Towards Field Theory of D-Particles

Tamiaki Yoneya

- ◇ Motivation for D-brane field theory:
  - ◇ New understanding of open-closed string duality?
- ◇ Two approaches to D-brane field theory
  - ◇ A Toy Theory: D3-brane field theory restricted to 1/2-BPS sector*
  - ◇ Second quantization of D-particle Yang-Mills quantum mechanics**

hep-th/0510114 (JHEP12-028) and a work** in progress, hopefully, hep-th/0703???*

String theory must be far deeper than what has been understood so far.

And there must be many facets yet to be discovered.

Perhaps one of the most important and mysterious properties of string theory is
the duality between open and closed strings

- the most important signature of unification of gravity with other interactions
  - ◇ origin of gauge/gravity correspondence

yet,

- poorly understood from the viewpoint of possible non-perturbative formulations of string/M theory such as
  - ◇ string field theory and matrix models

Could the open-closed string duality be a consequence of more fundamental principle, such as *wave-particle duality* of quantum theory?

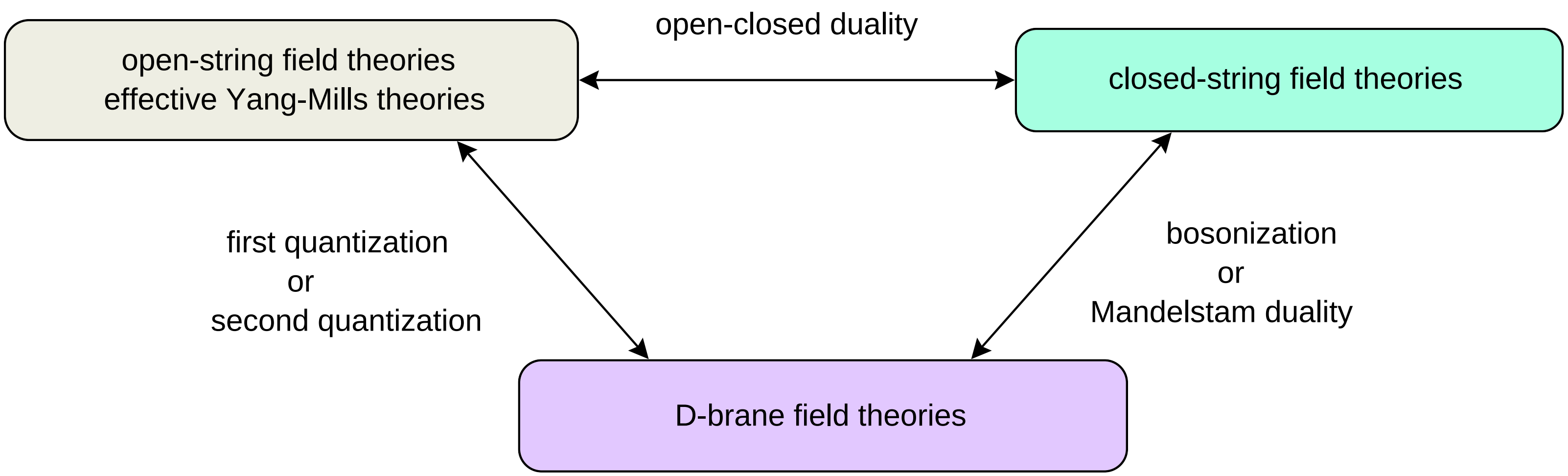
Why D-brane field theory ?

From this viewpoint, I propose

- to seek a possible generalization of the usual wave-particle duality to D-branes.

  Namely, it is natural to expect the existence of some appropriate generalization of the concept of quantum fields of particles to D-branes.

Project of D-brane field theories



An analogy on the right-hand side: soliton operator $\leftrightarrow$ Dirac field
in the duality between sine-Gordon model and massive Thirring model

$$\exp(\pi(x) \pm i\phi(x)) \leftrightarrow \psi(x), \quad \epsilon_{\mu\nu}\partial_\nu\phi \leftrightarrow \overline{\psi}\gamma_\mu\psi(x) \quad etc$$

second-quantized field theories for D-branes ?

◇ Two main approaches to D-brane dynamics :

- Open strings: Effective super Yang-Mills theories or open-string field theories of D-branes are

  ◇ configuration-space (first-quantized) formulations of D-branes

- Closed strings: D-branes $\sim$ soliton (or 'lump') solutions

  ('VSFT' also belongs to this category)

  ◇ difficult to treat fluctutations with respect to creation and annihilation of D-branes

Desirable to develop a *truly second-quantized* formulation of D-brane dynamics

quantum field theory of D-branes

$\Downarrow$

The whole set {N=0, 1, 2, 3, . . . } of U(N) super Yang-Mills theory should be treated in terms of some 'Fock space'-like representation?

This brings us to a new realm which has not been discussed before:

quantum-statistical symmetry (permutations) in particle quantum mechanics

$$(x_1, x_2) \leftrightarrow (x_2, x_1)$$

$$\Downarrow$$

$U(N)$ gauge symmetry (or Chan-Paton symmetry) of Yang-Mills theory

$$X_{ij} \leftrightarrow (UXU^{-1})_{ij}$$

continuous quantum-statistics !

(For diagonal matrix coordinates $X_{ij} = x_i \delta_{ij}$, gauge symmetry reduces to permutation symmetry)

Such a quantum statistics can never be treated in terms of usual bosonic or fermionic statistics (nor para-statistics).

This seems require an extension of mathematical framework of quantum field theories.

Two attempts to D-brane field theory

◇ A special toy model : $1/2$-BPS sector of D3-branes

◇ Direct second quantization of Yang-Mills quantum mechanics of D-particles

A Toy theory of D3-branes in 1/2-BPS sector

If we restrict ourselves to the 1/2-BPS sector of $\mathcal{N} = 4$ susy SYM$_4$, the (extremal) correlators of *generic* 1/2-BPS operators

$$\mathcal{O}^I_{(k_1,k_2,\ldots,k_n)}(x) \equiv w^I_{i_1\cdots i_r}\mathrm{Tr}\Big(\phi_{i_1}\cdots\phi_{i_{k_1}}\Big)\cdots\mathrm{Tr}\Big(\phi_{i_{r-k_n+1}}\cdots\phi_{i_r}\Big)$$

$$r = k_1 + k_2 + \cdots + k_n = \Delta$$

$$\{w^I_{i_1\cdots i_r}\} = \text{basis for totally symmetric traceless tensors}$$

(as discussed by Coley-Jevicki-Ramgoolam, hep-th/0111222)

can be evaluated using a free-field approximation, owing to the non-renormalization property of these correlators.

The extremal correlators of them are expressed in terms of $c = 1$ matrix model as

$$\langle\mathcal{O}^{I_1}_{(k_1,k_2,\ldots,k_n)}(\tau_1)\mathcal{O}^{I_2}_{(\ell_1,\ell_2,\ldots,\ell_n)}(\tau_2)\rangle = \underbrace{\langle w^{I_1}w^{I_2}\rangle}_{\text{invariant product}} \times G(\{k.\ell\}, N)e^{-r(\tau_1-\tau_2)}$$

$$G(\{k,\ell\}, N)\mathrm{e}^{-r(\tau_1-\tau_2)} = \text{correlator of hermitian 1-matrix free-field theory}$$

in the coherent-state ($\sim$ lowest Landau-level condition) representation.

(See also a related recent paper, Jevicki-T.Y. hep-th/0612262 "1/2-BPS correlators as $c = 1$ S-matrix")

As is well known, (the singlet sector of) the c=1 matrix model can be mapped to free fermions

$$\int \frac{[dX]}{[dU]} \langle \Psi_1 | X \rangle \langle X | \Psi_2 \rangle = \int \Big(\prod_{i=1}^{N} dx_i \Big) \langle \tilde{\Psi}_1 | X \rangle \langle X | \tilde{\Psi}_2 \rangle$$

$$\langle X | \tilde{\Psi} \rangle \equiv \underbrace{\Big(\prod_{i<j} (x_i - x_j) \Big)}_{\text{Vandermonde determinant}} \times \langle X | \Psi \rangle \quad \text{: completely antisymmetric}$$

This allows us to *partially* circumvent the problem of 'continuous' quantum statistics, using appropriate generalization of the ordinary fermion algebra.

The free fermion field might be regarded as quantum field for D3-branes,
restricted to 1/2-BPS sector,

but not quite !

How to represent SO(6) degrees of freedom?
There is no known extension of the fermion picture to multi-matrix models.

Various old attemps ($\sim$ the early 80s) of field theories
on the space of invariants (Wilson loops) do not fit for our purpose.

An answer

proposed in T. Y. hep-th/0510114

The extremal correlators can be represented as correlators of products of bilinear operators of D3-brane field whose normal modes are represented by *composite operators* of the following form:

$$(b_{n,I}, b^{\dagger}_{n,I}) = (c_I \otimes b_n, c^{\dagger}_I \otimes b^{\dagger}_n)$$

$(b_n, b^{\dagger}_n) =$ ordinary fermion creation and annhilation operator$n \sim$ 'energy' $=$ conformal dimension

$$\{b_n, b^{\dagger}_m\} = \delta_{nm}$$

$(c_I, c^{\dagger}_I) =$ satisfying *Cuntz-algebra*: $I=$ SO(6) indices

$$c_{I_1} c^{\dagger}_{I_2} = \delta_{I_1 I_2}, \quad \sum_{I=0}^{\infty} c^{\dagger}_I c_I = 1$$

- The single-trace operators

$$w^I_{i_1 i_2 \cdots i_k} \mathrm{Tr}(\phi_{i_1}\phi_{i_2}\cdots\phi_{i_k})$$

on the SYM_4 side are expressed as bilinear composite fields in terms of D-brane fields

$$B_k = \int [d^6\phi |d\alpha|^2] \sum_{n=-\infty}^{\infty} \Psi_n^{(-)}[\phi,\alpha,\overline{\alpha}] w^I_{i_1 i_2 \cdots i_k} \phi_{i_1}\phi_{i_2}\cdots\phi_{i_k}\alpha^k \Psi_n^{(+)}[\phi,\alpha,\overline{\alpha}].$$

The D-brane fields

$$\Psi_n^{(+)}[\phi,\alpha,\overline{\alpha}] = \sum_{k=0}^{\infty} \sqrt{\frac{2^{n+k}}{(n+k)!}} \mathrm{e}^{-|\alpha|^2 - |\phi|^2/4} \sqrt{k!}\phi_{i_1}\phi_{i_2}\cdots\phi_{i_k}\alpha^{n+k}\, b_{n+k}\, c_{i_1 i_2 \cdots i_k}, \quad \text{etc}$$

have definite charges ($Q_S = n$), corresponding to a superselection symmetry under the transformation ('S-charge' symmetry)

$$b_n^\dagger \to \mathrm{e}^{in\theta} b^\dagger, \quad c_I^\dagger \to \mathrm{e}^{-ik(I)\theta} c_I^\dagger,$$
$$k(I) = \text{rank of the representation } I$$

- The ground state (=$\mathrm{AdS}_5 \times \mathrm{S}^5$) of N D3-brane sector is the highest 'S-charge' state

$$|N\rangle \equiv (c_0^\dagger)^N \otimes b^\dagger_{N-1} b^\dagger_{N-2} \cdots b_0^\dagger |0\rangle$$

- The composite normal-mode operator

$$b_{n,I} = c_I \otimes b_n$$

implies that the quantum statistics of D3-branes in the 1/2-BPS sector are governed by a strong exclusion principle ('Dexclusion' principle or 'super' Pauli principle) that the D3-brane states with a fixed energy(=conformal dimension) cannot be occupied multiply even if they have different SO(6) indices "I".

This is consistent with the expected property of *entropy of the so-called 'superstar' solutions* of type IIB supergravity.

- Because of the 'free' nature of the Cuntz algebra, the algebra of D-brane fields are quite non-local (do not satisfy canonical commutation relations),

but

the non-locality is consistent with the space-time uncertainty relation, since the S-charge symmetry is equivalent with the scaling symmetry of super Yang-Mills theory

$$x^\mu \to \lambda^{-1} x^\mu, \quad \phi^i (\sim X^i) \to \lambda \phi^i \ (\sim \lambda X^i)$$

- The Hilbert space of the products of bilinear operators are equivalent with the space of multi-trace operators in the 1/2-BPS sector of the super Yang-Mills theory.

In other words, the 'bosonized' fields (*collective fields* for Yang-Mills theory) obey *effectively* the usual canonical quantum statistics.

- Hamiltonian is given as

$$H = \int [d^6\phi |d\alpha|^2] \sum_{n=-\infty}^{\infty} \Psi_n^{(-)}[\phi, \alpha, \overline{\alpha}](\alpha \frac{\partial}{\partial \alpha} + \alpha\overline{\alpha})\Psi_n^{(+)}[\phi, \alpha, \overline{\alpha}]$$

$$= \underbrace{\left(\sum_{(k)=0}^{\infty} c_{(k)}^{\dagger} c_{(k)} \right)}_{=\text{identity}} \left(\sum_{n=0}^{\infty} n b_n^{\dagger} b_n \right) = \sum_{n=0}^{\infty} n b_n^{\dagger} b_n$$

◇ ◇ ◇ ◇

Unfortunately, this approach depends crucially on the specialty of 1/2-BPS sector.

Lesson:

We have to deal directly with the problem of *continuous* quantum statistics,
and
should not afraid of introducing some *non-standard algebras** (and their representations) of fields.

We have to invent a new appropriate mathematical framework
to faithfully represent the D-brane statistics.

* *may be both non-commutative and non-associative !*

Second-quantization of D0-brane quantum mecahnics

As a first step towards this direction, I will report some preliminary results towards quantum field theory of D-particles.

- It is natural to try to define a Fock space of D-particles by treating all of the Hilbert spaces of (super) Yang-Mills quantum mechanics with different N (=the number of D-particles) in a completely unified way.

  So what we discuss now is the possibility of an extension of elementary *second quantization* in non-relativistic particle quantum mechanics to D-particles:

$$\Psi(x_1, x_2, \ldots, x_N; t)$$

$$\Downarrow$$

$$|\Psi(t)\rangle = \Big(\prod_{i=1}^{N}\int dx_i\Big)\Psi(x_1, x_2, \ldots, x_N; t)\psi^\dagger(x_N)\psi^\dagger(x_{N-1})\cdots\psi^\dagger(x_1)|0\rangle$$

$$\psi^\dagger(x) : \mathcal{H}_N \to \mathcal{H}_{N+1}$$

$$\psi(x) : \mathcal{H}_N \to \mathcal{H}_{N-1}$$

◇ $\psi^\dagger(x_N)\psi^\dagger(x_{N-1})\cdots\psi^\dagger(x_1)|0\rangle = \psi^\dagger(x_{P(N)})\psi^\dagger(x_{P(N-1)})\cdots\psi^\dagger(x_{P(1)})|0\rangle$

◇ $\psi(y)\psi^\dagger(x_N)\cdots\psi^\dagger(x_1)|0\rangle = \frac{1}{(N-1)!}\sum_P \delta(y-x_{P(N)})\psi^\dagger(x_{P(N-1)})\ldots\ldots\psi^\dagger(x_{P(1)})|0\rangle$

with $P : (12\ldots N) \longrightarrow (i_1 i_2 \ldots i_N), P(k) = i_k$. (bosons for definiteness)

These relations can also be expressed as canonical commutators.

$$[\psi(x), \psi^\dagger(y)] = \delta(x-y), \quad [\psi(x), \psi(y)] = 0 = [\psi^\dagger(x), \psi^\dagger(y)]$$

Actually, however, the above two relations (not CCR) are sufficient to represent physical operators as bilinears of qunatized fields.

In the present situation, by constrast,

◇ *Configuration space* :

$(x_1, x_2, \cdots, x_N) \quad \longrightarrow \quad X_{ab} = \overline{X}_{ba}$

Permutation symmetry $\quad \longrightarrow \quad X_{ab} \to (UXU^{-1})_{ab}$

◇ *Two nobel features* :

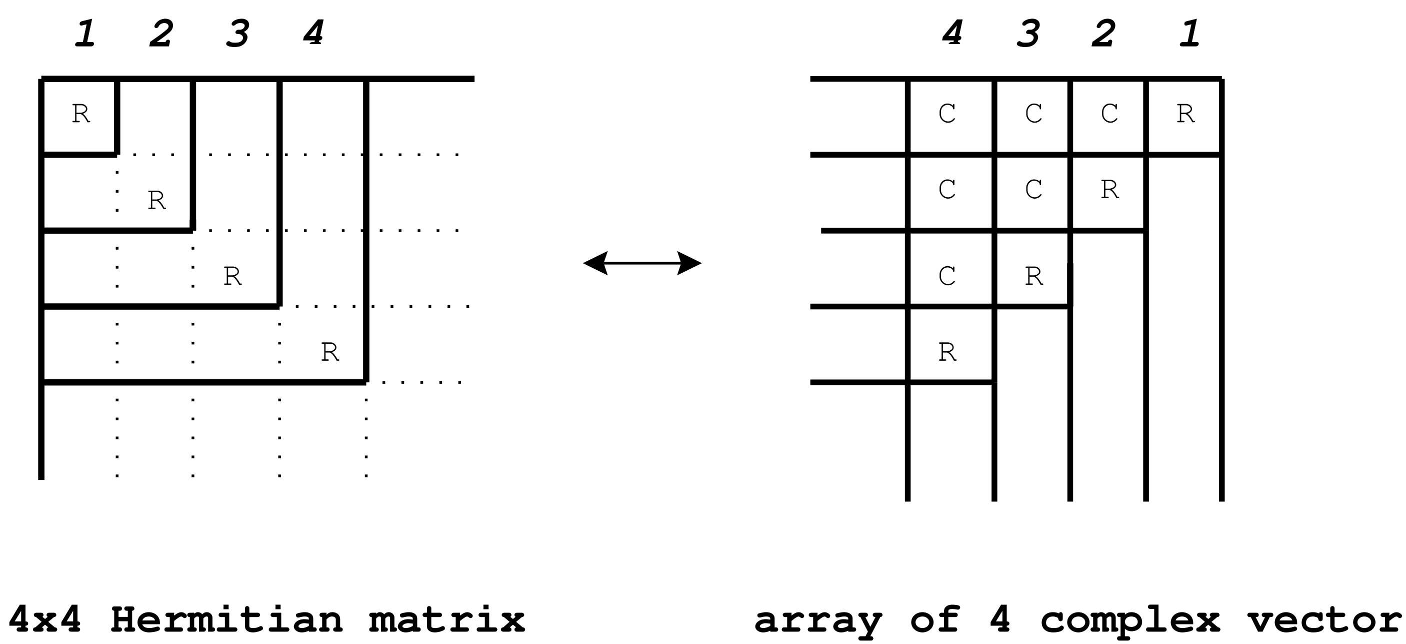
– The increase of the degrees of freedom in $\mathcal{H}_N \longrightarrow \mathcal{H}_{N+1}$ is

$$2N + 1 = (N+1)^2 - N^2. \qquad \big(\text{not } 1 = (N+1) - N\big)$$

– The statitical symmetry is the continuous group U(N), instead of S$_N$.

Usual canonical formalism is not applicable to D-brane field theory !

We can achieve a second quantization of this system by embedding the matrices into an (infinite) array of infinite-dimensional complex vectors $(z_1 = x_1 + iy_1, z_2 = x_2 + iy_2, \ldots)$ as



- D-brane field (suppressing time variable) : $\phi^-[z, \overline{z}], \phi^+[z, \overline{z}]$
- They act on the D-particle Fock space as (symbolically)

$$\phi^+ : |0\rangle \to \phi^+[z^{(1)}, \overline{z}^{(1)}]|0\rangle \to \phi^+[z^{(2)}, \overline{z}^{(2)}]\phi^+[z^{(1)}, \overline{z}^{(1)}]|0\rangle \to \cdots$$

$$\phi^- : 0 \leftarrow |0\rangle \leftarrow \phi^+[z^{(1)}, \overline{z}^{(1)}]|0\rangle \leftarrow \phi^+[z^{(2)}, \overline{z}^{(2)}]\phi^+[z^{(1)}, \overline{z}^{(1)}]|0\rangle \leftarrow \cdots$$

with the following conditions

◇ *'reduction'* conditions:

$$\partial_{y_1^{(1)}}\phi^+[z^{(1)},\overline{z}^{(1)}]|0\rangle = 0, \quad \partial_{z_k^{(1)}}\phi^+[z^{(1)},\overline{z}^{(1)}]|0\rangle = 0 \quad \text{for} \quad k \geq 2$$

$$\partial_{y_2^{(2)}}\phi^+[z^{(2)},\overline{z}^{(2)}]\phi^+[z^{(1)},\overline{z}^{(1)}]|0\rangle = 0, \quad \partial_{z_k^{(2)}}\phi^+[z^{(2)},\overline{z}^{(2)}]\phi^+[z^{(1)},\overline{z}^{(1)}]|0\rangle = 0 \quad \text{for} \quad k \geq 3$$

. . . .

◇ *'gauge-statistic'* conditions:

$$\phi^+[(UXU^{-1})_{12},(UXU^{-1})_{21},(UXU^{-1})_{22}]\phi^+[(UXU^{-1})_{11}]|0\rangle = \phi^+[z^{(2)},\overline{z}^{(2)}]\phi^+[z^{(1)},\overline{z}^{(1)}]|0\rangle$$

. . . .

where the identification between the complex vector coordinates z's and the matrices X's is made as being indicated by the figure of the previous page

$$X = \begin{pmatrix} x_1^{(1)} & z_1^{(2)} \\ \overline{z}_1^{(2)} & x_2^{(2)} \end{pmatrix} \quad \ldots\ldots$$

Extension to 3-body and higher states is straigtforward, but requires more precise notations, taking into account non-associativity of the field multiplication rules

Similarly, the action of annihilation operators is defined as

$$\phi^-[z,\overline{z}]|0\rangle = 0$$

$$\phi^-[z,\overline{z}]\phi^+[z^{(1)},\overline{z}^{(1)}]|0\rangle = \delta(x_1 - x_1^{(1)})\delta(y_1)\prod_{k\geq 2}\delta^2(z_k - z_k^{(1)})|0\rangle$$

$$\phi^-[z,\overline{z}]\phi^+[z^{(2)},\overline{z}^{(2)}]\phi^+[z^{(1)},\overline{z}^{(1)}]|0\rangle$$

$$= \int dU\,\delta^2(z_1 - (UXU^{-1})_{12}))\delta(x_2 - (UXU^{-1})_{22})\prod_{k\geq 3}\delta^2(z_k - z_k^{(2)})\,\phi^+[(UXU_{11}^{-1}]|0\rangle\,,$$

$$\ldots\ldots\ldots\ldots$$

$$etc$$

These definitions are sufficient to express ordinary gauge-invariant Yang-Mills operators in terms of *bilinears of D-brane fields in N-independent form.*

For example,

- The number operator :

$$N = \int [dz]\,\phi^+[z]\phi^-[z] \equiv \langle\phi^+,\phi^-\rangle\,.$$

$$\mathrm{Tr}(X^i X^j)\mathrm{Tr}(X^j X^i) + \mathrm{Tr}(X^i X^i X^j X^j) = (\langle\phi^+,\phi^-\rangle + 1)\langle\phi^+, (\bar{z}^i\cdot z^j)(\bar{z}^j\cdot z^i)\phi^-\rangle$$

$$etc$$

- *The Schrödinger equation :*

$$\mathcal{H}|\Psi\rangle = 0\,,$$

$$\mathcal{H} = i(4\langle\phi^+,\phi^-\rangle + 1)\partial_t$$

$$+2g_s\ell_s\Big((\langle\phi^+,\phi^-\rangle + 1)\langle\phi^+, \partial_{\bar{z}^i}\cdot\partial_{z^i}\phi^-\rangle + 3\langle\phi^+,\partial_{\bar{z}^i}\phi^-\rangle\cdot\langle\phi^+,\partial_{z^i}\phi^-\rangle\Big)$$
$$+\frac{1}{2g_s\ell_s^5}(4\langle\phi^+,\phi^-\rangle + 1)(\langle\phi^+,\phi^-\rangle + 1)\langle\phi^+, \Big((\bar{z}^i\cdot z^j)^2 - (\bar{z}^i\cdot z^j)(\bar{z}^j\cdot z^i)\Big)\phi^-\rangle$$

  - global U(∞) symmetry $z \longrightarrow \mathcal{U}z$.
  - supersymmetrization is straightforward
    $\longrightarrow$ D-particle superfield $\phi^{\pm}[z,\bar{z},\theta,\bar{\theta}]$ on infinite dimensional superspace of complex vectors.
  - scaling symmetry $\quad t \longrightarrow \lambda^{-1}t, \quad z \longrightarrow \lambda z, \quad g_s \longrightarrow \lambda^3 g_s$
    $\longrightarrow$ consistency with space-time uncertainty relation

– a simplication in the large $\langle N \rangle \longrightarrow \infty$ limit

$$i\partial_t|\Psi\rangle = \Big[-\frac{g_s \ell_s}{2}\langle \phi^+, \partial_{\bar{z}^i} \cdot \partial_{z^i}\phi^- \rangle - \frac{\langle N \rangle}{2 g_s \ell_s^5}\langle \phi^+, \Big((\bar{z}^i \cdot z^j)^2 - (\bar{z}^i \cdot z^j)(\bar{z}^j \cdot z^i) \Big)\phi^- \rangle \Big] |\Psi\rangle$$

Hamiltonian is *almost* bilinear.

$\Downarrow$

large N-planar limit reduces somehow to that of complex vector model?

not quite !

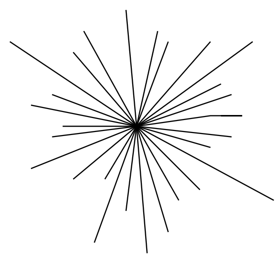
Complications are hided in the strange non-commutative and non-associative field algebra

Basic (*intuitive*) features of D-particle field theory:

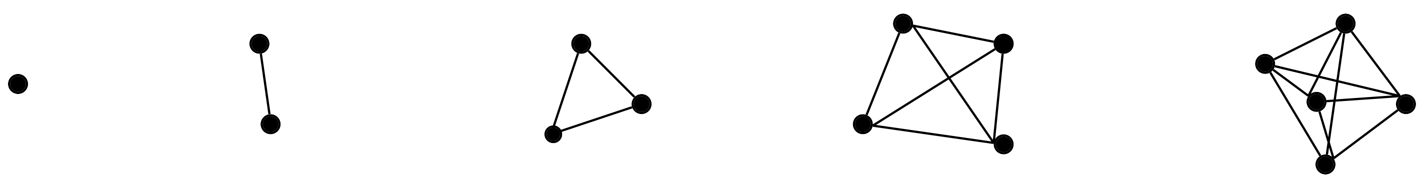
- The D-brane fields do not satisfy canonical commutation relations. Instead, they are characterized by an entirely new *'continuous' quantum statistics.*
- Putting aside this crucial difference, the concept of D-brane field is somewhat reminiscent of Yukawa's idea (1968) of *elementary domains*.

$$\Psi(D,t): \quad D \to \{z_n, \bar{z}_n\}$$

The 'elementary domains' $\{D\}$ of D-brane fields are represented by open-string degrees of freedom and as such are 'fuzzier' than Yukawa's. Our elementary domains are a sort of *"clouds of threads"* emanating from D-particles.



- Our domains are much more dynamical and obey the space-time uncertainty relation, and the theory encompasses, in principle, General Relativity through open-closed string duality.
- After acting on the states, the complexity of domains depends on the number of D-particles.



Problems:

- clarifications on the operator algebra of D-particle fields
  (or any appropriate mathematical framework)
- development of computational tools using the D-particle fields
  Is it really useful for studying dynamics in the large N limit?
- covariantization in 10 dimensions and 11 dimensional M-theory context ?
  $\longrightarrow$ treatment of both D and anti-D particles.
- background independence and any geometrical interpretation ?
  (A first step toward this goal would be to eliminate the dependence on $g_s \sim$ dilaton.)

and so forth ...