Gravitational Duals of 3d Cascading Gauge Theories and Dynamical SUSY Breaking

Prof. Yoichi Kazama 60th Birthday Celebration

Happy Birthday!

“Kazamania”, Komaba 2010, Tokyo

Shinji Hirano

Based on a work to appear (with Aki Hashimoto, Peter Ouyang) and discussions with AH, PO, Ofer Aharony, Oren Bergman

AdS$_4$/CFT$_3$ (ABJ $\frac{1}{2}$M $\pm$ $\frac{1}{2}$M)

M-theory on $AdS_4 \times S^7/Z_k$ with (discrete) torion C_3	$= U(N{+}l)_k \times U(N)_{-k}$ Chern-Simons-Matter

◁ valid when $k \ll N^{1/5}$ and large N and $G_4 = dC_3 = 0$

$\int_{S^3/Z_k \subset S^7/Z_k} C_3 = \frac{l}{k}$ (with $l = 1, \cdots, k-1$)

representing l fractional M2-branes = M5-branes wrapped on S^3/Z_k

Type IIA on $AdS_4 \times CP^3$ with NSNS B_2	$= U(N{+}l)_k \times U(N)_{-k}$ Chern-Simons-Matter

◁ valid when $N^{1/5} \ll k \ll N$ and large N (1st inequality $\leftrightarrow$ weak type IIA coupling $\leftrightarrow$ small M-theory circle) and $H_3 = dB_2 = 0$

$\int_{CP^1 \subset CP^3} B_2 = \frac{l}{k}$ (with $l = 1, \cdots, k-1$),

representing l fractional D2-branes = D4-branes wrapped on CP^1

(i) $\mathcal{N} = 8$ SUSY for $k = 1, 2$ and $\mathcal{N} = 6$ SUSY for $k > 2$

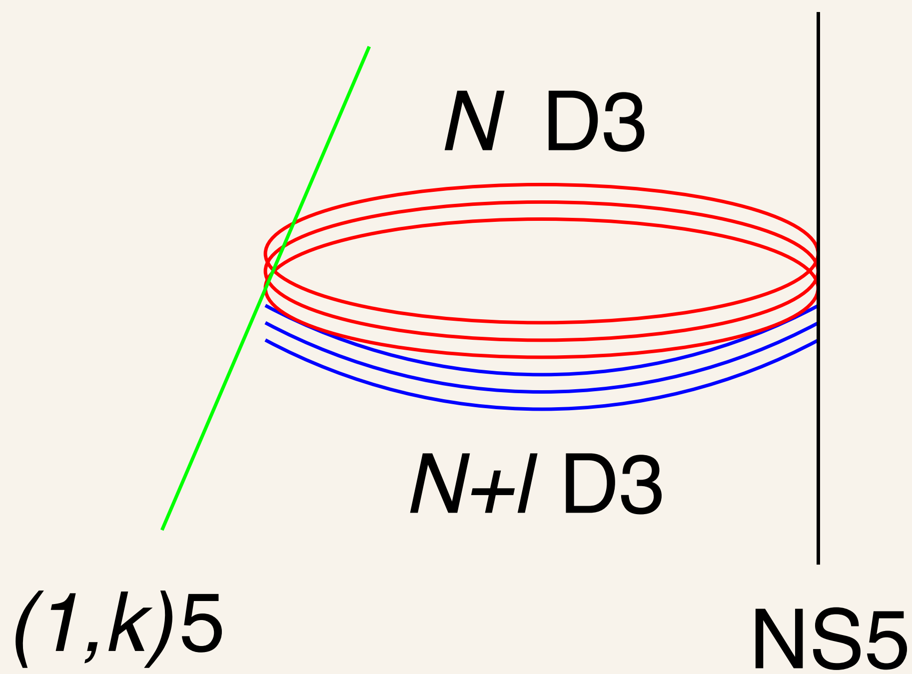
(ii) CSM theory 'tHooft coupling $\lambda = N/k$ strong in SUGRA descriptions

UV embedding (Hashimoto, Ouyang)

• 3d gauge theory super-renormalizable $\Longrightarrow$ a simple UV embedding of ABJ/M exists (in contrast to 4d gauge theory such as KS, not to be confused with Kazama-Suzuki):

▷ $\mathcal{N} = 3$ SYM+CS, flowing to the IR fixed point $\mathcal{N} = 6, 8$ ABJM CS+Matter theory

$=$ LEET of IIB dual brane configuration

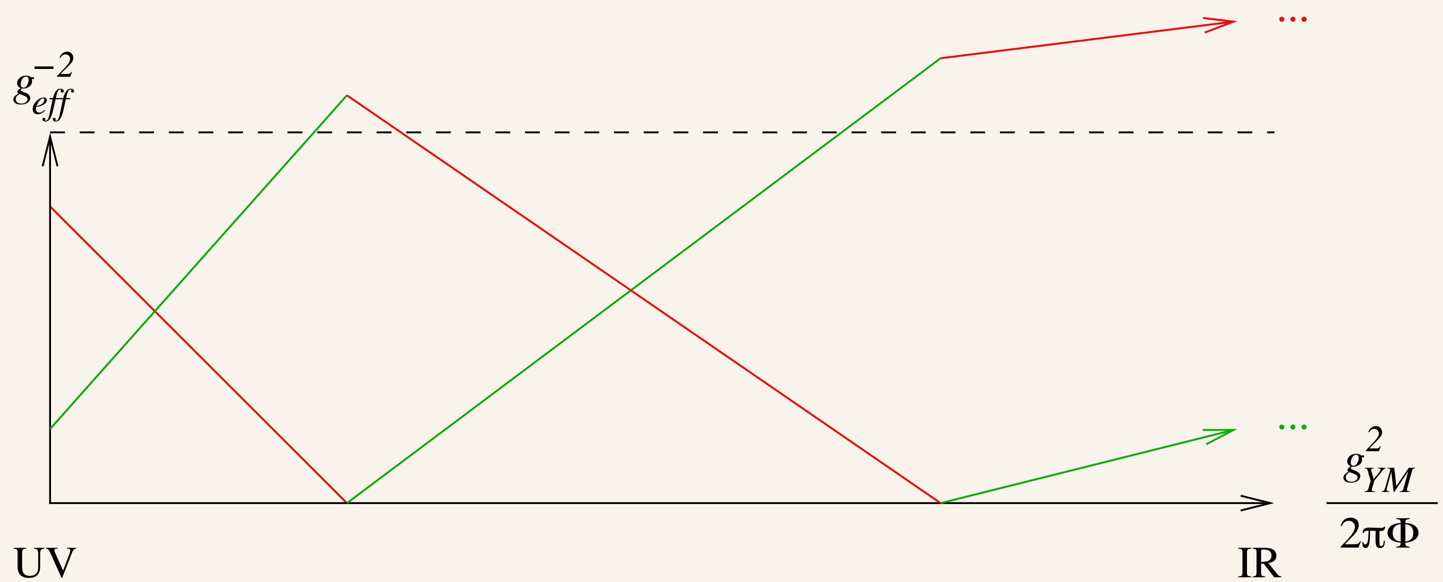


▷ M-theory lift (Gauntlett-Gibbons-Papadopoulos-Townsend) $\Longrightarrow$ M2-branes in LWY (Lee-Weinberg-Yee) space $\sim$ nontrivial product of two TN's

$=$ SUGRA dual of entire RG flow from $\mathcal{N} = 3$ SYM+CS to ABJM !

Cascade (Aharony, Hashimoto, Hirano, Ouyang)

- Typical theories in ABJM and its generalizations are A_1 quiver $U(N_2+N_4)_k \times U(N_2)_{-k} \implies$ Two gauge couplings run with duality cascade



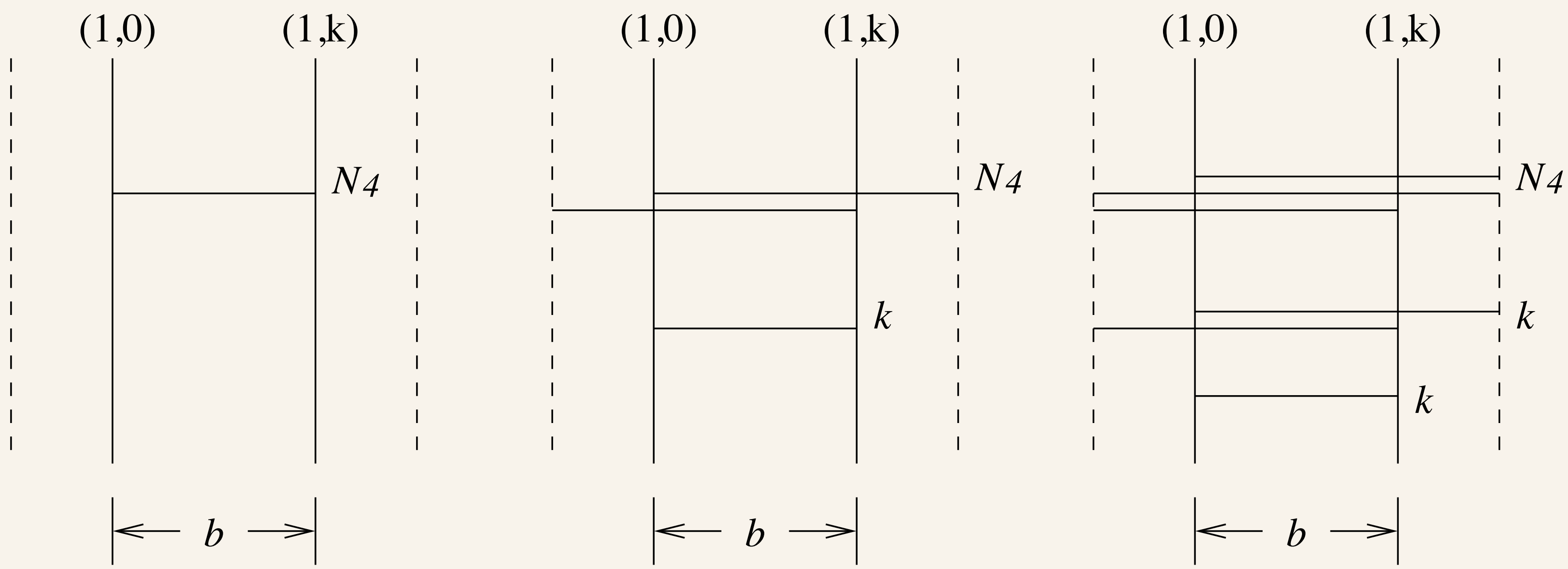
One-loop running

$$\frac{1}{g_{eff1}^2(\Phi)} = b_\infty - \frac{g_{YM}^2 N_4}{2\pi\Phi}\,, \qquad \frac{1}{g_{eff2}^2(\Phi)} = (1-b_\infty) + \frac{g_{YM}^2 N_4}{2\pi\Phi}$$

Seiberg-like duality

$$N_4 \to N_4 - k\ , \qquad N_2 \to N_2 - N_4$$

▷ Can be understood as NS5-brane moves and Hanany-Witten effect



(A) Cascade terminates in UV (super-renormalizable)

(B) Generalized s-rule: $N_4(\text{IR}) \leq k \implies$ if $N_4 > k$ is related to $N_4(\text{IR})$ by cascade steps, N_4 D3-branes are allowed to end on $(1, k)$ 5-brane (more than one fractional D3 states winding around S^1)

(C) Two ways to end cascade in IR: (1) $N_4(\text{IR}) < k$ and $N_2(\text{IR}) > 0$, (2) $N_2(\text{IR}) < 0$ and $N_4(\text{IR}) > k$,

Case (1) $\implies$ IR theory SCFT ABJ

Case (2) $\implies$ IR theory SUSY breaking (presumably with mass gap) [Note: $\mathcal{N} = 1$ SYM+CS Witten index = 0 for $N_4(\text{IR}) > k$]

Gravitational duals (AHHO)

- Geometry and flux

$$ds^2 = H^{-2/3} ds^2_{R^{1,2}} + H^{1/3} ds^2_{LWY}$$
$$G_4 = dt \wedge dx_1 \wedge dx_2 \wedge dH^{-1} + G$$

- G-flux needs to be turned on

$$C_3 = (k\, b_\infty + \hat{N}_4) f(r) (\text{nontrvial}) - \hat{N}_4\, J \wedge dx_{11}$$

with $f(\infty) = 1$ (UV) and $f(0) = 0$ (IR) (J $=$ Kähler form on CP^3)

▷ In type IIA

(1) exact (singular) part $\Longrightarrow$ fractional D2 (wrapped D4) quantization

$$\hat{N}_4 = Q_4^{Page} = \int_{CP^2} (\tilde{F}_4 + (B_2 + F) \wedge F_2) = N_4 - \frac{k}{2}$$

(2A) nontrivial part $\Longrightarrow$ NS5-(1,k)5 separation

$$b_\infty = \int_{CP^1} B_2(\infty)$$

(2B) nontrivial part $\Longrightarrow$ cascade (varying flux)

$$Q_2^{Maxwell} = \int_{CP^3} * \tilde{F}_4 = N_2 + b(r) \left(N_4 - \frac{k}{2} \right) + \frac{1}{2} b(r)^2 k$$

♠ Wish to explore two phases (1) SUSY and (2) SUSY breaking, but hard to construct G_4 explicitly in LWY

$\mathcal{N}=1$ theories

• M2-branes in Spin(7) manifolds simpler than in LWY

Our focus: ALC (asymptotically locally conical) cones over squashed S^7 (Cvetic-Gibbons-Lu-Pope) $\sim$ analogous to Taub-NUT and Taub-BOLT

$$ds^2 = \frac{(r\pm\ell)^2}{(r\pm 3\ell)(r\mp\ell)}dr^2+\frac{1}{4}(r\pm 3\ell)(r\mp\ell)\,(D\mu_i)^2+\frac{\ell^2(r\pm 3\ell)(r\mp\ell)}{(r\pm\ell)^2}\sigma^2+\frac{1}{2}(r^2-\ell^2)d\Omega_4^2$$

(A) A_8 : R^8/Z_k (IR$_{r=\ell}$) and (cone over squashed CP^3) $\times$ S^1 (UV)

(B) $B_8(a)$: $R^4/Z_k\times S^4$ (IR$_{r=3\ell}$) and (cone over squashed CP^3) $\times$ S^1 (UV)

(1) M2-branes in A_8 = alternative UV embedding of ABJM

[IIB brane config (GT+k)= almost $\mathcal{N}=3$ but w/ 3 angles $=\frac{2\pi}{3}+\frac{1}{3}\arctan(k)$]

(2A) A line of deformations parametrized by a (= size of S^4). In the limit $a\to 0$, IR (core) geometry is a singular cone over squashed S^7

(2B) IR of M2-branes in ALC $B8(0)$ conifold dual to OP (Ooguri-Park), $\mathcal{N}=1$ SCFT with $Sp(2)\times U(1)$

♠ Can find (A)SD G-flux explicitly $\Longrightarrow$ a good setup to study two phases

SUSY bound and repulson

- SUSY bound $N_4(\text{IR}) \le k$ and $N_2(\text{IR}) \ge 0$ (from generalized s-rule)

$\Longrightarrow$ $N_2(\text{IR}) = N_2 - \frac{N_4(N_4-k)}{2k} \ge 0$ w/ cascade steps $[N_4/k]$

- Repulson-free (from gravitational dual)

$$\triangle H = -\frac{|G|^2}{48} - \left[N_2 - \frac{N_4(N_4-k)}{2k} + \frac{C^2}{2k}\left(kb_\infty + N_4 - \frac{k}{2}\right)^2 \right] \delta^8(x)$$

[Note: RHS = bulk + brane source charges], (i) $C \neq 0$ if flux nonvanishing at $r = r_0$ (typically for BOLT), (ii) 2nd sign + for ASD,

(1) $C = 0$ for SUSY SD 4-form on A_8, repulson-free if

$$N_2(\text{IR}) = N_2 - \frac{N_4(N_4-k)}{2k} \ge 0$$

(2) IR geometry

(i) $N_2(\text{IR}) > 0 \implies AdS_4 \times S^7/Z_k$ (ABJ SCFT)

(ii) $N_2(\text{IR}) = 0 \implies R^{1,2} \times R^8/Z_k$ (mass gap in accordance with Witten)

♠ naive (unphysical) continuation to $N_2(\text{IR}) < 0$ prohibited, but adding negative $N_2(\text{IR})$ objects (wrapped M5/D4 and/or anti-M2/D2) allowed

$\Longrightarrow$ SUSY dynamically broken (analogous to Maldacena-Nastase)

(3) What to expect in SUSY breaking phase $N_2(\text{IR}) < 0$?

A mass gap (Witten) $\Longrightarrow$ IR geometry $R^{1,2} \times$ BOLT

▷ This is possible if the brane source charge

$$N_2(\text{IR}) + \frac{C^2}{2k}\left(kb_\infty + N_4 - \frac{k}{2}\right)^2 = 0 \implies N_2(\text{IR}) < 0$$

- Without accidental luck, we would need to relax the ansatz

$$ds^2 = e^{2A} ds^2_{R^{1,2}} + e^{2B} ds^2_{A_8 + \delta A_8}$$

$$G_4 = dt \wedge dx_1 \wedge dx_2 \wedge de^F + G \qquad \text{w/ (non-)SD } G$$

▷ However, curiously, SD 4-form SUSY-breaking on $B_8(a)$ (w/ same orientation as A_8) + UV asymptotics both geometry and flux same

$^{??}$(iii)$^{??}$ $N_2(\text{IR}) < 0 \implies$ IR $R^{1,2} \times R^4/Z_k \times S^4$ (mass gap SUSY broken)

▷ If simply $N_2(\text{IR}) < 0$ and flux contribution $C = 0$, SUGRA solution would have been singular. With $C \neq 0$ (adding wrapped M5/D4), the flux holds S^4 finite and closes off the space before $N_2(\text{IR})$ cascades down to negative $\Longrightarrow$ singularity resolved

- Caveat: Too opportunistic. Can't rule out that warped $B_8(a)$ and A_8 with SD differ by relevant deformations. Not clear if UV field theories are really the same with the only difference $N_2(\text{IR})$

Summary and conlusions

(1) AdS_4/CFT_3 admits simple UV embeddings of CSM SCFT into SYM + CS.

(2) IIB dual brane configurations typically involve a (1,k) 5-brane and fractional D3-branes as well as NS5 and regular D3-branes (LEET = SYM + CS)

▷ (i) duality cascade and (ii) a simple criterion for SUSY breaking (generalized s-rule)

(A) A close connection between SUSY bound and repulson-free ($N_2(\mathrm{IR}) \geq 0$)

(B) Examples of SUGRA duals: (i) $N_2(\mathrm{IR}) > 0$ SCFT, (ii) $N_2(\mathrm{IR}) = 0$ SUSY w/ mass gap (like KS), $^{??}$(iii)$^{??}$ $N_2(\mathrm{IR}) < 0$ SUSY breaking w/ mass gap

♠ General idea: Adding negative $N_2(\mathrm{IR})$ objects (wrapped M5/D4 and/or anti-M2/D2) $\Longrightarrow$ DSB (analogous to MN)