Baryons from Instantons in holographic QCD

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- On the occasion of T.Yoneya's 60th birthday -

Plan

1. Introduction + brief review
2. Baryons as Instantons
3. Baryon spectrum
4. Outlook

1 Introduction + brief review

Can string theory be (again) a theory of hadrons?

- Difficulties found in the old days.
  - Consistent in 10 dim.
  - $\exists$ Massless graviton, gauge field.
  - QCD looks better
    $\Rightarrow$ string is not fundamental.
- The situation has drastically changed since the discovery of AdS/CFT.

Lessons from gauge/string duality

If a holographic dual of QCD exists, we expect ...

4 dim QCD $\overset{\text{dual}}{\longleftrightarrow}$ 10 dim string theory in a certain curved b.g.

(Cf) 4 dim N=4 SYM $\overset{\text{dual}}{\longleftrightarrow}$ string theory in $AdS_5 \times S^5$)

Massive hadrons in 4 dim $\longleftrightarrow$ massless graviton, gauge field in higher dim

- Recently, we proposed

[Sakai-S.S. 2004]

Type IIA string theory in Witten's D4 background

+ N_f Probe D8-branes

(assuming $N_c \gg N_f$)

dual

at low energy

4 dim QCD with N_f massless quarks

The meson sector is described by the 5 dim $U(N_f)$ Yang-Mills-Chern-Simons theory in a curved background.

$$S_{\text{5dim}} \simeq S_{\text{YM}} + S_{\text{CS}}$$

$K(z) = 1 + z^2$

$$S_{\text{YM}} = \kappa \int d^4x dz \, \text{Tr} \left(\frac{1}{2} K(z)^{-1/3} F_{\mu\nu}^2 + K(z) F_{\mu z}^2 \right)$$

$\mu, \nu = 0 \sim 3$

$$S_{\text{CS}} = \frac{N_c}{24\pi^2} \int_5 \omega_5(A)$$

(CS 5-form)

($M_{\text{KK}} = 1$ unit)

Meson
s

- mode expansion

$$A_\mu(x^\mu, z) = \sum_{n\geq 1} B_\mu^{(n)}(x^\mu)\psi_n(z)$$
$$A_z(x^\mu, z) = \sum_{n\geq 0} \varphi^{(n)}(x^\mu)\phi_n(z)$$

Some complete sets

- We interpret $\varphi^{(0)} \sim$ pion $\quad B_\mu^{(1)} \sim \rho$ meson $\quad B_\mu^{(2)} \sim a_1$ meson $\cdots$

  ($\varphi^{(n)}$ $(n = 1, 2, \cdots)$ are eaten by $B_\mu^{(n)}$)

$\pi, \rho, a_1, \cdots$ are unified in the 5 dim gauge field !

- Masses and couplings are roughly consistent with the experimer

e.g.

mass	ρ	a_1	ρ'	(a'_1)	ρ''
exp.(MeV)	776	1230	1465	(1640)	1720
our model	[776]	1189	1607	2023	2435
ratio	[1]	1.03	0.911	(0.811)	0.706

input

Highlights

[Sakai-S.S. 2004, 2005] [See also Son-Stephanov 2003]

- Geometric realization of the chiral symmetry breaking

$$U(N_f)_L \times U(N_f)_R \longrightarrow U(N_f)_V$$

- Structure of interaction

  ➡ consistent with
  - hidden local symmetry [Bando-Kugo-Uehara-Yamawaki-Yanagida 1985]
  - vector meson dominance [Gell-Mann-Zachariasen 1961, Sakurai 1969]
  - GSW model [Gell-Mann -Sharp Wagner 1962]

- Numerical estimate of the masses and couplings

  ➡ roughly agrees with the various experimental data

- Anomalies in QCD is reproduced

  ➡
  - an easy derivation of WZW term [Wess-Zumino 1971, Witten 1983]
  - Witten-Veneziano formula [Witten-Veneziano 1979]

- Baryon

D4 wrapped on S^4 $\simeq$ instanton on D $\simeq$ Skyrmion

[Witten, Gross-Ooguri 1998] [Atiyah-Manton 1989] [Skyrme 1961]

Relation to Skyrme model

- Define $U(x^\mu) \equiv P\exp\left\{-\int_{-\infty}^{\infty} dz\, A_z(x^\mu, z)\right\} \in U(N_f)$

  ➡ behaves as the pion field in the chiral lagrangian.

➡ We obtain

$$S_{\text{YM}} \simeq \int d^4x \left[\frac{f_\pi^2}{4}\text{Tr}(U^{-1}\partial_\mu U)^2 + \frac{1}{32e_S^2}\text{Tr}[U^{-1}\partial_\mu U, U^{-1}\partial_\nu U]^2 \right] + \cdots$$

This is the Skyrme model ! $\left(f_\pi^2 = \frac{4\kappa}{\pi} M_{\text{KK}}^2 \,,\quad e_S^{-2} \simeq 2.51 \cdot \kappa \right)$

- Skyrme proposed [Skyrme 1961]

Baryon $\simeq$ Soliton in Skyrme model (Skyrmion)

baryon # winding #

$$N_B = \frac{1}{24\pi^2}\int \text{Tr}(U dU^{-1})^3$$

➡ Let us generalize this idea to our 5 dim description.

Summary of the rest of the talk

- We propose a new way to analyze baryons
  that extends Skyrme's old idea
  including contributions from vector mesons.
- Baryons are described as (4 dim) instantons
  in the 5 dim gauge theory.
- Quantum mechanics on the instanton moduli space,
  gives the baryon spectrum.
- The quantitative tests are not good enough yet.
  Please be generous !

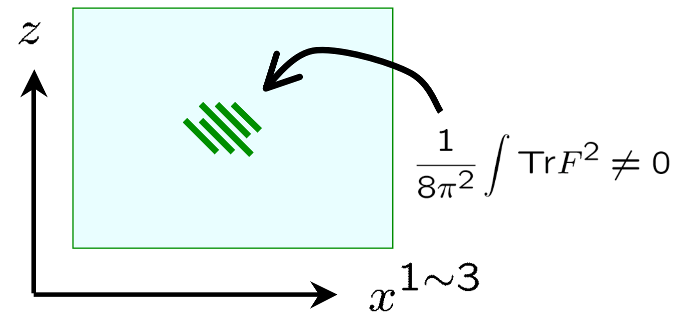
② Baryons as instantons

$M = 1, 2, 3, z$

- Consider an instanton config. in $x^M = (\vec{x}, z) \in \mathbf{R}^4$

  ➡ behaves as a point-like particle

  ➡ Interpreted as a baryon



- In fact,

baryon # Instanton #

$$\boxed{N_B = \frac{1}{8\pi^2}\int \mathrm{tr} F \wedge F}$$

- Note

D4 wrapped on S^4 $\simeq$ instanton on D8 $\simeq$ Skyrmion

[Witten, Gross-Ooguri 1998] [Atiyah-Manton 1989] [Skyrme 1961]

Realization of Atiyah-Manton: $U(x^\mu) \equiv P\exp\left\{-\int_{-\infty}^{\infty} dz A_z(x^\mu, z)\right\}$

Skyrmion Instanton

Classical solution

- The instanton solution for

$K(z) = 1 + z^2$

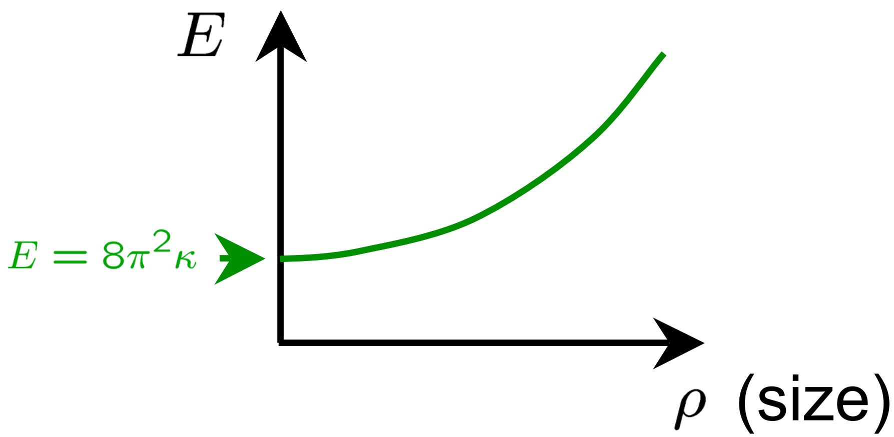
$$S_{\mathrm{YM}} = \kappa \int d^4x dz \, \mathrm{Tr} \left(\frac{1}{2} K(z)^{-1/3} F_{\mu\nu}^2 + K(z) F_{\mu z}^2 \right)$$

$\kappa = \dfrac{\lambda N_c}{216\pi^2}$

λ : 't Hooft coupling (assumed to be large)

shrinks to zero size !

(Even though the pion effective action contains the Skyrme term !)



The BPST instanton configuration with $\rho \to 0$ is the minimum energy configuration.

- The effect of the Chern-Simons term:

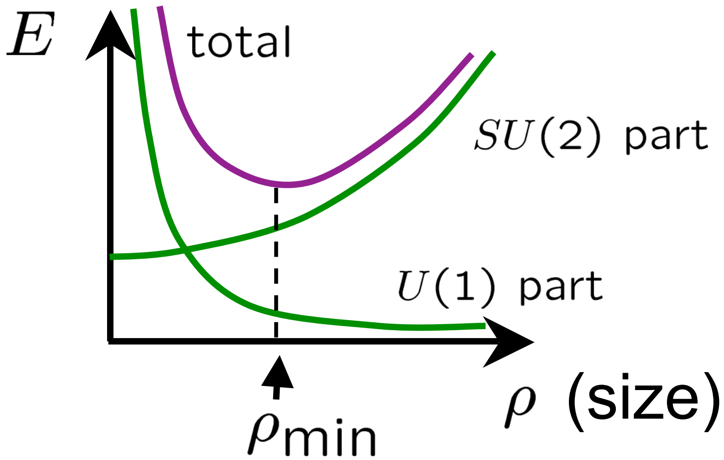
$$S_{\mathrm{CS}} = \frac{N_c}{24\pi^2}\int_5 \omega_5(A) = \frac{N_c}{16\pi^2}\int d^4x dz\, A_0^{U(1)}\, \underbrace{\epsilon^{ijk}\mathrm{Tr}F_{ij}F_{kz}}_{\text{Non-zero for instanton}} + \cdots$$

$U(1)$ part

➡ source of the U(1) charge

➡ point-like charge costs energy

➡ The size will be stabilized with a non-zero finite value.



This is the same mechanism as the stabilization of Skyrmions via ω meson. [Adkins-Nappi 1984]

- We can show $\rho_{\rm min} \sim \mathcal{O}(\lambda^{-1/2})$

  It is convenient to rescale as

$$x^M \to \lambda^{-1/2} x^M \qquad A_M \to \lambda^{1/2} A_M \qquad (M = 1,2,3,z)$$

  Then, we have

$$\mathcal{L}_{\rm YM} \sim \kappa\, {\rm Tr}\left(\frac{1}{2} F_{MN}^2 + \mathcal{O}(\lambda^{-1})\right)$$

YM in flat space

➡ The leading order classical solution is the BPST instanton with $\rho = \rho_{\rm min}$ and $Z = 0$

$$A_M^{\rm cl} = -i \frac{\xi^2}{\xi^2 + \rho^2}\, g \partial_M g^{-1}$$

$$g = \frac{(z - Z) - i(\vec{x} - \vec{X}) \cdot \vec{\tau}}{\xi} \qquad \xi = \sqrt{(\vec{x} - \vec{X})^2 - (z - Z)^2}$$

ρ : size $\quad (\vec{X}, Z)$: position of the instanton

3 Baryon spectrum

- Consider a slowly moving (rotating) baryon configuration.
  Use Manton's moduli space approximation method :

  Instanton moduli $\mathcal{M} \ni X^\alpha \rightarrow X^\alpha(t)$

  time

  $\boxed{A_M(t,x) \sim A_M^{\text{cl}}(x; X^\alpha(t))}$

  $S_{\text{5dim}} \rightarrow$ Quantum Mechanics for $X^\alpha(t)$

- For SU(2) one instanton,

$$\mathcal{M} \simeq \{(\underbrace{\vec{X}, Z}_{\text{position}}, \rho)\} \times SU(2)/\mathbf{Z_2} \qquad \mathbf{Z}_2 : \mathbf{a} \to -\mathbf{a}$$

size

Ψ $\mathbf{a}$ ← *SU(2)* orientation

$$\rightarrow \quad L_{\text{QM}} = \frac{G_{\alpha\beta}}{2}\dot{X}^\alpha \dot{X}^\beta - U(X^\alpha) \qquad U(X^\alpha) = 8\pi^2\kappa\left(1 + \lambda^{-1}\left(\frac{\rho^2}{6} + \frac{3^6\pi^2}{5\,\rho^2} + \frac{Z^2}{3}\right) + \mathcal{O}(\lambda^{-2})\right)$$

<u>Note</u> We include (ρ,*Z*) since they are light
compared to the other massive modes.

- Solving the Schrodinger equation for this Quantum mechanics,
  we obtain the baryon spectrum
  → Generalization of Adkins-Nappi-Witten [Adkins-Nappi-Witten1983]
  including vector mesons and ρ, Z modes

Results

- Only $I = J$ states appear. (Just as in the ANW)

  isospin spin

- Parity odd states appear. (Unlike in the ANW!)

  $\text{parity} = (-1)^{n_z}$ n_z : excitation of the Z mode

- Mass spectrum

$$M \simeq M_0 + \left(\sqrt{\frac{(l+1)^2}{6} + \frac{2}{15} N_c^2} + \sqrt{\frac{2}{3}} (n_\rho + n_z) \right) M_{\text{KK}}$$

$$l = 2I = 2J = 1, 3, 5, \cdots \quad n_\rho = 0, 1, 2, \cdots \quad n_z = 0, 1, 2, \cdots$$

numerical values (just for illustration !)

$$M \simeq M_0 + \left(\sqrt{\frac{(l+1)^2}{6} + \frac{2}{15} N_c^2} + \sqrt{\frac{2}{3}} (n_\rho + n_z) \right) M_{\rm KK}$$

$l = 2I = 2J = 1, 3, 5, \cdots \quad n_\rho = 0, 1, 2, \cdots \quad n_z = 0, 1, 2, \cdots \quad \text{parity} = (-1)^{n_z}$

- If we choose $M_{\rm KK} \simeq 500$ MeV
  and use nucleon mass ($\simeq 940$ MeV) to fix the consta M_0
  (we only consider the mass difference), we obtain

(n_ρ, n_z)	$(0,0)$	$(1,0)$	$(0,1)$	$(1,1)$	$(2,0)/(0,2)$	$(2,1)/(0,3)$	$(1,2)/(3,0)$
$N\,(l=1)$	$\boxed{[940]^+}$	1348^+	1348^-	1756^-	$1756^+, 1756^+$	$2164^-,\ 2164^-$	$2164^+, 2164^+$
$\Delta\,(l=3)$	$\boxed{1240^+}$	1648^+	1648^-	2056^-	$2056^+, 2056^+$	$2464^-, 2464^-$	$2464^+, 2464^+$

States appeared in the Skyrme model

($\pm$: parity)

- $I = J$ states from Particle Data Group look like….

(n_ρ, n_z)	$(0,0)$	$(1,0)$	$(0,1)$	$(1,1)$	$(2,0)/(0,2)$	$(2,1)/(0,3)$	$(1,2)/(3,0)$
$N\,(l=1)$	940^+	1440^+	1535^-	1655^-	1710^+, ?	2090^-_*, ?	2100^+_*, ?
$\Delta\,(l=3)$	1232^+	1600^+	1700^-	1940^-_*	1920^+, ?	?, ?	?, ?

(? : not found, $*$: evidence of existence is poor)

Comments

The predicted baryon spectrum looks nice,

but there are a lot of reasons that
you should NOT trust these values.

- $1/\lambda$ expansion may not work well.
- Higher derivative terms are neglected.
- $N_c = 3$ is not large enough especially for $l \geq 3,\ n_\rho + n_z \geq 3$
- The model deviates from real QCD at high energy $\sim M_{\rm KK}$
- $M_{\rm KK} \simeq 950$ MeV is the value consistent with ρ meson mass

➡ Need more investigation for the quantitative tests.

Comments (large N_c behavior)

For $N_c \gg l$, the mass formula becomes

$$M \simeq \widetilde{M}_0 + \frac{1}{4}\sqrt{\frac{5}{6}}\frac{l(l+2)}{N_c}M_{\mathrm{KK}} + \sqrt{\frac{2}{3}}(n_\rho + n_z)M_{\mathrm{KK}}$$

$(\widetilde{M}_0 \sim \mathcal{O}(N_c))$

The N_c dependence is consistent with that known in large N_c QCD. [Witten1979] [Adkins-Nappi-Witten1983]

Cf) The mass formula in Adkins-Nappi-Witten

$$M = M_0 + \frac{l(l+2)}{8\lambda} \qquad (M_0 \sim \mathcal{O}(N_c),\ \lambda \sim \mathcal{O}(N_c))$$

4 Outlook

- Baryons are described as (4 dim) instantons in a 5 dim gauge theory.
- We proposed a new way to analyze baryons that extends Skyrme's old idea including contributions from vector mesons.
- There are a lot more to do to improve the analysis.
  (solve EOM numerically, include higher derivative terms etc.
- It would be interesting to investigate other static properties of baryons.
  (charge radii, magnetic moments etc.)[See Hong-Rho-Yee-Yi 2007]