Lattice QCD with dynamical overlap fermion

- understanding chiral dynamics with exact chiral symmetry -

Tetsuya Onogi (YITP, Kyoto U.)

Talk at Komaba 2007,
Recent Developments in Strings and Fields
On the occasion of Prof. Yoneya's 60th birthday

Fukaya and T.O. Phys.Rev.D70(2004)054508

Fukaya et al., Phys.Rev.D74(2006)095405,
hep-lat/0702003

Why is exact lattice chiral symmetry important?

There are fundamental problems in particle physics and cosmology where nonperturbative QCD effect with chiral symmetry is important.

- Precise weak matrix elements for flavor physics

  Lack of chiral symmetry gives large systematic errors

  - Chiral extrapolation $f_B, B_B, \cdots$
  - Operator mixing from wrong chirality B_K

- Finite temperature phase transition for early universe

  Critical behavior is quite subtle

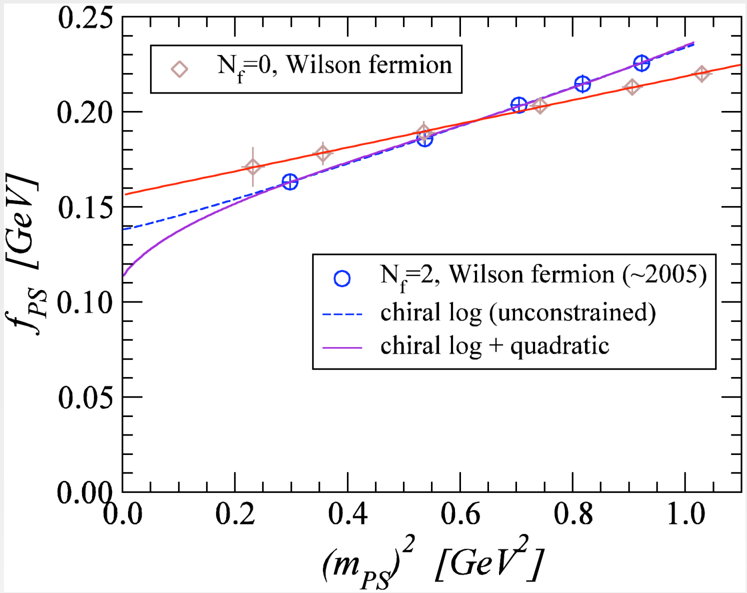
  - Order of phase transition?
  - Critical exponents?

In both cases, chiral symmetry plays crucial role.

Weak matrix elements for flavor physics

Uncertainty in chiral extrapolation

example: decay constant $f_\pi, f_B, \cdots$



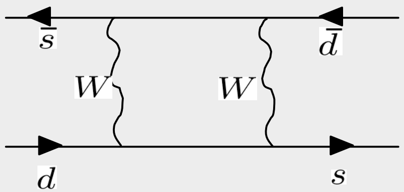
quark mass was far from the chiral regime

chiral extrapolation gives 10-15% error

We must reporduce chiral log from lattice

Operator mixing with wrong chirality

- Bag parameter B_K for $K-\bar{K}$ mixing

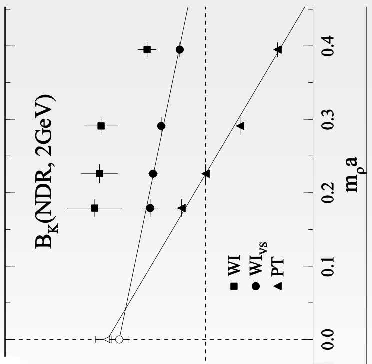


$$\langle K|(\bar{s}\gamma^\mu d)_L(\bar{s}\gamma_\mu d)_L|\bar{K}\rangle = \frac{8}{3}m_K^2 f_K^2 B_K$$

$$\begin{aligned}(\bar{s}_L\gamma^\mu d)_L(\bar{s}\gamma_\mu d_L)^{\overline{MS}}(\mu) &= Z_1(\bar{s}_L\gamma^\mu d_L)(\bar{s}_L\gamma_\mu d_L)^{lat}(a) \\ &\quad + Z_2(\bar{s}_L d_R)(\bar{s}_R d_L)^{lat}(a) + \cdots\end{aligned}$$

Without chiral symmetry, lattice operators can mix with operators having wrong chirality

(huge corrections due to chiral enhancement)



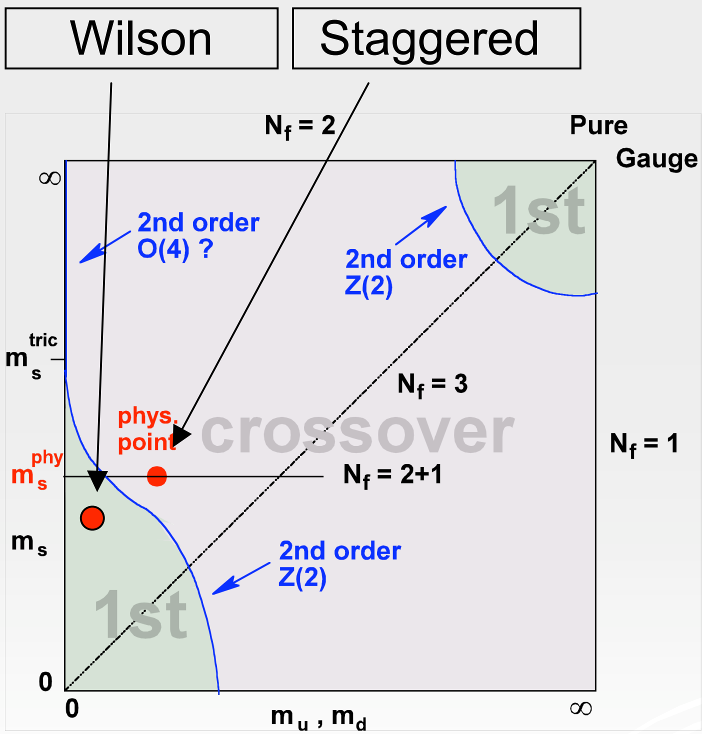
BK from Wilson fermion
S. Aoki et a (1999)
PRD60:114504

B_K is uncertain due to huge radiative corrections.

Finite temperature phase transition

Phase diagram for 2+1 flavor (u,d,s)

- Staggered fermion = crossover
- Wilson fermion = 1st order



Results from two actions do not agree

Scaling test for 2-flavor QCD

- Staggered fermion

	O(4)	O(2)	QCD
$1/\beta\delta$	0.537(7)	0.602(2)	0.64(2)
$1-1/\delta$	0.7939(9)	0.7928(3)	1.03(9)
$(1-\beta)/\beta\delta$	0.331(7)	0.394(2)	0.83(12)

Exponents do not agree with O(4) or O(2)

- Wilson fermion

  consistent with O(4) scaling

Results from two actions do not agree

Chiral symmetry on the lattice

- Nielsen-Ninomiya's theorem

  Nielsen and Ninomiya, Nucl.Phys.B185(1981) 20

  Consider a lattice fermion action $S_F = \bar{\psi} D \psi$ satisfying

  - Translational invariance $D\gamma_5 + \gamma_5 D = 0$
  - Chiral symmetry:
  - Hermiticity
  - Locality

  Then, doublers must exist

Wilson fermion : broken chiral symmetry, symmetry recovered in continuum.

Staggered fermion: 4 spinors $\otimes$ 4 "tastes" (doublers)

to apply QCD (u,d,s) one must take "the fourth root trick" $det(D) \sim det(D_{staggered})^{1/4}$

Very dangerous compromise! Even locality is doubtful.

We develop a method for lattice QCD with dynamical overlap fermion.

First large scale simulation.

Short term goal:

Derive the chiral dynamics from 1st principle QCD

- Prediction from Chiral Random Matrix theory
- Chiral behavor of light hadrons

Long term goal:

Apply our method to flavor physics, finite temperature QCD

Ginsparg-Wilson fermion

- Ginsparg-Wilson relation

Ginsparg and Wilson, Phys.Rev.D 25(1982) 2649.

$$D\gamma_5 + \gamma_5 D = aD\gamma_5 D$$

Exact chiral symmetry on the lattice (index theorem)

Hasenfratz, Laliena and Niedermayer, Phys.Lett. B427(1998) 125

Luscher, Phys.Lett.B428(1998)342.

$$\begin{aligned} \psi &\rightarrow \psi + i\gamma_5(1-aD)\psi = \psi + i\hat{\gamma}_5\psi \\ \bar{\psi} &\rightarrow \bar{\psi} + i\bar{\psi}\gamma_5 \end{aligned}$$

- Overlap fermion (explicit construction)

$$D = \frac{1}{a}\left[1 + \gamma_5 sign(H_W)\right], \qquad H_W \equiv \gamma_5(D_W + M_0)$$

D_W : Wilson Dirac op., M_0 : negative mass

Pioneering studies on Dynamical Overlap fermion

- Fodor, Katz, Szabo JHEP 08(2004), 003;
  Nucl. Phys. B(Proc.Suppl.)140(2005)704; JHEP 01 (2006) 049.

- Cundy, Krieg, Arnold, Frommer, Lippert, Schilling
  Nucl.Phys.B(Proc.Suppl.)140(2005)841; arXiv: hep-lat/0502007;
  Cundy, Krieg, Lippert PoS(LAT2005)107;

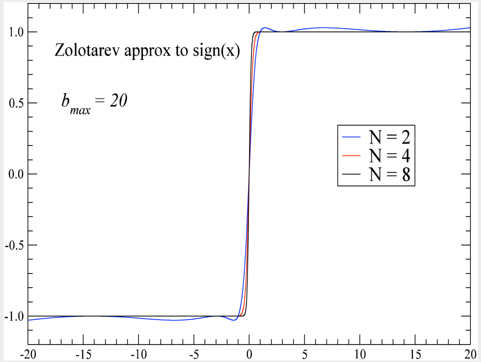
- DeGrand, Shafer Phys. Rev. D71(2005)034507;
  Phys.Rev.D72(2005)054503; PoS(LAT2005)140.

Dirac operator becomes singular at the zeros of H_{W}

Conclusion: Large scale simulation is hopeless.

Problems (all related to the zeros of Hw)

- We make rational approximation with completely controlled error except near zero mode.



$$sign(H_W) = \frac{H_W}{\sqrt{H_W^2}} \sim H_W(p_0 + \sum_{l=1}^{N} \frac{p_l}{H_W^2 + q_l})$$

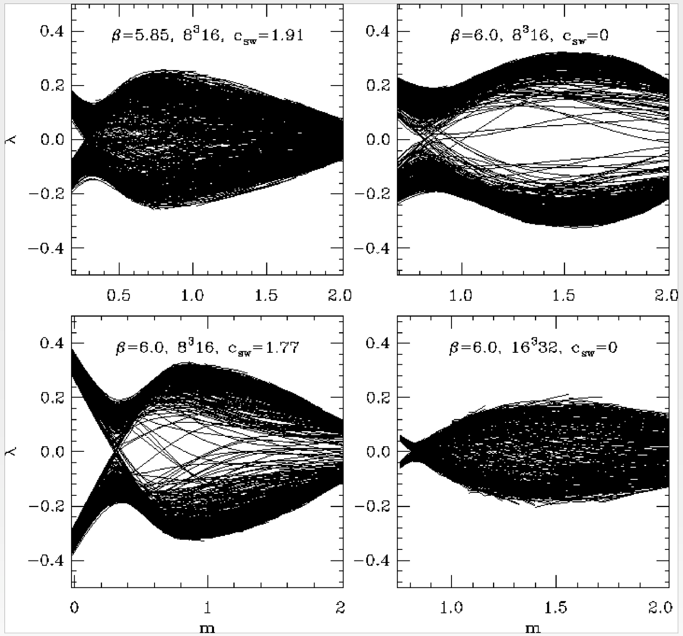
- Dov makes a discontinous jump when an eigenmode of Hw crosses zero. Hybrid Monte Carlo breaks down.

- A method to cure this problem has been developed. One has to monitor the zero crossing at much higher precision and include correction terms at the exact point of crossing. (Hopelessly huge numerical cost)

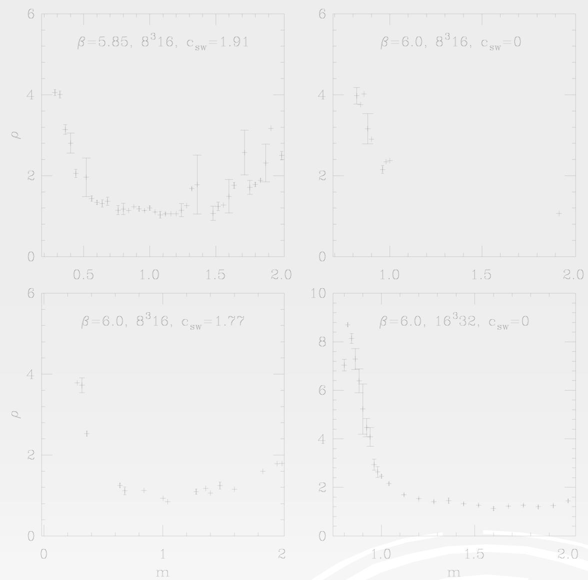
Low mode and topology change

- Zeros of Hw(m) arise when the topology changes through localized modes. $Tr[\hat{\gamma}_5] = -Tr[\gamma_5 D_{ov}] = -Tr[\epsilon(H_W)]$

Edwards, Heller, Narayanan Nucl.Phys.B535(1998)403.



Spectral flow of Hw



Localization size of the crossing mode

Our strategy:

use topology conserving gauge action to suppress the low modes

Det(Hw) term (Fukaya, Vranas)

Fukaya et al. hep-lat/0607020

Introduce negative heavy mass wilson fermion as a UV regulator field, whose mass is exactly the same as that appears in Dov. Infrared physics is unchanged.

$$Z = \int DU\, det(H_W^2) det(D_{\mathrm{ov}})^2 e^{-S}$$

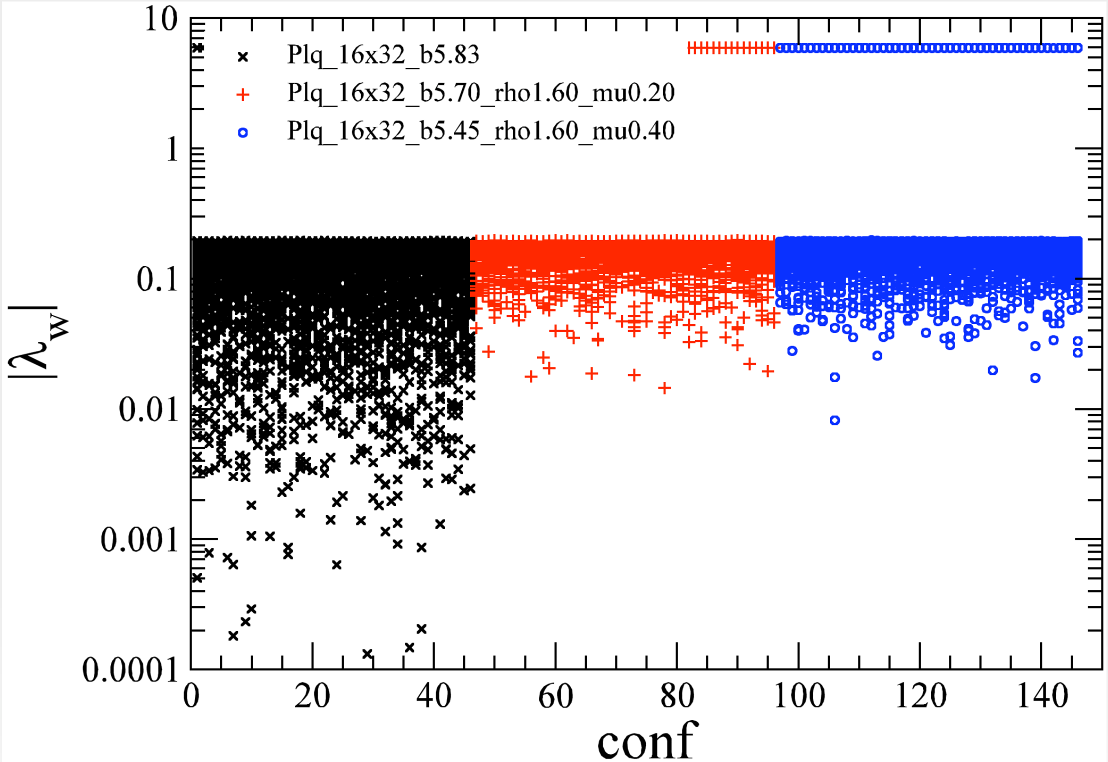
or

$$\int DU \frac{det(H_W^2)}{det(H_W^2+\mu^2)} det(D_{\mathrm{ov}})^2 e^{-S}$$

This term should kill the breakdown of locality topology change, and blow-up of numerical cost simultaneouly.

Distribution of low modes of Hw in real simulations

Fukaya et al.



Suppression of low modes by $\frac{det(H_W^2)}{det(H_W^2+\mu^2)}$ is confirmed.

JLQCD GW project

KEK BlueGene (10 racks, 57.3 TFlops)

- Started on March 1, 2006
- 1rack=1024 nodes=2048CPU

JLQCD collaboration:

KEK: S. Hashimoto, T. Kaneko, H. Matsufuru, J. Noaki, M. Okamoto, N. Yamada

RIKEN: H.Fukaya

YITP: T. Onogi

Tsukuba: S. Aoki, K.Kanaya,A.Ukawa,T.Yoshie

Hiroshima: K.-I.Ishikawa, M.Okawa

Taiwan National U: T.W.Chiu collaborators (TWQCD)

Numerical simulation

Dynamical simulation with Nf=2 overlap fermion

Run1 (epsilon-regime)

- 16^3 x 32 , 0.11 fm
- quark mass around 3MeV!!
- Ffixed topology

Run2 (normal regime)

- 16^3 x 32, a=0.12 fm
- quark mass 6 values in the range of ms/6-ms
- fixed topology
- At Q=0 accumulated 10,000 trajectories

QCD in epsilon regime

Gasser, Leutwyler Phys.Lett.188(1987)477

Epsilon-regime: special kinematical situation

$$\Lambda_{QCD}^{-1} \ll L \leq m_\pi^{-1} \quad \text{(usually, } \Lambda_{QCD}^{-1} \ll m_\pi^{-1} \ll L\text{)}$$

Different expansion applies in ChPT

$$L = \underbrace{f^2 tr(\partial_\mu U \partial_\mu U^\dagger)}_{\text{nonzero mode only} \sim f^2 L^{-2}} + \underbrace{m\Sigma tr(U + U^\dagger)}_{\text{zero mode}} + \cdots$$

(zero mode integral is described by Bessel function)

Low eigenvalues of Dirac operator can be evaluated from Chiral Random Matrix Theory(ChRMT)

Damgaard and Nishigaki Phys.Rev.D63(2001)045012

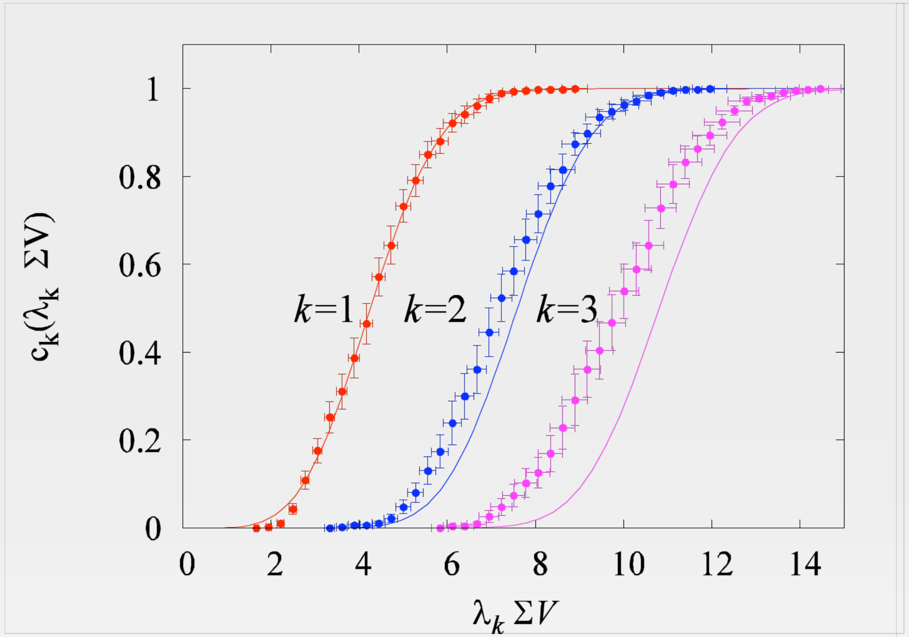
Low energy constant Σ can be determined.

Nonzero mode contribution give finite size corrections.

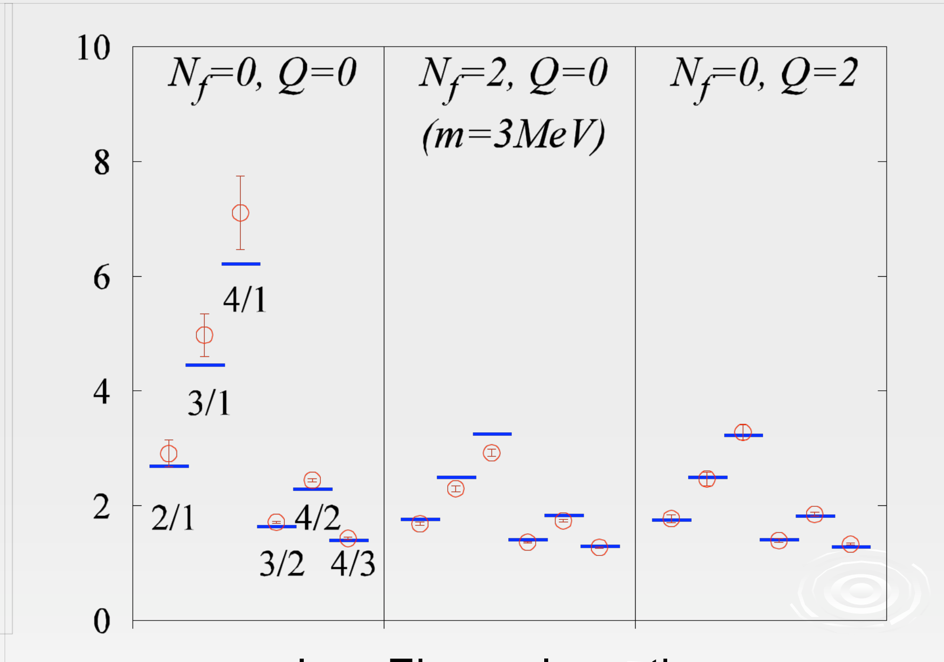
In principle, f_π can also determined from corrections.

QCD in ϵ regime (Run1)

- Eigenmode distribution is consistent with Chiral Random Matrix model up to finite volume corrections.



Cumulative distribution of low eigenvalues



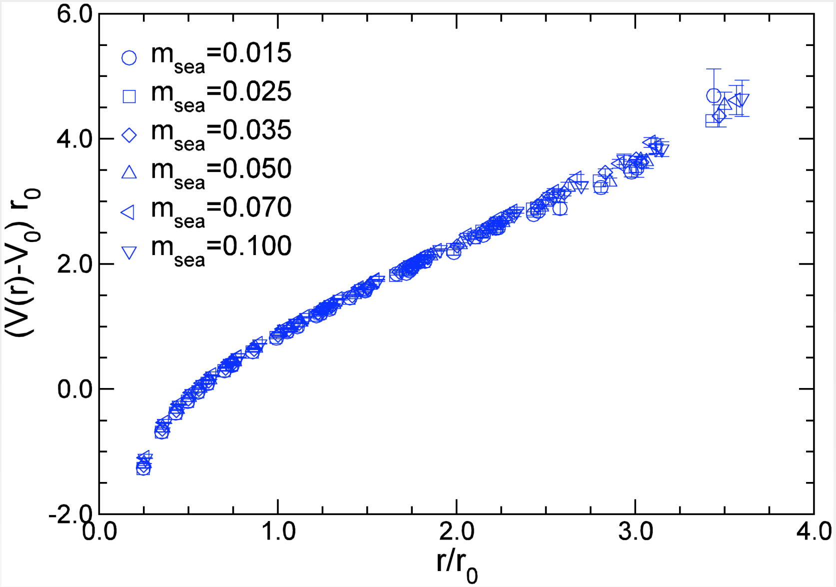
Low Eigenvalue ratios

$$\Sigma^{\overline{MS}}(2GeV) = (251 \pm 7 \pm 11\text{MeV})^3$$

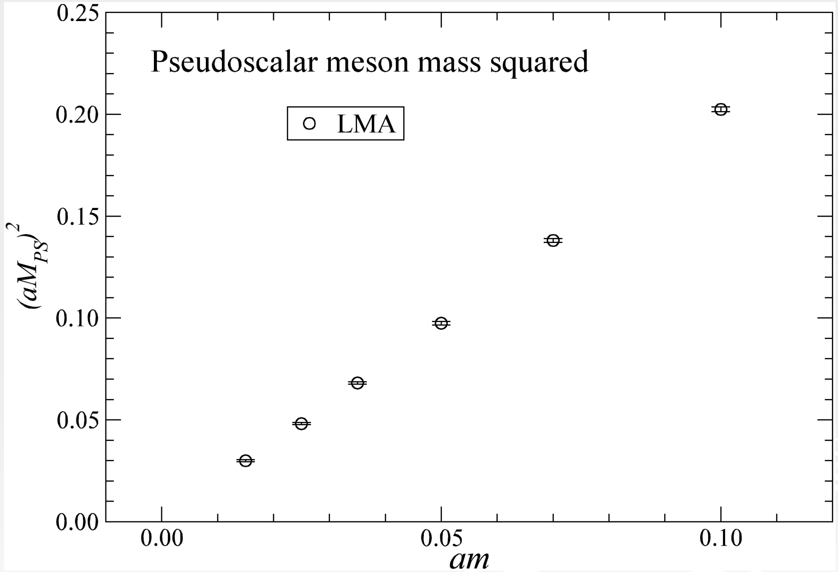
Fukaya et al. hep-lat/0702003

QCD in normal regime (Run2)

Preliminary results



the static quark potential



Quark mass dependence of the pion mass

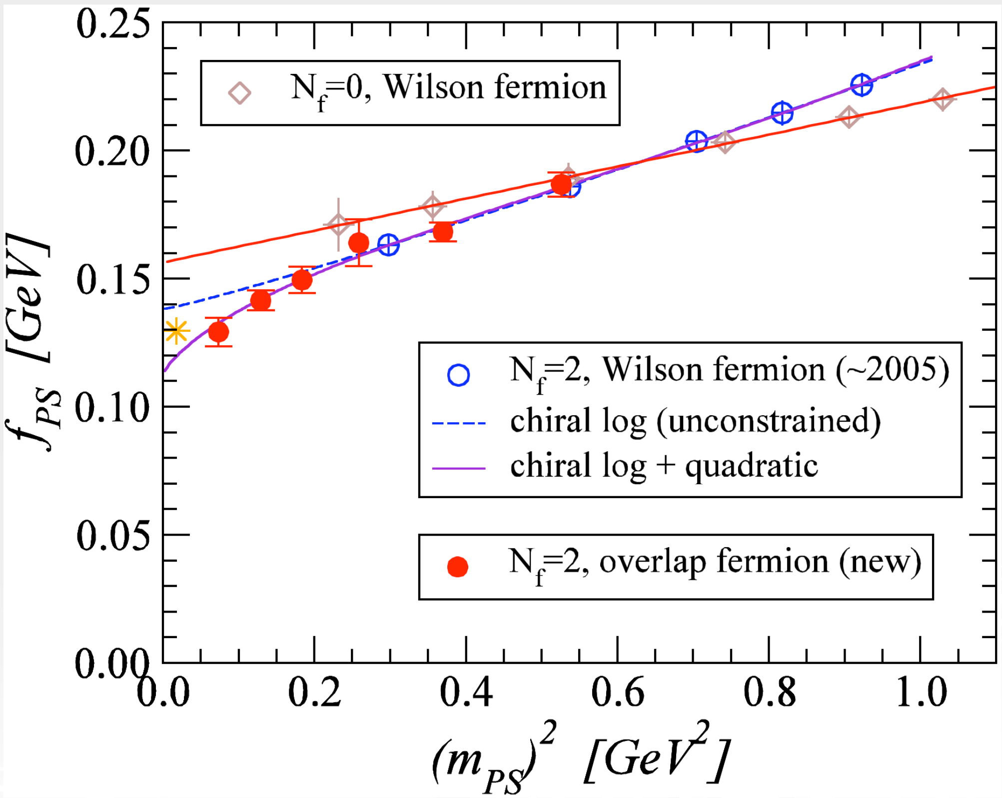
Preliminary results

Chiral log visible?

$$\frac{f}{f_0} = 1 - \frac{N_f}{2} y \log y + \alpha y$$

α : fit parameter
y : $m_\pi^2/(4\pi f_0)$
f_0 : decay const. in chiral limit

Chiral behavior consistent with chiral log is reproduced.



Quark mass dependence of f_π

How to extract physics from fixed topologies

- In principle, fixing the topology does not affect physics for large enough volume for $\theta = 0$

  (Sum over topologies is needed for $\theta \neq 0$)

- However, there may be finite volume effects which should be estimated.

- Also, it is not obvious the local fluctuation of the topology via instanton anti-instanton pair creation is sufficiently thermalized with topology conserving action.

⟹ Measuring topological susceptibility is important.

What is the fixed Q effect?

Brower, Chandrasekaran, Negele, Wiese,
Phys.Lett.B560(2003)64 "QCD at fixed topology"

- $M(\theta)$:hadron mass in θ vacuum

  M^Q :hadron mass at fixed Q

$$M^Q = M(0) + \frac{1}{2} M''(0) \frac{1}{V\chi_t} \left(1 - \frac{Q^2}{V\chi_t}\right)$$

Example: θ dependence from Chiral Perturbation Theory

$$M_\pi(\theta) = M_\pi(0) cos(\theta/N_f), \quad M''_\pi(0) = -M_\pi(0)/N_f^2, \quad \chi_t = \frac{F_\pi^2}{2N_f} M_\pi^2(0)$$

$$\Longrightarrow \quad M_\pi^{Q=0} = M_\pi(0)[1 - \underbrace{\frac{1}{N_f V M_\pi^2(0) F_\pi^2}}_{\text{correction}}]$$

The correction from fixing the topology is 3%-1%

for $M_\pi(0) = 300\text{MeV} - 500\text{MeV}$ with (2fm)^4

Topological susceptibility

- Measure the topological susceptibility
  - check thermal equilibrium in topology
  - Useful for estimate the finite size effects

Definitions

- Giusti, Rossi, Testa Phys.Lett.B587(2004)157

  disconnected loop $\sum_x \langle \langle P(x) \rangle_F \langle P(0) \rangle_F \rangle_A$

- Luescher, Phys.Lett.B(2004)296

  n-point function without div. $\sum_{x_1,x_2,x_3,x_4} \langle \langle P_{12}(x_1) S_{23}(x_2) S_{31}(x_3) \rangle_F \langle P_{45}(x_4) S_{54}(0) \rangle_F \rangle_A$

- Asymptotic value Fukaya, T.O. Phys.Rev.D70(2004)054508

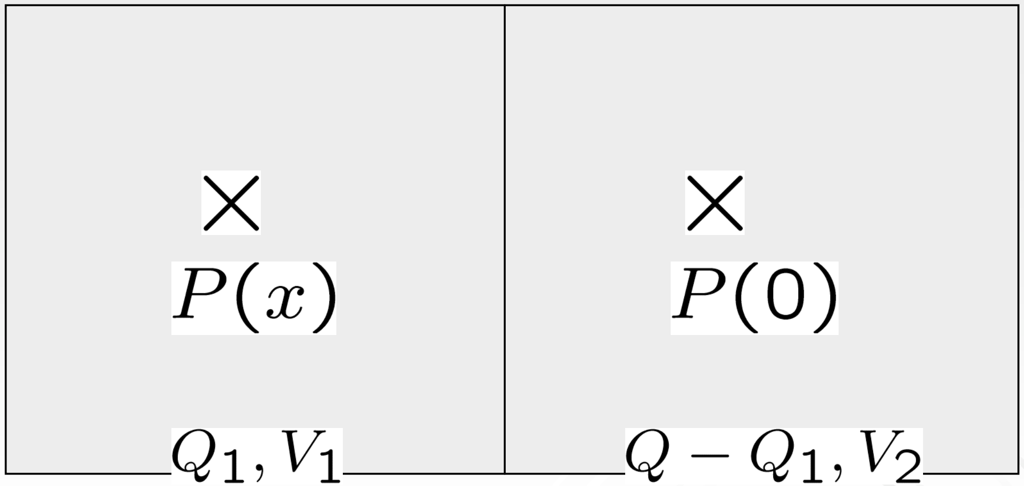
$$-\langle P(x)P(0) \rangle_{Q,V} \rightarrow_{x\rightarrow\infty} 4(\chi_t/V - Q^2/V^2)/m^2$$

(1) Ward-Takahashi identity $\langle P(x)\rangle_{Q,V} = \frac{2Q}{mV}$

(2) Cluster Property $R(Q,V)$ Q distribution

$$\langle P(x)P(0)\rangle_{Q,V} \quad (|x| \text{ large}) \\ \sim \sum_{Q_1} R(Q_1,V_1)\langle P(x)\rangle_{Q_1,V_1} R(Q-Q_1,V_2)\langle P(0)\rangle_{Q-Q_1,V_2}$$

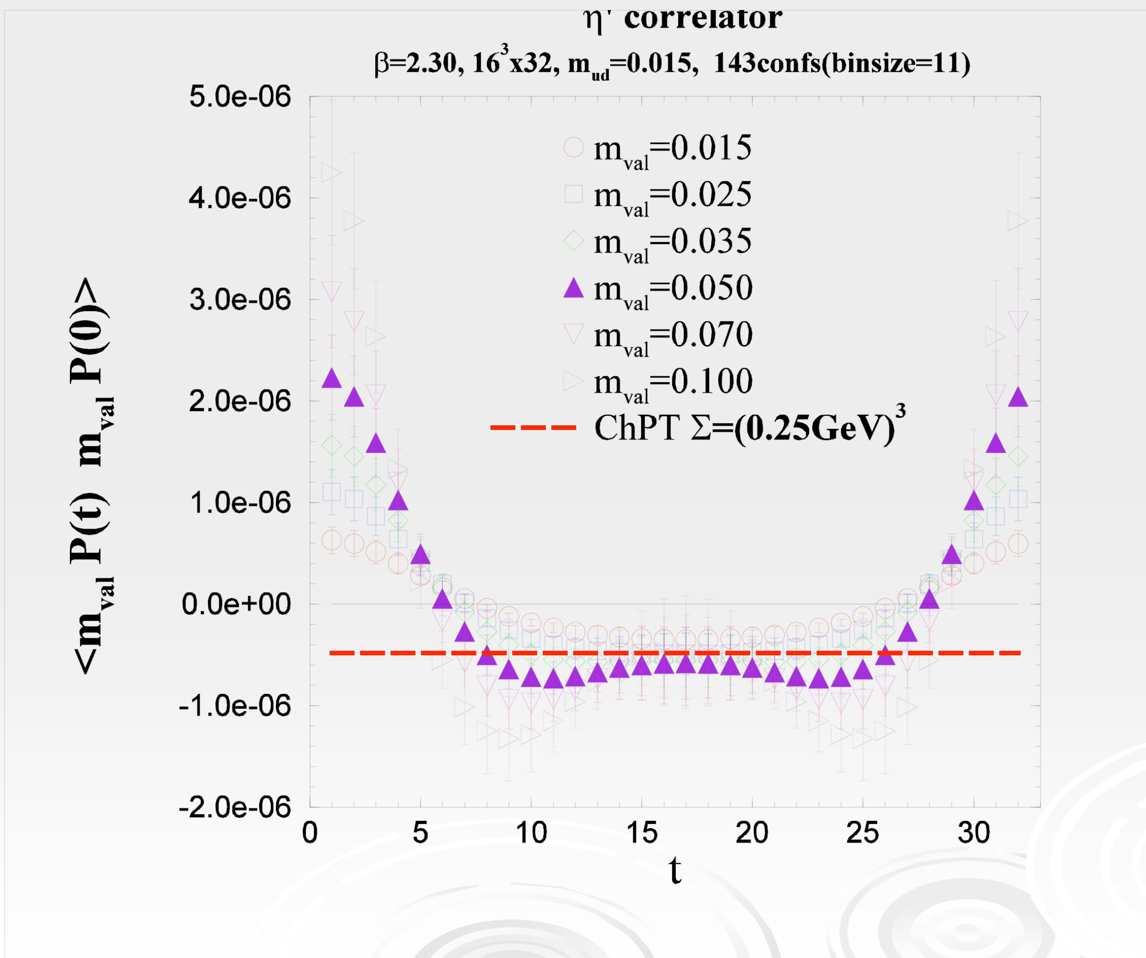
(1)&(2) ⇒ $-\langle P(x)P(0)\rangle_{Q,V} \sim \frac{4}{m^2}\left(\frac{\chi_t}{V} - \frac{Q^2}{V^2}\right)$



Topological suscpetibility can be measured indirectly from asymptopic values of Pseudoscalar 2-pt ftn

Topological susceptibility

preliminary



Summary

- Our proposal for Topology conserving action realized the first large scale simulation with dynamical Ginsparg-Wilson fermion.

  A new era for lattice QCD.
- We reproduced the ChRMT prediction from 1st principles of QCD and determined the chiral condensate/
- Chiral behavoir of the light meson is consistent with ChPT.
- Topological susceptibility is also consistent with ChPT.

Futture prospects

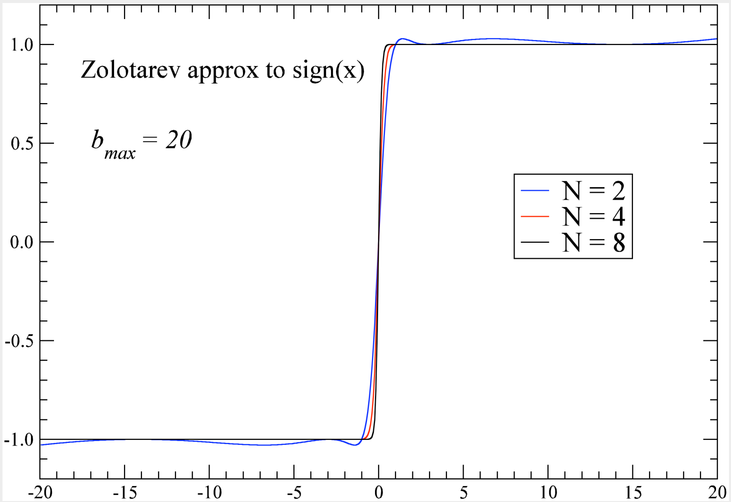
- 2+1 flavor QCD
- Weak matrix elements $B_K, \cdots$
- Finite temperature QCD

- Backup slides

projects of unquenched QCD simulations

Many unquenched simulations are performed or starting now.
In addition to rooted staggered by MILC collab.,
Wilson-type fermions and Ginsparg-Wilson fermions are in progress.
Important for cross-check and theoretically clean

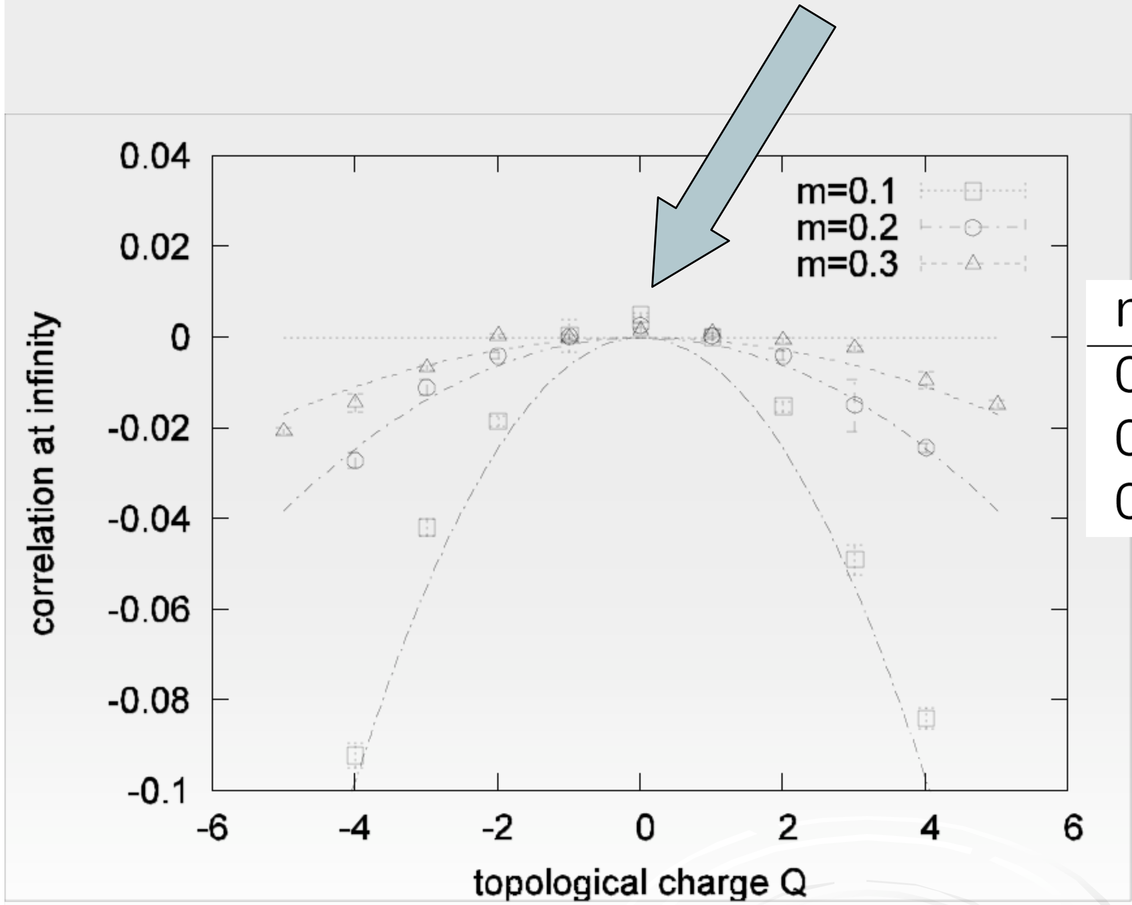
Group	Action	n_f	a (fm)	m_π (MeV)
MILC	Staggered	2+1	0.09, 0.12	$\geq$300
Del Debbio et al.	Wilson, O(a)-imp Wilson	2	0.052-0.075	$\geq$ 300
CP-PACS/JLQCD	O(a)-imp Wilson	2+1	0.07,0.10,0.12	$\geq$600
PACS-CS	O(a)-imp Wilson	2+1	0.07, 0.10, 0.12	$\geq$200
ETMC	twisted Wilson	2	0.075, 0.096	$\geq$270
JLQCD	Overlap	2 (2+1)	0.11	$\geq$300
RBC UKQCD	Domain wall	2+1	0.09-0.13	$\geq$ 600



Zolotarev approx to sign(x)
$b_{max} = 20$
N = 2
N = 4
N = 8
1.0
0.5
0.0
-0.5
-1.0
-20
-15
-10
-5
0
5
10
15
20

Schwinger model case (fixed topology simulation)

There is indeed a nonzero constant for $-\lim_{|x|\to large}\langle P(x)P(0)\rangle_{Q=0,V}$



This constant gives topological susceptibility consisitent with direct measurement

m	χ_t direct	χ_t indirect
0.1	0.0036(4)	0.0033(3)
0.2	0.0066(6)	0.0069(13)
0.3	0.0095(8)	0.0093(22)