram [e.g. Kac]

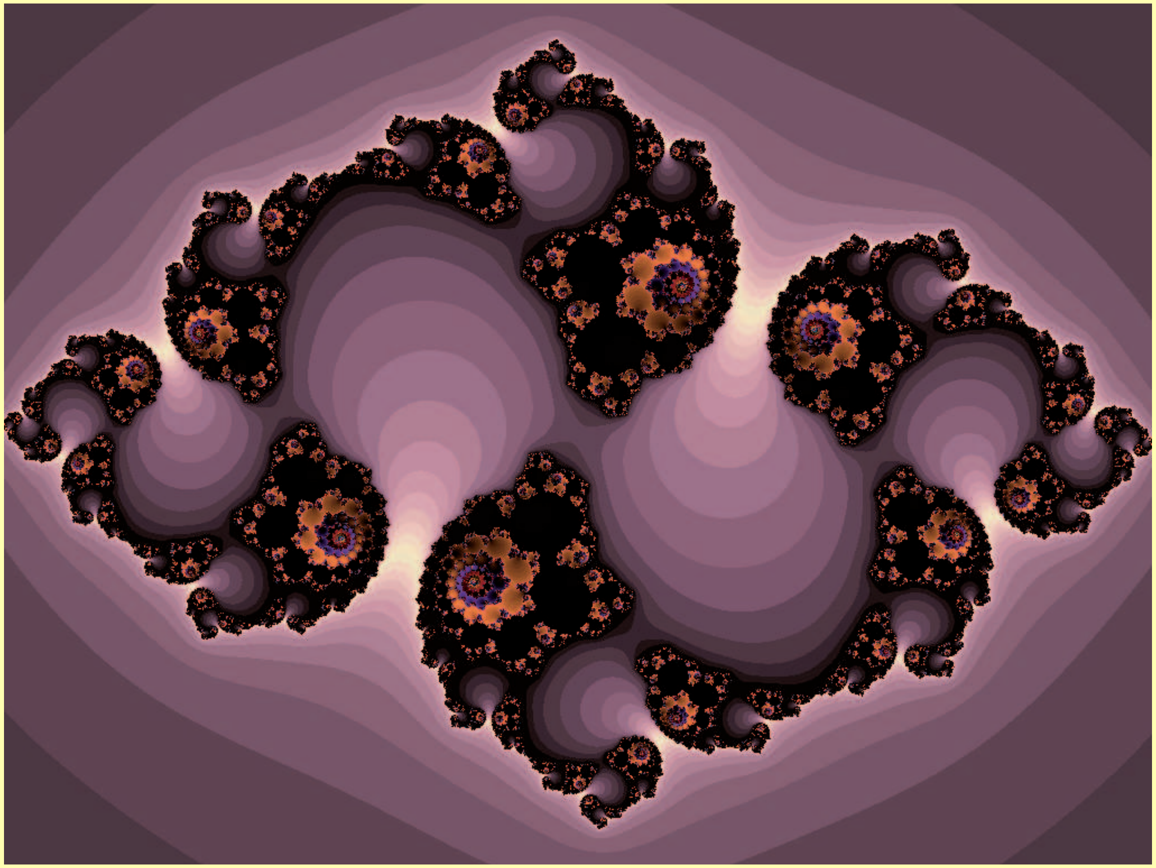


Defined by generators $\{e_i, f_i, h_i\}$ and relations via Cartan matrix A_{ij} ('Chevalley-Serre presentation')

$$\begin{aligned}
[h_i, h_j] &= 0, & [e_i, f_j] &= \delta_{ij} h_i, \\
[h_i, e_j] &= A_{ij} e_j, & [h_i, f_j] &= -A_{ij} f_j, \\
(\mathrm{ad}\, e_i)^{1-A_{ij}} e_j &= 0 & (\mathrm{ad}\, f_i)^{1-A_{ij}} f_j &= 0.
\end{aligned}$$

$\mathfrak{e}_{10}$ is the free Lie algebra generated by $\{e_i, f_i, h_i\}$ modulo these relations $\rightarrow$ infinite dimensional as A_{ij} is *indefinite* $\rightarrow$ Lie algebra gets *infinitely complicated* !

Infinite Complexity from simple recursion



A Mandelbrot set generated from $z_{n+1} = f_c(z_n)$.

Some Key Properties

- **Root space decomposition** $\alpha \in Q(E_{10}) = \mathrm{II}_{1,9}$

$$\mathfrak{g}_\alpha = \{x \in \mathfrak{g} : [h, x] = \alpha(h)x \ \text{ for } \ h \in \mathfrak{h}\}$$

  where $\mathfrak{g} \equiv \mathfrak{e}_{10}$ and $\mathfrak{h} \equiv$ Cartan subalgebra of E_{10}. Real roots ($\alpha^2 = 2$) and imaginary roots ($\alpha^2 \leq 0$).

- **Triangular decomposition** $\longrightarrow$ **Computability**

$$\mathfrak{g} = \mathfrak{n}_- \oplus \mathfrak{h} \oplus \mathfrak{n}_+ \qquad \textbf{with } \mathfrak{n}_\pm := \bigoplus_{\alpha \gtrless 0} \mathfrak{g}_\alpha$$

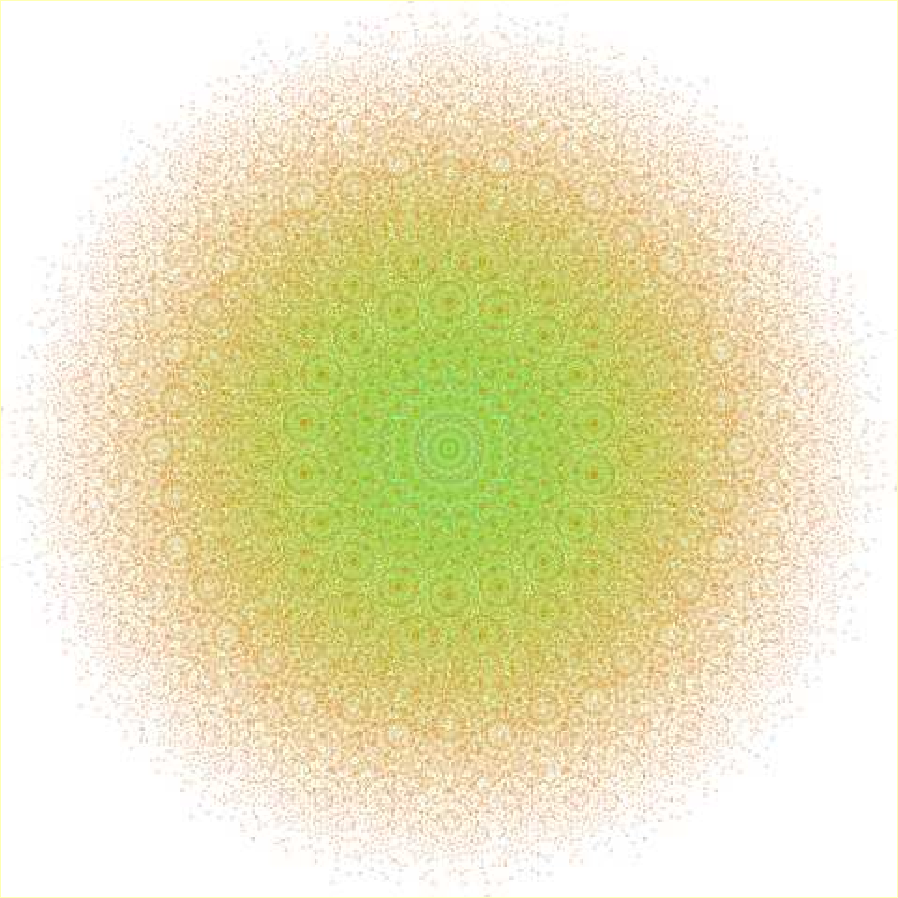
- **Invariant bilinear form** $\longrightarrow$ **Action Principle**

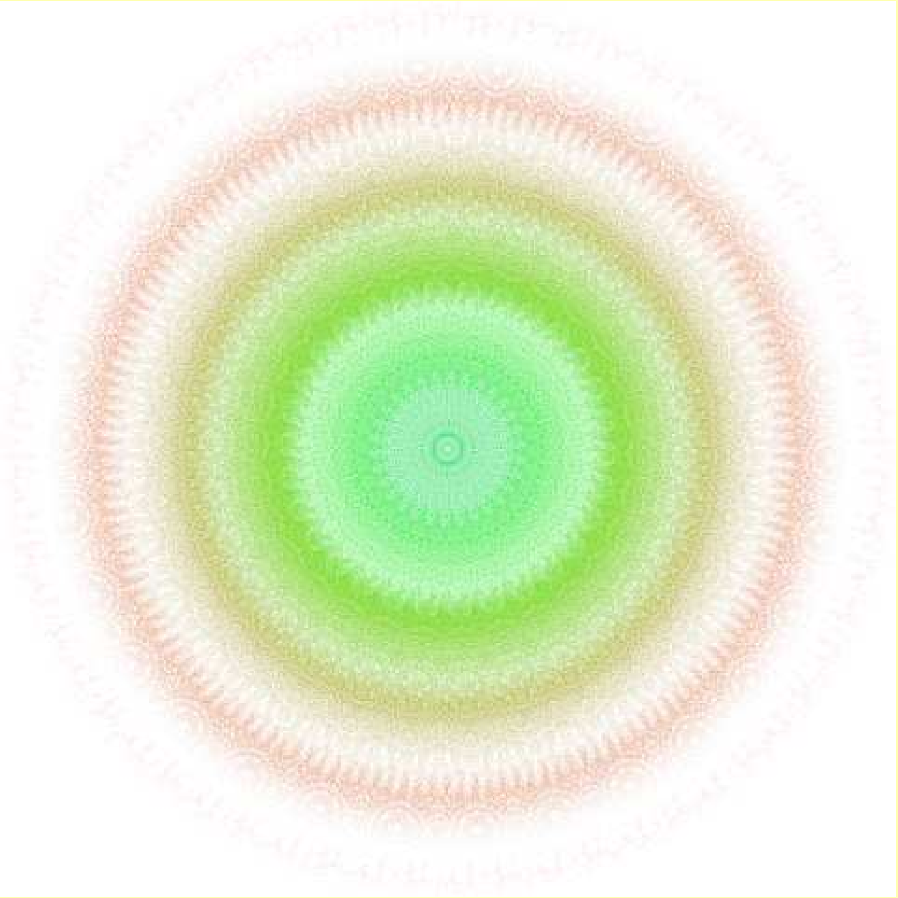
$$\langle h_i | h_j \rangle = A_{ij} \ \ , \quad \langle e_i | f_j \rangle = \delta_{ij} \ \ , \quad \langle [x, y] | z \rangle = \langle x | [y, z] \rangle$$

- **Even Weyl group** $W^+(E_{10}) = PSL_2(\mathbf{O})$ **(where $\mathbf{O} \equiv$ integral octonions = 'octavians')** `[FKN, math.RT/0805.3018]`

  [Pure gravity: $W^+(AE_3) = PSL_2(\mathbb{Z})$ = *modular group*]

Vistas into E_{10}...

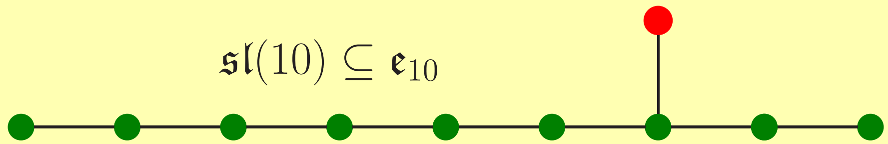




[from: Teake Nutma (University of Groningen)]

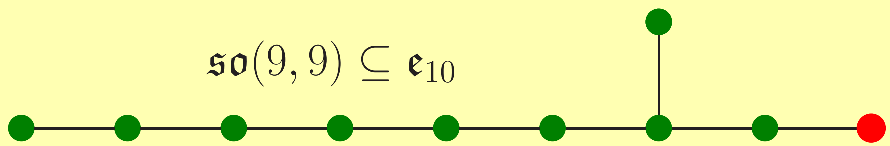
E_{10} Versatility

$\mathfrak{sl}(10) \subseteq \mathfrak{e}_{10}$



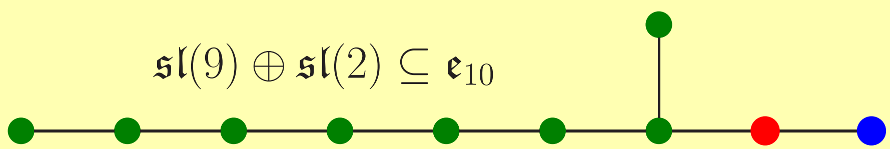
$D = 11$ **SUGRA**

$\mathfrak{so}(9,9) \subseteq \mathfrak{e}_{10}$



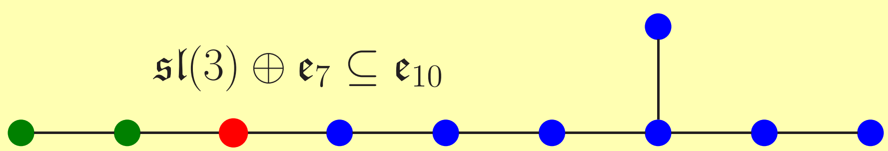
mIIA $D = 10$ **SUGRA**

$\mathfrak{sl}(9) \oplus \mathfrak{sl}(2) \subseteq \mathfrak{e}_{10}$



IIB $D = 10$ **SUGRA**

$\mathfrak{sl}(3) \oplus \mathfrak{e}_7 \subseteq \mathfrak{e}_{10}$



$\mathcal{N} = 8$**,** $D = 4$ **SUGRA**

Cosmological quantum billiards

- ***Mini-superspace*** **quantization for diagonal degrees of freedom $\equiv$ quantization of $E_{10}/K(E_{10})$ model *restricted to Cartan subalgebra* $\Rightarrow$ WDW equation**

$$\mathcal{H}\Psi\big|_{\mathfrak{h}} = G^{ab}\partial_a\partial_b\Psi \Rightarrow \left\{-\frac{\partial}{\partial\rho}\left(\rho^{d-1}\frac{\partial}{\partial\rho}\right) + \rho^{d-3}\triangle_{\mathbf{H}}\right\}\Psi[\rho,\omega] = 0$$

with variables projecting onto unit hyperboloid H

$$\beta^a \equiv \rho\omega^a \quad \textit{with} \quad \omega^a G_{ab}\omega^b = -1 \;\Rightarrow\; \rho^2 = -\beta^a G_{ab}\beta^b$$

and $\triangle_{\mathbf{H}}$ is Laplace-Beltrami operator on H.

- **Reduced equation studied for pure gravity in $D = 4$**

[Misner(1972);Graham,Szepfalusy(1990);Forte(2008)]

- **Factorize $\Psi[\rho,\omega] = R(\rho)F(\omega)$ with $-\triangle_{\mathbf{H}}F(\omega) = EF(\omega) \;\Rightarrow$**

$$R_{\pm}(\rho) = \rho^{-\frac{d-2}{2}} e^{\pm i\sqrt{E-\left(\frac{d-2}{2}\right)^2}\log\rho}$$

- **For** E_{10} **use 'upper half plane' coordinates** $\omega \equiv \omega(z)$

$$z := u + iv \quad : \quad u \in \mathbb{O} \equiv \mathbb{R}^8 \, , \; v \in \mathbb{R}_{>0} \qquad (iu = \bar{u}i)$$

- $W(E_{10})$ **acts by** ***modular transformations*****:**

$$w_{-1}(z) = 1/\bar{z} \; , \;\; w_0(z) = -\theta\bar{z}\theta + \theta \; , \;\; w_j(z) = -\varepsilon_j \bar{z} \varepsilon_j$$

with ε_j **= simple roots of** $E_8 \subset$ **unit octavians** $\Rightarrow$

- **Billiard domain** $\equiv$ **fundamental domain for** $W(E_{10})$

$$(\Psi_1|\Psi_2) = i \int_{\mathcal{F}} d\Sigma^a \Psi_1^* \overleftrightarrow{\partial}_a \Psi_2 \quad , \qquad \mathcal{F} = \mathbf{H}/W(E_{10})$$

- $\Rightarrow$ **a new theory of** ***automorphic forms*****(?): solutions to WDW equation are** ***odd Maass wave forms*** **for arithmetic group** $W^+(E_{10}) = PSL_2(\mathbf{O})$**.**

- $E \geq \frac{1}{4}(d-2)^2$ **implies** $\Psi \to 0$ **for** $\rho \to \infty$**, and wave function cannot be continued beyond singularity** $\to$ **Singularity avoidance in quantum cosmology?**

[Kleinschmidt, Koehn, HN: arXiv:0907.3048[gr-qc]; Kleinschmidt, HN: arXiv:0912.0854[gr-qc]]

Beyond the billiard approximation

- **Expected form of wave functional in BKL limit:**

$$\mathbf{\Psi} \sim \prod_{\mathbf{x}} \Psi_{\mathbf{x}}(\rho_{\mathbf{x}}, z_{\mathbf{x}}) \quad , \quad \mathbf{x} \in \Sigma$$

  $\rightarrow$ inhomogeneities and spatial decoupling.

- **Main task: replace this *formal* expression by a wave function depending on infinite tower of E_{10} d.o.f.'s.**

- **Conversion of 'small tension' expansion in spatial gradients into (algebraic) expansion in heights of E_{10} roots would effectively implement *de-emergence of space and time* near the initial singularity.**

- **E_{10} Cartan–Killing form $\rightarrow$ *unique* Hamiltonian**

- **Conserved E_{10} Noether charges supply infinitely many observables *à la* Dirac (whereas none are known in standard canonical gravity).**

Outlook

- **Symmetry by no means exhausted as a *guiding principle of physics* but many open questions remain.**
- **E_{10} is a *uniquely distinguished* Lie algebra, but to find a manageable representation for it remains an outstanding mathematical challenge (after 40 years).**
- **Near cosmological singularity (as $\rho \to \infty$) life becomes 'infinitely complicated' as we expect *all* E_{10} *degrees of freedom to get excited.***
- **Main new features of quantized $E_{10}/K(E_{10})$ model:**
  - **Wave function generically vanishes at singularity**
  - **Wave function is generically complex and oscillating ...**
  - **... and cannot be continued beyond singularity**
- **An element of *non-computability* for $T \to 0$?**

風間先生、お誕生日おめでとうございます。