Searching for predictions in corners of the landscape

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Yoneya-Fest

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Outline

1. Predictions from string theory
2. Case study: Intersecting brane models on toroidal orientifold
3. Proof of finiteness, estimates for $\mathcal{N}_G$ & generation numbers
4. Generalization to other Calabi-Yau orientifolds
5. Conclusions

1. Predictions from string theory

Can string theory make predictions for particle physics?

If we could do experiments at $> 10^{19}$ GeV,
answer would probably be Yes.

String theory predicts (in any macroscopic space-time):

—Gravity + gauge fields + matter in $D \leq 11$

—SUSY (at some energy scale)

But there is a big gap between

Planck scale	10^{19} GeV
LHC	10^{4} GeV

Better question today: Can string theory make predictions relevant for the LHC, or for other current particle/astro/cosmo expts.?

If we are lucky, may see distinctive signals of string theory physics:

—KK modes (large extra dimensions)

—Cosmic strings

—Black holes @ LHC

But we probably won't be so lucky. If not, what can we say?

Even if

—String theory is a correct theory of nature

—We knew how to completely define it

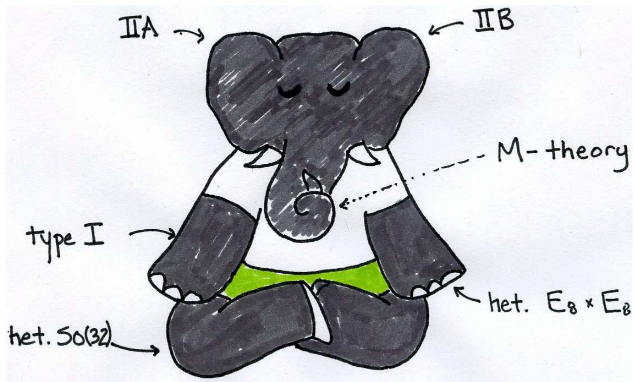
—We could calculate properties of configurations to high precision

The answer might practically still be no for near-term experiments.

Answering the question, however, may now be within reach

What is string theory?

- We don't know



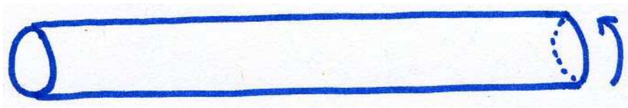
- Described in special limits by supergravity, perturbative strings

- Not even a perturbative description exists in the most interesting backgrounds with R-R fluxes, $\Lambda > 0$, etc.

- No background independent description – but progress via SFT

String theory and 4D physics

- String theory in 10D/11D



compact 6D

$\longleftrightarrow$

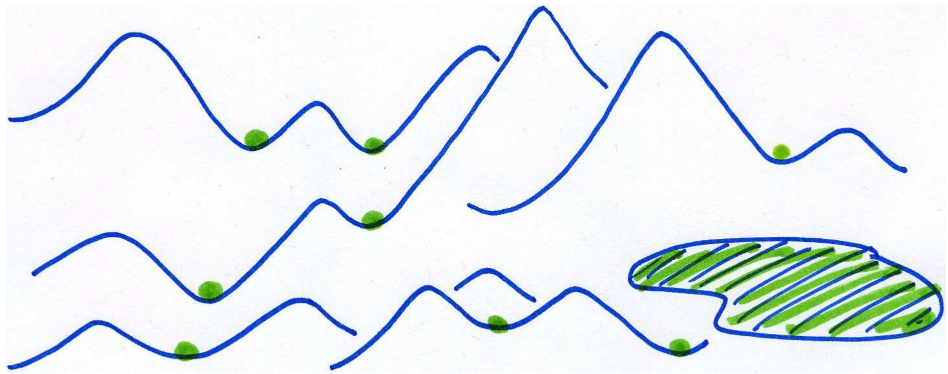
$\Rightarrow$ 4D by "compactification"

- Can study compactifications using supergravity (KK)
  $\Rightarrow$ 4D GR + gauge fields + matter

- Calabi-Yau: 4D SUSY backgrounds

- Fluxes:
  —stabilize moduli
  —expand range of geometries

String landscape



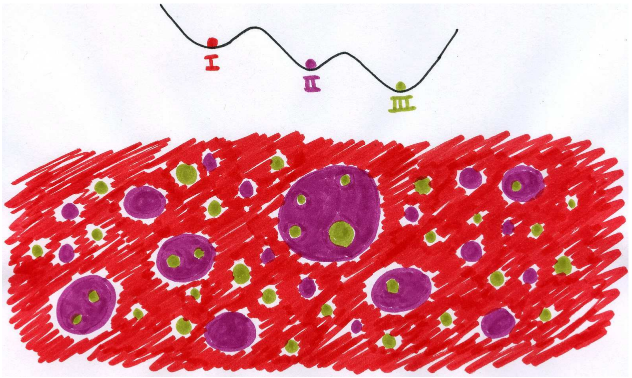
- Lots of vacua

- Many CY (∞?), flux combos (∞)

- Previous decades: Hope for dynamical selection
  —no hint yet for mechanism

- Now: use to get small cc
  (Weinberg/eternal inflation/metaverse)

Metaverse



Multitudes of vacua raise potential problems for predictability

Q: what field theories at scales < TeV
come from string compactifications? ("swampland")

If all, or "almost all", & vacua uniformly distributed
then predictions difficult

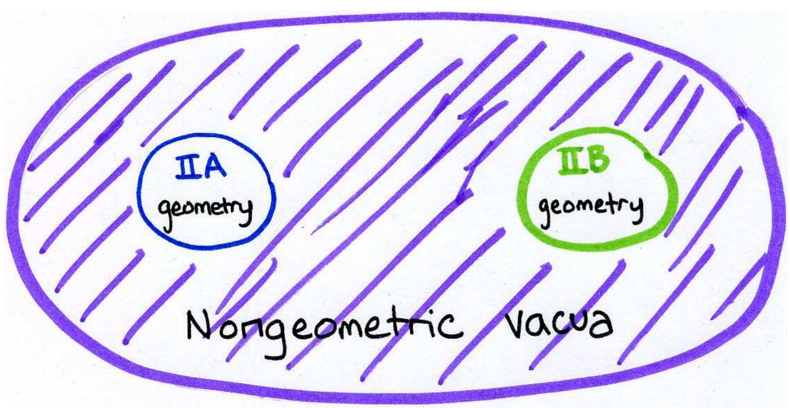
Need $\gg 10^{(\#\text{ observed digits})}$ vacua for no predictivity
(necessary condition)

How big is the landscape?

- Early estimates $\sim 10^{600}$ from IIB
  (from CY with ~ 200 fluxes $\Rightarrow 1000^{200}$)

- But in IIA, infinite families of highly controlled vacua
  [de Wolfe/Giryavets/Kachru/WT]
  —Maybe finite with simple cutoffs though

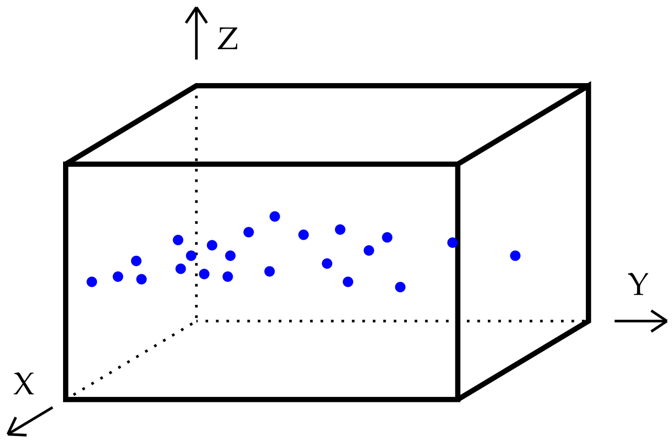
- May be dominated by ∞ numbers of "nongeometric" vacua
  [Shelton/WT/Wecht]



- No good handle on extent
- Probably most vacua are not in known or controllable corners of ST

Difficult to argue for predictions from discretization

- Program: find "correlations in corners"



– Find family of vacua with computable EFT parameters X, Y, Z

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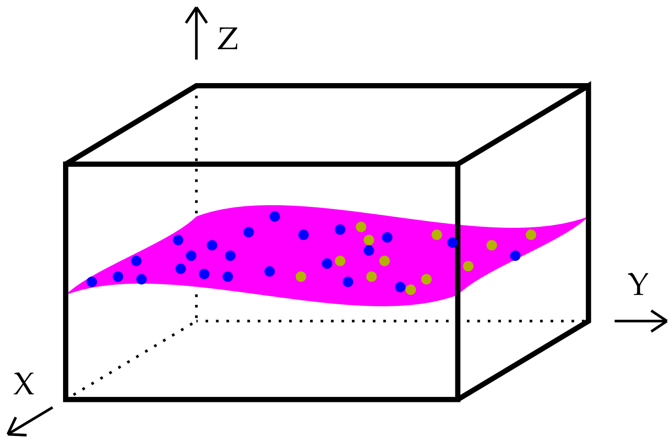


– Find family of vacua with computable EFT parameters X, Y, Z

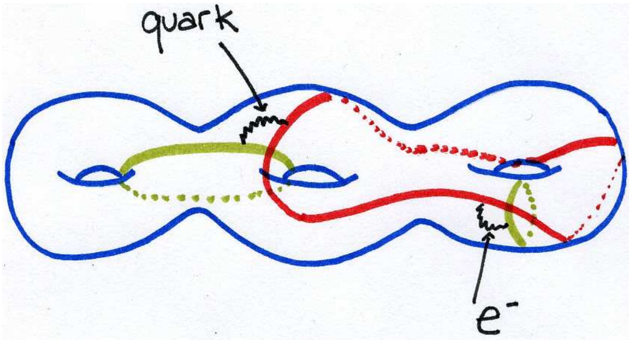
– Find constraints/correlations

– Compare with other families of vacua

2. Case study: Intersecting brane models

M. Douglas, WT: `hep-th/0606109`

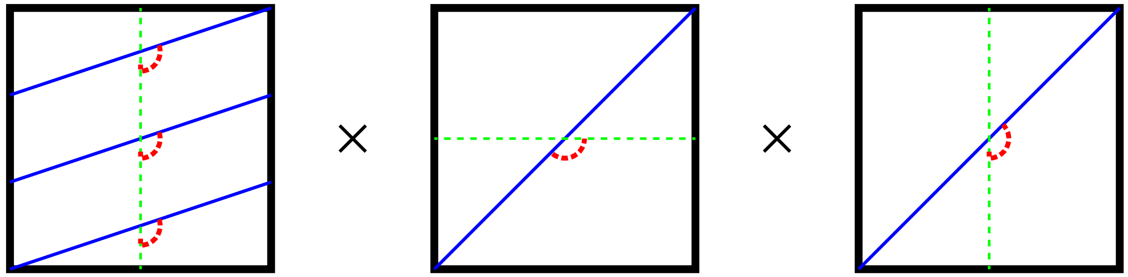
General Intersecting Brane Models



Branes $\Rightarrow$ gauge groups,

Intersections $\Rightarrow$ chiral fermions

Intersecting Brane Models on a toroidal orientifold



- IIA: Factorizable D6-branes with windings (n_i, m_i)
- IIB: Magnetized D9-branes with fluxes
- U(N) on multiple branes
- Chiral fermions on intersecting branes

$$I = \prod_{i=1}^{3} (n_i \hat{m}_i - \hat{n}_i m_i)$$

Most studied orientifold: $T^6/\mathbf{Z}_2 \times \mathbf{Z}_2$

$\mathbf{Z}_2 \times \mathbf{Z}_2$ orbifold action

$$\begin{aligned}(z_1, z_2, z_3) &\rightarrow (-z_1, -z_2, z_3)\\(z_1, z_2, z_3) &\rightarrow (z_1, -z_2, -z_3)\end{aligned}$$

Orientifold $z_i \rightarrow \bar{z}_i$

- Used to construct 3-generation models
  containing standard model gauge group
  [Cvetic/Shiu/Uranga, Cvetic/Li/Liu, Cremades/Ibanez/Marchesano]
- Moduli not stabilized
  but promising arena for real flux compactification
  [Marchesano/Shiu, ...]

Questions to address:

- How many intersecting brane models on fixed orientifold?
  finite/infinite?
- Distribution of gauge group G, # generations, Yukawas, ...
  correlations/constraints?

Previous statistical analysis:

[Blumenhagen/Gmeiner/Honecker/Lust/Weigand]

- Based on one-year computer search
- Suggested gauge group, # generations are fairly independent
- Suggests $\sim 10^{-9}$ of models have $SU(3) \times SU(2) \times U(1)$
  and 3 generations of chiral fermions

Outline of basic results

Technical advances in `hep-th/0606109`:

- Analytic proof of finiteness
- Focus on models *containing* G
  w/ possible extra ("hidden sector") branes

Physics results

- Analytic estimates for $\mathcal{N}_G = \#$ models containing G
- G, # generations essentially independent
  (modulo some number theoretic features,
  + bound on G, large $G \Rightarrow$ bounds smaller # generations)

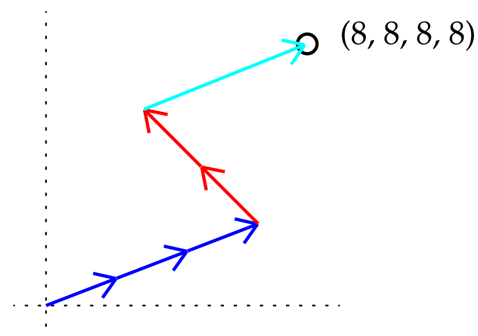
3. Proof of finiteness, estimates for $\mathcal{N}_G$

Finding SUSY IBM models $\sim$ partition problem

$$\sum_a (P_a, Q_a, R_a, S_a) \quad = \quad (T, T, T, T) = (8, 8, 8, 8)$$

$$P = n_1 n_2 n_3 \qquad Q = -n_1 m_2 m_3$$

$$R = -m_1 n_2 m_3 \qquad S = -m_1 m_2 n_3$$



SUSY conditions (when $P, Q, R, S > 0$):

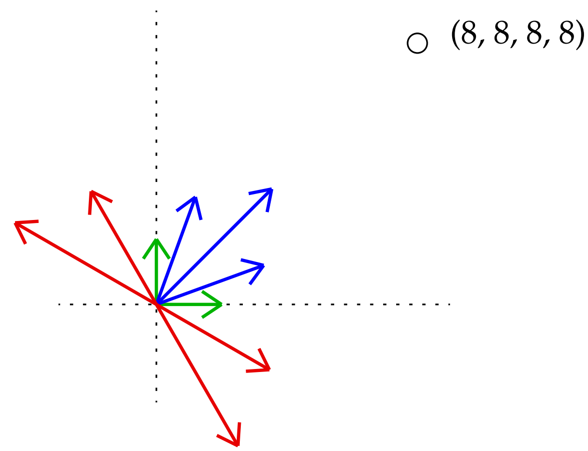
$$\frac{1}{P} + \frac{j}{Q} + \frac{k}{R} + \frac{l}{S} = 0, \qquad P + \frac{1}{j} Q + \frac{1}{k} R + \frac{1}{l} S > 0\,.$$

Why proof of finiteness is nontrivial

Can have negative tadpoles, *e.g.*

$$n = (3, 1, -1), \qquad m = (1, 1, -1)$$

$$\Rightarrow (P, Q, R, S) = (-3, 3, 1, 1)$$



3 kinds of branes (up to S_4 symmetry):

A: $- + + +$, **B**: $+ + 0\ 0$, **C**: $+ 0\ 0\ 0$

Proof of finiteness: example piece of proof

Take **A**-brane with $S_a < 0$,

$$\begin{aligned} P_a + \frac{1}{j}Q_a + \frac{1}{k}R_a + \frac{1}{l}S_a &= P_a + \frac{Q_a}{j} + \frac{R_a}{k} - \frac{1}{\frac{1}{P_a} + \frac{j}{Q_a} + \frac{k}{R_a}} \\ &\geq \frac{2}{3}\left(P_a + \frac{1}{j}Q_a + \frac{1}{k}R_a\right) \end{aligned}$$

using

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} > \frac{3}{x+y+z}$$

So for a general configuration positive tadpoles give

$$\sum_{a+} P_a + \frac{1}{j}\sum_{a+} Q_a + \frac{1}{k}\sum_{a+} R_a + \frac{1}{l}\sum_{a+} S_a \leq \frac{3}{2}T(1 + \frac{1}{j} + \frac{1}{k} + \frac{1}{l})$$

implies, assuming wlog $1 \leq j \leq k \leq l$

$$\sum_{a+} P_a \leq 6T\,.$$

So negative P's at most sum to $\mathcal{O}(T)$; similar arg for Q.

General results on scaling

$$
\begin{aligned}
1\text{A} &: \quad \sim (-T^3, T, T, T) \\
2\text{A} &: \quad \sim \left.\begin{array}{c} (-T^5, T^3, T, T) \\ (T^5, -T^3, T, T) \end{array}\right\} \rightarrow (-T^3, T, T, T)
\end{aligned}
$$

Worst scaling $\sim (T^5, T^3, T, T)$

$\Rightarrow$ Finite number of SUSY configurations

Also: can estimate numbers of configurations with fixed gauge group

Counting configurations with gauge subgroup G

- Expect $> \sim \mathcal{O}(e^T)$ solutions to partition problem
- Look at configurations given G undersaturating tadpole
  $\Rightarrow$ polynomial number of solutions

e.g. $U(N)$ from N **A**-branes (allow "hidden sector" **B**, **C**'s)

$$\sim (-T^3/N^3, T/N, T/N, T/N)$$

$$\mathcal{N}_{NA} \sim \frac{\pi^6 T^3}{6^4(\zeta(3))^3 N^3}$$

e.g. $U(N) \times U(M)$ from $N\mathbf{A} + M\mathbf{B}$

$$A \sim (-T^3/N^3, T/N, T/N, T/N), \qquad B \sim (T^3/(N^2 M), T/M)$$

$$\mathcal{N}_{NA+MB} \sim \mathcal{O}(\frac{T^7}{N^5 M^2})$$

Generally, $U(N)$ factor suppressed by $1/N^{\nu}, \nu \geq 2$.

e.g., expect

$$\mathcal{N}_{aa} \sim (T/N)^6, \quad \mathcal{N}_{ab} \sim (T/N)^7, \quad \mathcal{N}_{bb} \sim (T/N)^4,$$

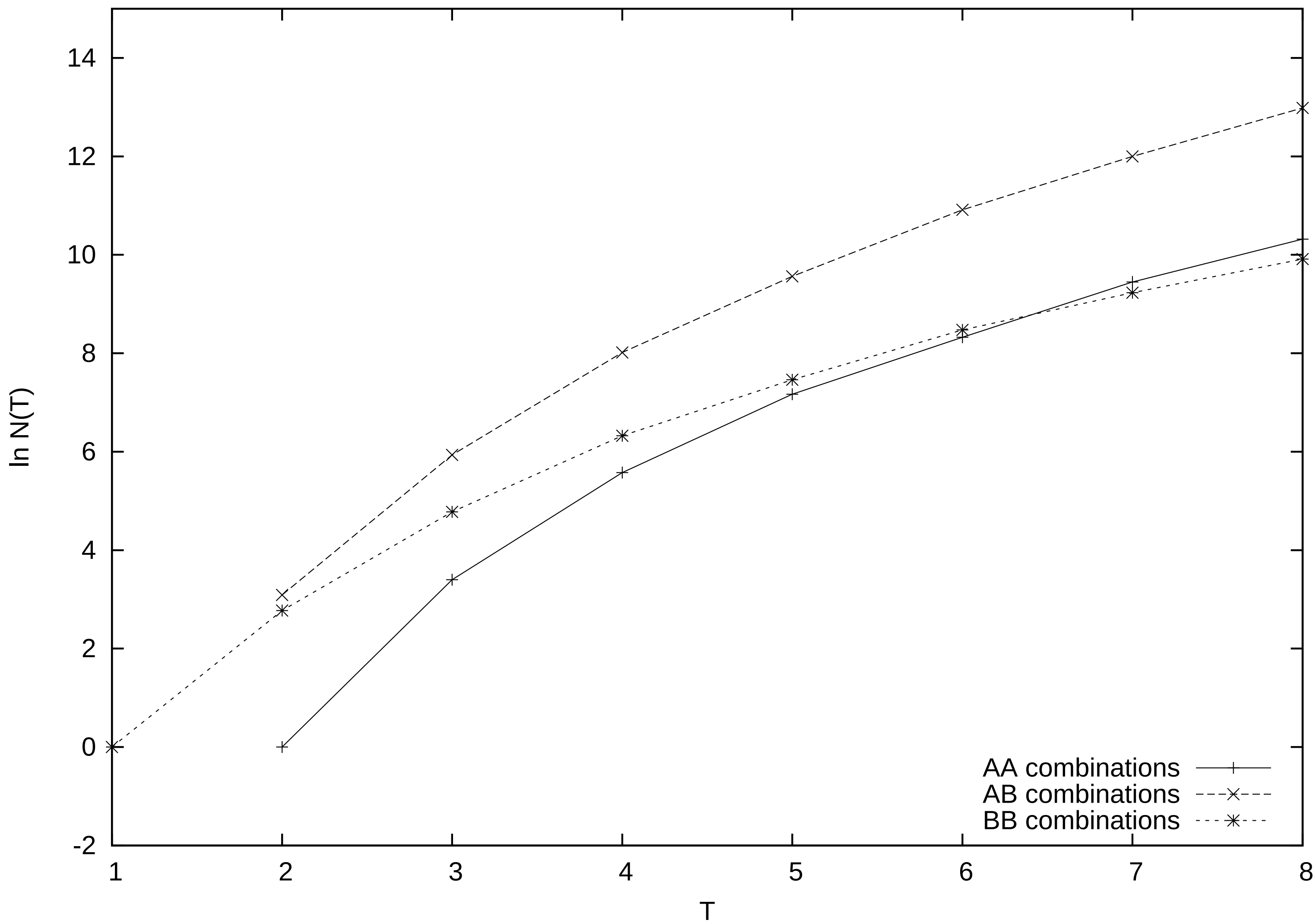
at $T = 8$ $[U(1) \times U(1)]$

$$\begin{aligned} \mathcal{N}_{aa}(8) &= 30,255 \\ \mathcal{N}_{ab}(8) &= 434,775 \\ \mathcal{N}_{bb}(8) &= 20,244 \end{aligned}$$

at $T = 4$ $[U(2) \times U(2)$ at $T = 8]$

$$\begin{aligned} \mathcal{N}_{aa}(4) &= 264 \\ \mathcal{N}_{ab}(4) &= 3,029 \\ \mathcal{N}_{bb}(4) &= 558 \end{aligned}$$

Growth as expected (contains extra logs)



(Log of) number of type **AA**, **AB**, **BB** branes for varying T

Estimates for $\mathcal{N}_G$

- Analytic estimates for # configurations with gauge subgroup G
- Efficient algorithms to scan all valid configurations
- Expect

$$\begin{aligned} \mathcal{N}_{SU(3)\times SU(2)\times U(1)} &\sim 10^{10} \\ \mathcal{N}_{SU(4)\times SU(2)\times SU(2)} &\sim 10^{7} \end{aligned}$$

- Each configuration admits $\sim e^{\Delta T}$ hidden sectors

Distribution of generation numbers

Generations of chiral fermions between $(n, m), (\hat{n}, \hat{m})$ branes from

$$I = \prod_{i=1}^{3} (n_i \hat{m}_i - \hat{n}_i m_i)$$

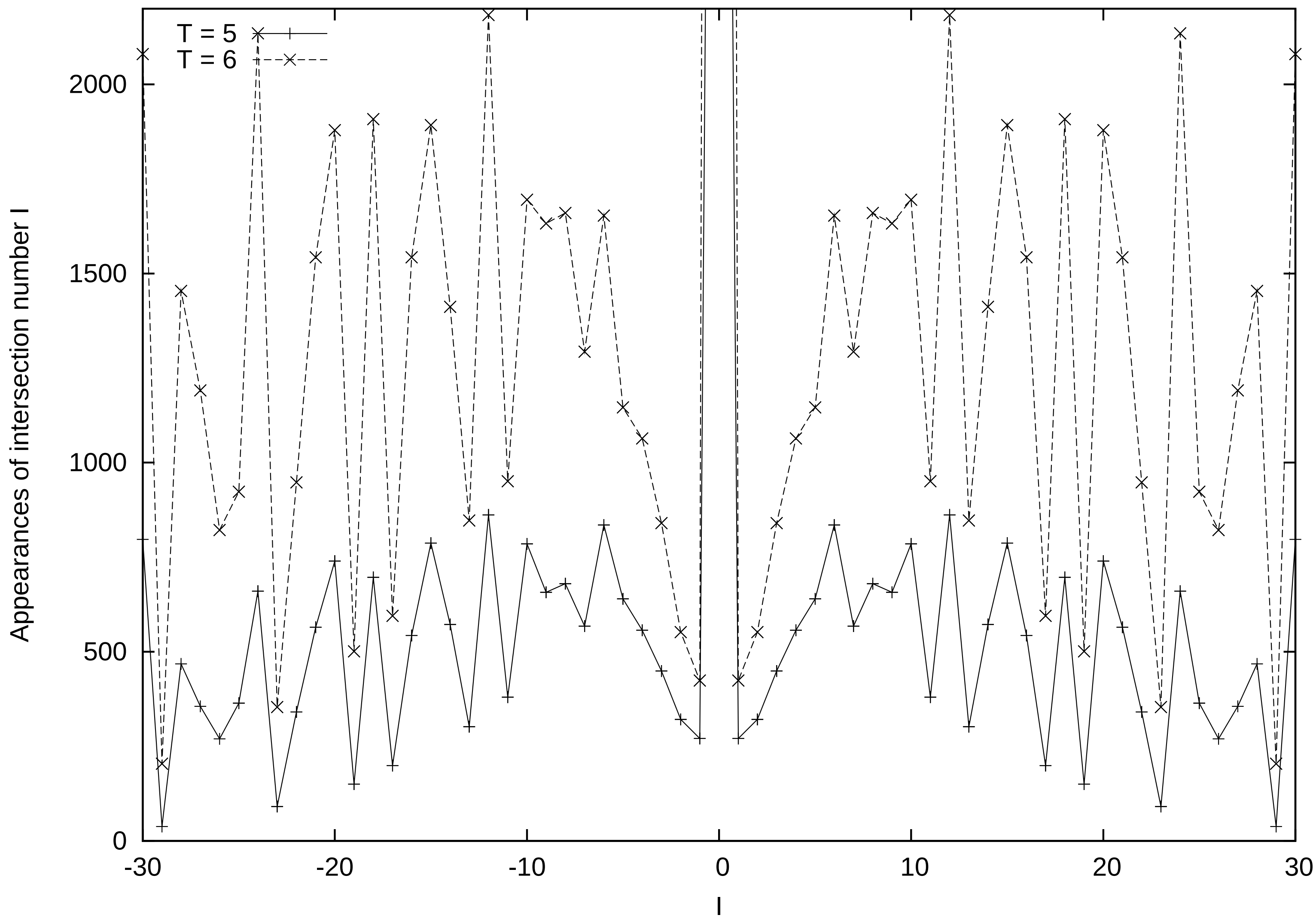
Look at distribution of I

- Intersection #'s for *e.g.* $U(N) \times U(N) \sim 10^3/N^5$ w/o extra **A**'s
  generally expect $\sim (T/N)^7$

- Mild enhancement of composite I's

- Intersection numbers distributed quite independently

  e.g. for $\mathbf{AB\hat{B}}$, $T = 3, \quad H(I_{ab}) = H(I_{a\hat{b}}) = 4.553$
  mutual entropy $2H(I_{ab}) - H(I_{ab}, I_{a\hat{b}}) \sim 0.085$



Frequencies of small intersection numbers

Specific models

- For 3-generation $SU(3) \times SU(2) \times U(1), SU(4) \times SU(2) \times SU(2)$
  expect $\mathcal{O}(10 - 100)$ models

- Parity constraint from image branes $I_{xy} + I_{xy'} \equiv 0 (\text{mod } 2)$
  $\Rightarrow$ odd generations only w/ **C** branes, discrete B/skew tori

- $\mathcal{O}(10)$ models found in previous constructions
  [Cvetic/Shiu/Uranga, Cvetic/Li/Liu, Cremades/Ibanez/Marchesano]

- We identified 2 additional models:

$$
\begin{aligned}
4\mathbf{A} &: \quad n = (3, 1, -1), \quad m = (1, 1, -1), \quad (P, Q, R, S) = (-3, 3, 1, 1) \\
2\mathbf{C} &: \quad R = 1 \\
2\mathbf{C} &: \quad S = 1
\end{aligned}
$$

$+$ two distinct hidden sector $\mathbf{A} + \cdots$ combinations

4. Generalization to other Calabi-Yau orientifolds

Notions from T^6 generalize– Work in IIB picture

D6 $\rightarrow$ D9 + fluxes $F \in H^2(M, \mathbf{Z})$

Kähler form $J \in H^2(M, \mathbf{Z})$, $\cap$ form $C(\alpha, \beta, \gamma) = \int_M \alpha \wedge \beta \wedge \gamma$.

SUSY conditions

$$0 = -3C(J, J, F_\alpha) + C(F_\alpha, F_\alpha, F_\alpha)$$

$$0 < C(J, J, J) - 3C(J, F_\alpha, F_\alpha).$$

Tadpole conditions generalize, using D_i = basis of divisors

$$8 = \sum_\alpha N_\alpha, \qquad T_i = c_2(M) \cdot D_i - \sum_\alpha N_\alpha C(D_\alpha, D_\alpha, D_i)$$

Positivity: Mori cone $MC(M) \subset H_2(M, \mathbf{Z})$ of effective classes

Example: Elliptic fibration over $\mathbf{P}^1 \times \mathbf{P}^1$

$D_{1,2}$ denote divisors associated w/ elliptic fibrations over each $\mathbf{P}^1$.
B = section of total fibration, $D_0 = B + 2D_1 + 2_2$

Nonzero intersection numbers:

$$D_0^3 = 8, \quad D_0^2 D_1 = D_0^2 D_2 = 2, \quad D_0 D_1 D_2 = 1.$$

With $J = \sum_{i=0,1,2} j_i D_i, \qquad F \to \sum_{i=0,1,2} m_i D_i,$

SUSY conditions

$$\begin{aligned} &8m_0^3 + 6m_0^2 m_1 + 6m_0^2 m_2 + 6m_0 m_1 m_2 \\ &\quad = \quad 3\left[(8j_0^2 + 4j_0 j_1 + 4j_0 j_2 + 2j_1 j_2)m_0 + (2j_0^2 + 2j_0 j_2)m_1 + (2j_0^2 + 2j_0 j_1)m_2\right] \end{aligned}$$

$$\begin{aligned} &8j_0^3 + 6j_0^2 j_1 + 6j_0^2 j_2 + 6j_0 j_1 j_2 \\ &\quad > \quad 3\left[(8m_0^2 + 4m_0 m_1 + 4m_0 m_2 + 2m_1 m_2)j^0 + (2m_0^2 + 2m_0 m_2)j^1 \right. \\ &\qquad\qquad \left. + (2m_0^2 + 2m_0 m_1)j^2\right] \end{aligned}$$

On general Calabi-Yau, classification of branes generalizes

C) D9 and single SUSY D5 in Mori cone

B) Brane with tadpole in Mori cone

A) Generic branes, tadpole outside Mori cone

Given an orientifold, analysis proceeds in similar fashion

Technical issues:

- Need more general proof of finiteness
- Generic orientifold is not just O9 but $\sim$ "O9 + O5"

5. Conclusions I

• String theory need not make predictions for particle physics below 100 TeV

• We can't define string theory yet

• The number of suspected solutions is enormous, and growing fast

• Nonetheless, constraints on low-energy physics correlated between calculable corners of the landscape may lead to predictions

• If not, probably need major conceptual breakthrough to have any possibility of predictivity for low-energy particle physics.

• *Raison d'etre* for string theory: quantum gravity.
Remarkably, also gives new insight into gauge theory.
Suggests interesting new structures for phenomenological models.
Specific low-energy physics prediction would be unexpected bonus.

Conclusions II

- Proved finiteness for IBM on toroidal orientifold
- Estimates for numbers of models with fixed gauge group
- Analyzed generation numbers, no significant correlations
- No strong constraints on G, # generations, mild bounds
- Expect $\mathcal{O}(10 - 100)$ 3-generation G_{SM} models for this orientifold, found 2 new
- Generic models have many "hidden" U(1) factors, extra matter

Future directions

- Look at other orientifolds, CY
- Find larger families (10^9?) of 3-generation G_{SM} models
- Compute more detailed properties (Yukawas etc.), look for constraints
- Include fluxes, stabilize closed + open moduli
- Consider other corners (RCFT, heterotic, M-theory, etc. – SVP)