Fiber bundles and matrix models

-gauge/gravity correspondence for SU(2|4) theories-

A. Tsuchiya (Osaka U.)

In collaboration with

T. Ishii, G. Ishiki, S. Shimasaki (Osaka U.)

and Y. Takayama (NBI)

- JHEP 0611(2006)089 (hep-th/0610038)
- hep-th/0702xxx

Introduction

- Emergence of space-time is one of the key concepts of matrix models as nonperturbative definition of superstring

  e.g.) Matrix theory BFSS IIB matrix model IKKT
- Eigenvalues of matrix in c=1 matrix model ~ Liouville direction

  Das-Jevicki, Gross-Klebanov, Polchinski
- Bubbling AdS geometry LLM, Jevicki's talk
- It should also be useful to study relationship between field theories and matrix models
- large N reduction Eguchi-Kawai
- quenched reduced model

  Bhanot-Heller-Neuberger, Parisi, Gross-Kitazawa

  twisted reduced model Gonzalez-Arroyo-Okawa

  ~ noncommutative field theory
- T-duality for D-brane effective theories `matrix T-duality' Taylor
- It is important to understand how curved space-time is realized in matrix models ~ our problem cf.) Hanada-Kawai-Kimura

Here we focus on the gauge/gravity correspondence for the SU(2|4) theories proposed by Lin-Maldacena

1) N=4 SYM on R x S^3/Z_k

dim. reduction

2) 2+1 SYM on R x S^2

dim. reduction

3) plane wave matrix model

BMN

large N

extension of matrix T-duality to that on nontrivial fiber bundle

large N

extension of twisted reduced model to that on curved space

doubly interesting

- different vacua of one theory ~ different geometries
- suggest relations between fields theories on curved spaces and matrix model `inverse' of dimensional reductions

We show these relations directly on gauge theory side

SU(2|4) theories and gauge/gravity correspondence

SU(2|4) theories

Start with N=4 SYM on R x S^3

$$SU(2,2|4) \supset SO(2,4) \times SO(6) \supset R \times SO(4) \times SO(6)$$

$$SO(4) = SU(2)_L \times SU(2)_R$$

Divide by subgroup of $SU(2)_R$ ⟹ SU(2|4) theories

16 supercharges

Z_k ⟶ 1) N=4 SYM on R x S^3/Z_k

$$S_{S^3/Z_k} = \int dt d\Omega_3 \text{Tr}\left(\frac{1}{4}F_{ab}^2 + \cdots\right)$$

U(1) ⟶ 2) 2+1 SYM on R x S^2

$$S_{S^2} = \int dt d\Omega_2 \text{Tr}\left(-\frac{1}{2}(f_{12} - \phi)^2 - \frac{1}{2}(D_a\phi)^2 + \cdots\right)$$

SU(2) ⟶ 3) plane wave matrix model

Kim-Klose-Plefka

$$S_{PW} = \text{Tr}\left(-\frac{1}{2}(Y_i - \frac{i}{2}\epsilon_{ijk}[Y_j, Y_k])^2 + \cdots\right)$$

Dim. reduction

Common features: mass gap, discrete spectrum,

many discrete vacua

Gravity duals

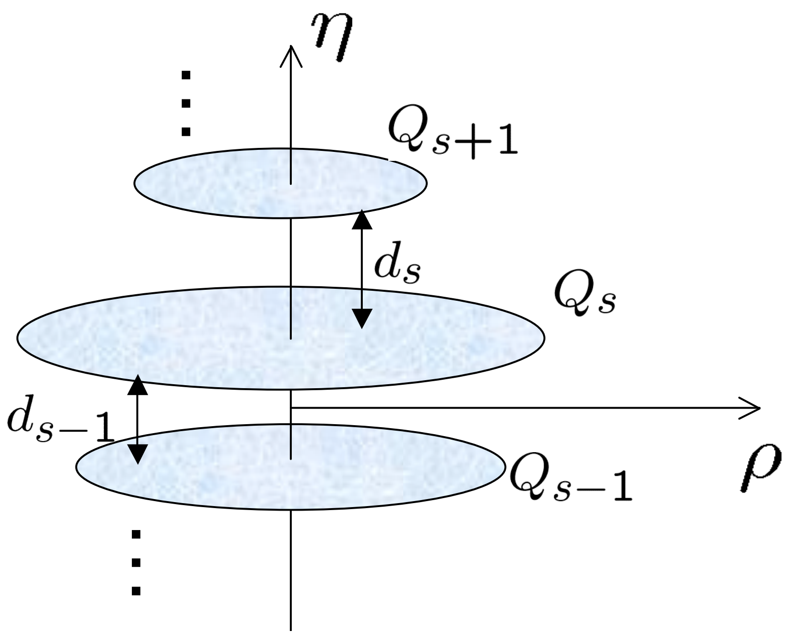
Lin-Maldacena

general form of smooth solutions of type IIA SUGRA
that have SU(2|4) isometry

$$ds^2_{10} = \left(\frac{\ddot{V} - 2\dot{V}}{-V''}\right)\left\{-4\frac{\ddot{V}}{\ddot{V} - 2\dot{V}}dt^2 + \frac{-2V''}{\dot{V}}(d\rho^2 + d\eta^2) + 4d\Omega_5^2 + 2\frac{V''\dot{V}}{\Delta}d\Omega_2^2\right\}$$

$$\Delta = (\ddot{V} - 2\dot{V})V'' - (\dot{V}')^2, \qquad \dot{} = \rho\partial_\rho, \quad ' = \partial_\eta$$

$V(\rho, \eta)$: electrostatic potential for axially symmetric system



# of conducting disks T ~ topology

Q_s: D2-brane charge

d_s: NS 5-brane charge

asymptotic behavior of V
specifies a theory

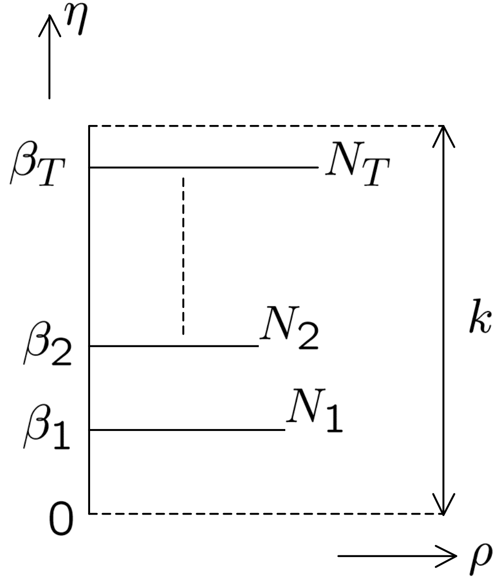
config. of disks specifies a vacuum

Vacua

1) SYM on R x S^3/Z_k

holonomy around generator of $\pi_1(S^3/Z_k) = Z_k$

$$W = \mathrm{diag}(\underbrace{e^{i\frac{2\pi}{k}\beta_1}, \cdots, e^{i\frac{2\pi}{k}\beta_1}}_{N_1}, \underbrace{e^{i\frac{2\pi}{k}\beta_2}, \cdots, e^{i\frac{2\pi}{k}\beta_2}}_{N_2}, \cdots, \underbrace{e^{i\frac{2\pi}{k}\beta_T}, \cdots, e^{i\frac{2\pi}{k}\beta_T}}_{N_T})$$

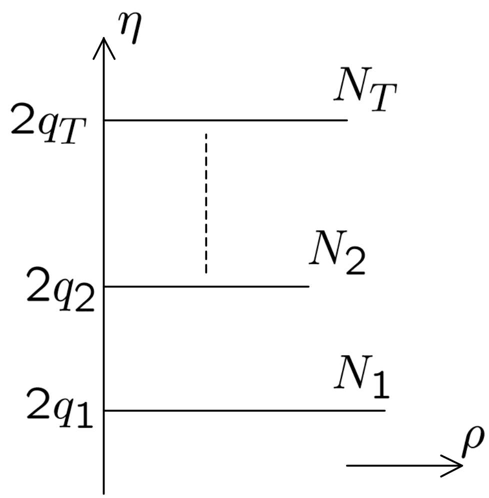


2) SYM on R x S^2 Dirac monopoles

$$\phi = \mathrm{diag}(\underbrace{q_1, q_1, \cdots, q_1}_{N_1}, \underbrace{q_2, q_2, \cdots, q_2}_{N_2}, \cdots, \underbrace{q_T, q_T, \cdots, q_T}_{N_T})$$

$$A_\theta = 0$$

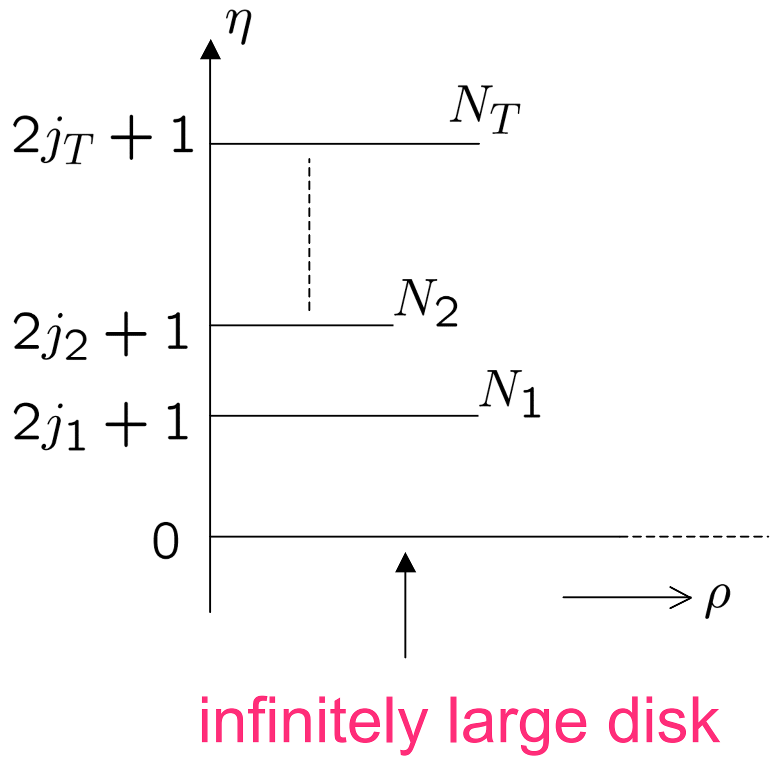
$$A_\varphi = \begin{cases} (1 - \cos\theta)\,\phi & \text{for } 0 \le \theta < \pi \\ (-1 - \cos\theta)\,\phi & \text{for } 0 < \theta \le \pi \end{cases}$$



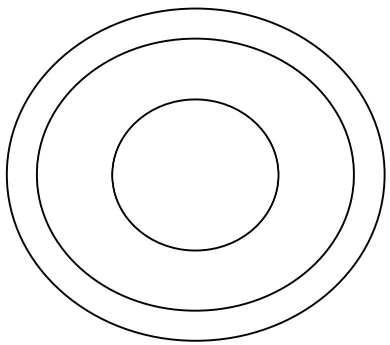
3) PWMM

concentric fuzzy spheres with different radii

$$Y_i = -\begin{pmatrix} \overbrace{\begin{matrix} L_i^{[j_1]} & & \\ & \ddots & \\ & & L_i^{[j_1]} \end{matrix}}^{N_1} & & & \\ & \overbrace{\begin{matrix} L_i^{[j_2]} & & \\ & \ddots & \\ & & L_i^{[j_2]} \end{matrix}}^{N_2} & & \\ & & \ddots & \\ & & & \overbrace{\begin{matrix} L_i^{[j_T]} & & \\ & \ddots & \\ & & L_i^{[j_T]} \end{matrix}}^{N_T} \end{pmatrix} \quad \Longleftrightarrow$$

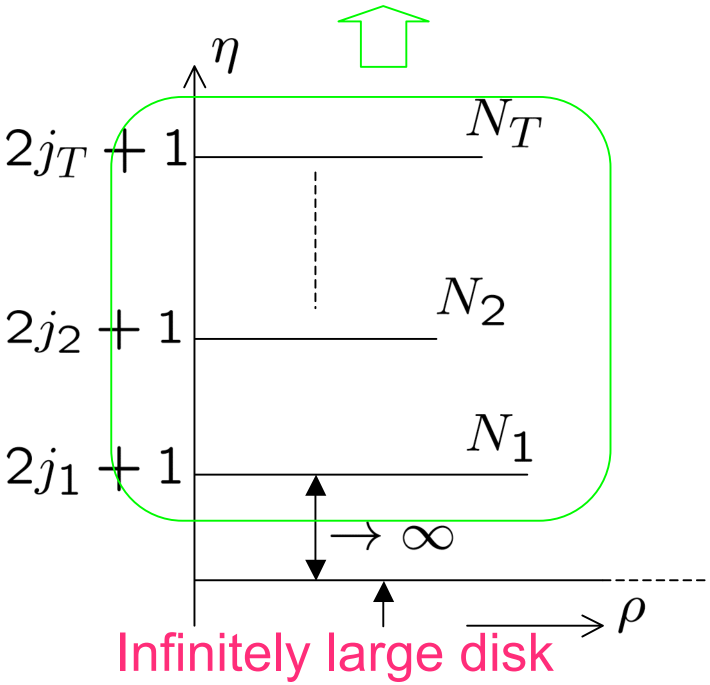


$L_i^{[j_s]}$: spin j_s representation of $SU(2)$

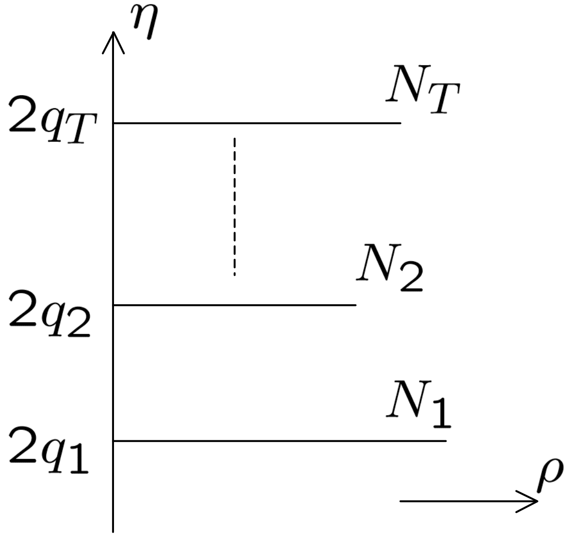


PWMM vs SYM on R x S^2

PWMM



SYM on R x S^2



$$j_s,\ j_t \to \infty$$
$$j_s - j_t = q_s - q_t$$

theory around

$$Y_i = -\begin{pmatrix} \overbrace{L_i^{[j_1]} \ \ddots \ L_i^{[j_1]}}^{N_1} & & \\ & \ddots & \\ & & \overbrace{L_i^{[j_T]} \ \ddots \ L_i^{[j_T]}}^{N_T} \end{pmatrix}$$

=

theory around

$$\phi = \mathrm{diag}(\underbrace{q_1,\cdots,q_1}_{N_1},\cdots,\underbrace{q_T,\cdots,q_T}_{N_T})$$
$$A_\theta = 0$$
$$A_\varphi = \begin{cases} (1-\cos\theta)\,\phi & \text{for } 0 \le \theta < \pi \\ (-1-\cos\theta)\,\phi & \text{for } 0 < \theta \le \pi \end{cases}$$

Outline of proof

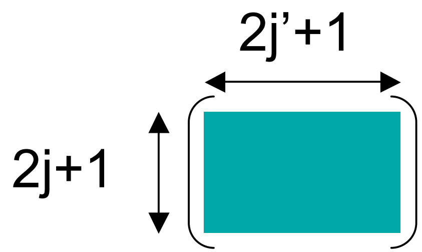
See agreement of harmonic expansions of the theories around the vacua of two theories including int. terms

monopole harmonics by Wu-Yang $Y_{Jmq}(\theta, \varphi) \quad (|q| \leq J)$

|| $j, j' \to \infty,\ j - j' = q$

\`fuzzy' spherical harmonics $\hat{Y}^{(jj')}_{Jm} \sim \sum_{rr'} C^{Jm}_{jr\,jr'} |jr\rangle\langle j' - r'| \quad (|j - j'| \leq J \leq j + j')$

~basis for rectangular matrices



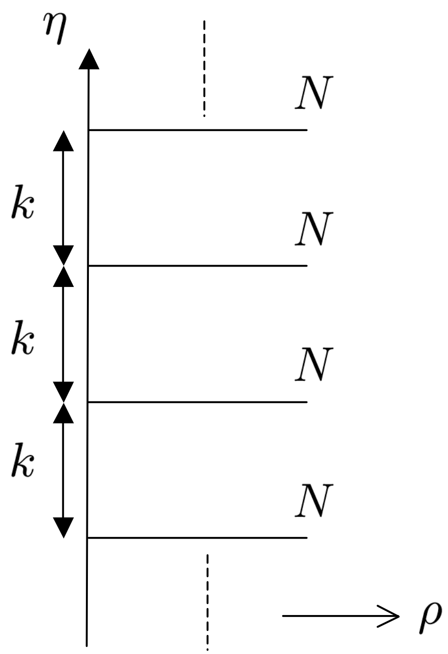
Topologically nontrivial configuration

continuum limit of concentric fuzzy spheres with different radii = monopoles on S^2

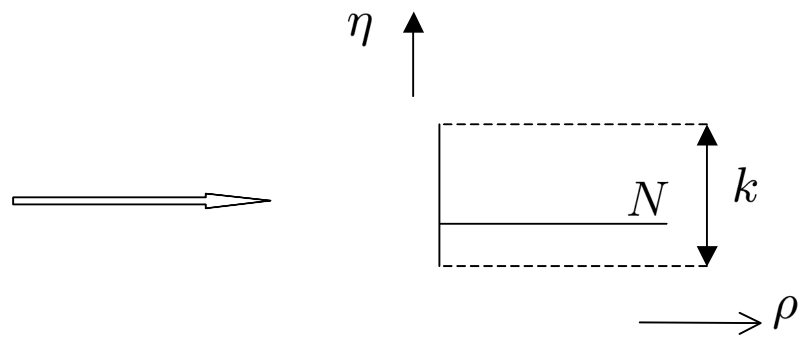
c.f.) Grosse-Klimcik-Presnajder, Baez-Balachandran-Ydri-Vaidya,Landi, Balachandran-Vaidya, Valtancoli, Steinacker, Karabali-Nair-Polychronakos, Carow-Watamura-Steinacker-Watamura, Aoki-Iso-Nagao, Madore, Balachandran-Govindarajan-Ydri, Ydri,Balachandran-Immirzi, Aoki-Iso-Maeda-Nagao, Aoki-Iso-Maeda,…..

SYM on R x S^2 vs SYM on R x S^3/Zk

SYM on R x S^2



SYM on R x S^3/Z_k



theory around $-\infty < s < \infty$

$$\phi = k\,\mathrm{diag}\left(\cdots, \underbrace{\frac{s-1}{2},\cdots,\frac{s-1}{2}}_{N}, \underbrace{\frac{s}{2},\cdots,\frac{s}{2}}_{N}, \underbrace{\frac{s+1}{2},\cdots,\frac{s+1}{2}}_{N},\cdots\right)$$

$$A_\theta = 0$$

$$A_\varphi = \begin{cases} (1-\cos\theta)\,\phi & 0 \le \theta < \pi \\ (-1-\cos\theta)\,\phi & 0 < \theta \le \pi \end{cases}$$

with periodicity $X^{(s+1,t+1)} = X^{(s,t)}$

=

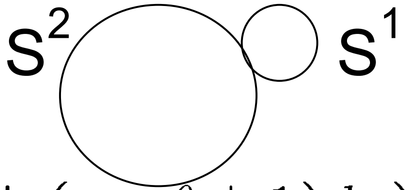
theory around trivial vacuum

extension of matrix T-duality to nontrivial fiber bundle S^3/Z_k as S^1/Z_k on S^2

Extension of matrix T-duality

Matrix T-duality on principal S^1 bundle

S^3 as S^1 on S^2 (Hopf bundle)



metric of S^3 $\quad ds^2_{S^3} = d\theta^2 + \sin^2\theta d\varphi^2 + (d(\psi \pm \varphi) + (\cos\theta \pm 1)d\varphi)^2$

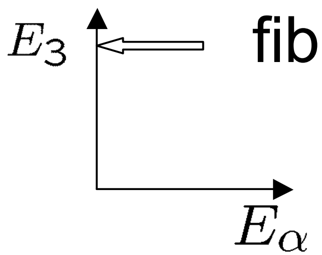
fiber direction

$y_{[1]} = \psi + \varphi \quad$ for $0 \leq \theta < \pi \quad$ patch 1

$y_{[2]} = \psi - \varphi \quad$ for $0 < \theta \leq \pi \quad$ patch 2

connection 1-form $\quad \omega = d(\psi \pm \varphi) + (\cos\theta \mp 1)d\varphi$

vertical-horizontal decomposition of tangent space of S^3



fiber direction

$$\omega(E_\alpha) = 0 \quad (\alpha = 1, 2)$$

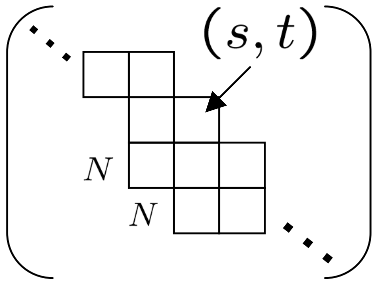
$$S_{S^3} = \int d\Omega_3 \mathrm{Tr}\left(\frac{1}{4}F_{ab}^2\right) \Longrightarrow S_{S^2} = \int d\Omega_2 \mathrm{Tr}\left(\frac{1}{2}(f_{12} - \phi)^2 + \frac{1}{2}(D_\alpha\phi)^2\right)$$

$$A_\alpha = a_\alpha \quad (\alpha = 1, 2)$$
$$A_3 = \phi$$

matrix T-duality between S^2 and S^3

theory around the vacuum of S_{S^2} with U(N×∞) and periodicity
= S_{S^3} with U(N)

U(N) ⟶ U(N×∞)



vacuum

$$\hat{\phi} = \frac{1}{2}(\cdots, s-1, s, s+1, \cdots)$$
$$\hat{a}_1 = 0$$
$$\hat{a}_2^{[1]} = \tan\frac{\theta}{2}\hat{\phi}, \quad \hat{a}_2^{[2]} = -\cot\frac{\theta}{2}\hat{\phi}$$

s/2: monopole charge

periodicity for fluctuations $\tilde{a}_\alpha^{[I](s,t)} = \tilde{a}_\alpha^{[I](s-t)}, \quad \tilde{\phi}^{[I](s,t)} = \tilde{\phi}^{[I](s-t)}$

Fourier transf. on each patch (T-duality)

$$A_\alpha = \sum_s \tilde{a}_\alpha^{[1,2](s)} e^{-\frac{i}{2}s(\psi\pm\varphi)}$$
$$A_3 = \sum_s \tilde{\phi}^{[1,2](s)} e^{-\frac{i}{2}s(\psi\pm\varphi)}$$

⇨ relation between monopole harmonics and harmonics on S^3

monopole charge =momentum

$/Z_k$ easy

We can extend matrix T-duality to that on general S^1 principal bundles

Ex.)

S^5 as S^1 on CP^2 (gravitational and electromagnetic instanton)

Heisenberg nilmanifold as S^1 on T^2 (constant magnetic flux on T^2)

Conclusion & Discussions

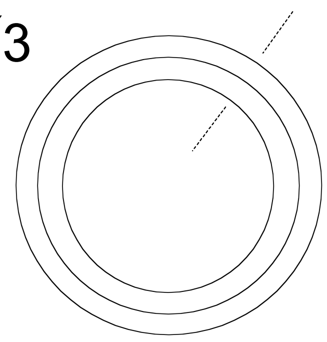
- We showed

theory around each vacuum of SYM on R x S^3/Z_k
= theory around a vacuum of SYM on R x S^2 with periodicity

theory around each vacuum of SYM on R x S^2
= theory around a vacuum of PWMM

- Nontrivial check of gauge/gravity correspondence for SU(2|4) theories
- Continuum limit of concentric fuzzy spheres~ monopoles on S^2
- Extension of matrix T-duality to that on S^1 principal bundles
- Theory around every vacuum of SU(2|4) theories is realized in PWMM
- N=4 SYM on R x S^3 is realized in PWMM
- S^3 is realized in terms of three matrices Y_1 Y_2 Y_3

$$Y_i = -\begin{pmatrix} \ddots & & & & \\ & L_i^{[j_{s-1}]} & & & \\ & & L_i^{[j_s]} & & \\ & & & L_i^{[j_{s+1}]} & \\ & & & & \ddots \end{pmatrix} \otimes \mathbf{1}_N$$



Outlook

- CS on $S^3 \longleftrightarrow$ pure Yang Mills on S^2 with K. Ohta
- Nonabelian case e.g.) S^7 as S^3 on S^4 , $SU(3)$ as $U(2)$ on CP^2
- Lattice theory for N=4 SYM on R x S^3 cf.) Kaplan
- Relate our findings to (flux) compactification in string theory

Review of matrix T-duality

Taylor

Dimensional reduction

U(N) YM on R^p x S^1

$$S_{p+1} = \int d^{p+1}x \, \frac{1}{4} \mathrm{Tr}(F_{MN}^2)$$

$$x^M = (x^\mu, y) \qquad \mu = 1, \cdots, p$$

y~S^1 with radius R

Dp-brane

$A_\mu = a_\mu$

$A_y = \phi$

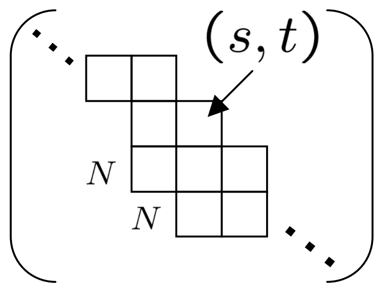
drop y-dep.

U(N) YM with Higgs on R^p

$$S_p = \int d^p x \, \mathrm{Tr} \left(\frac{1}{4} f_{\mu\nu}^2 + \frac{1}{2} (D_\mu \phi)^2 \right)$$

D(p-1)-brane

$U(N) \to U(N \times \infty)$



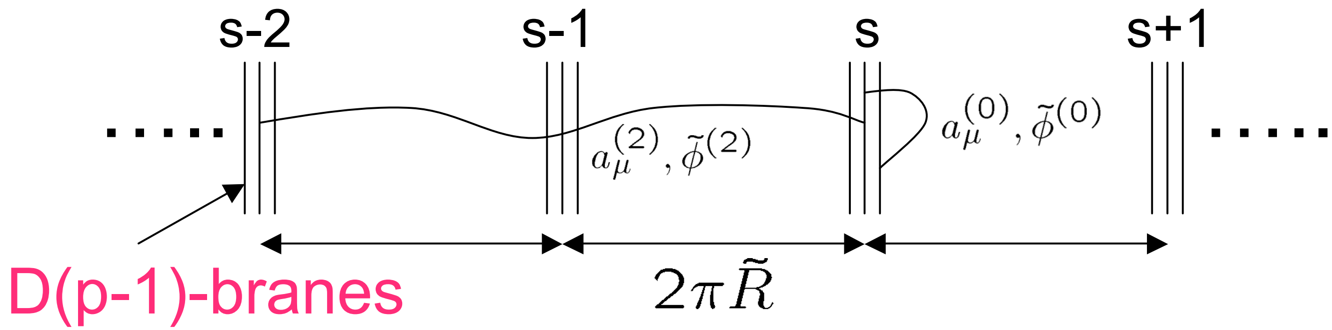
S^1 compactification

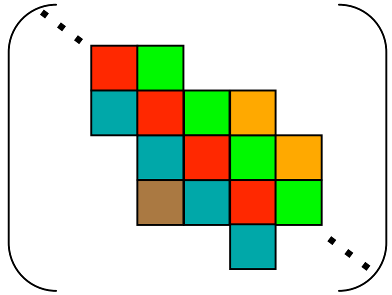
S^1 with radius $\tilde{R}$

$\phi = \hat{\phi} + \tilde{\phi}$ $\qquad \hat{\phi} = 2\pi\tilde{R}\,\mathrm{diag}(\cdots, s-1, s, s+1, \cdots) \otimes 1_N$

periodicity

$$\begin{cases} a_\mu^{(s,t)} = a_\mu^{(s-t)} \\ \tilde{\phi}^{(s,t)} = \tilde{\phi}^{(s-t)} \end{cases}$$

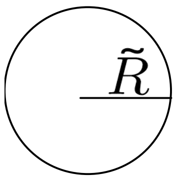




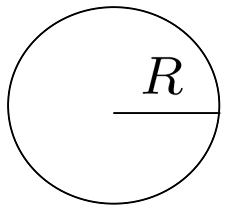
T-duality

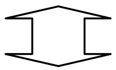
$\hat{R} = \frac{1}{2\pi R}$

winding #: $s - t$ ⟺ momentum: $s - t$



D(p-1)-brane ⟺ Dp-brane





$$\begin{cases} A_\mu(x,y) = \sum_s a_\mu^{(s)}(x) e^{-i\frac{s}{R}y} \\ A_y(x,y) = \sum_s \tilde{\phi}^{(s)}(x) e^{-i\frac{s}{R}y} \end{cases}$$

⇨ recovery of Sp+1