Lattice QCD
with dynamical overlap fermion
- understanding chiral dynamics
with exact chiral symmetry -

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Talk at Komaba 2007,
Recent Developments in Strings and Fields
On the occasion of Prof. Yoneya’s 60th birthday


Fukaya et al., Phys.Rev.D74(2006)095405,
hep-lat/0702003
Why is exact lattice chiral symmetry important?

There are fundamental problems in particle physics and cosmology where **nonperturbative QCD effect with chiral symmetry** is important.

- **Precise weak matrix elements for flavor physics**
  - Lack of chiral symmetry gives large systematic errors
    - Chiral extrapolation $f_B, B_B, \cdots$
    - Operator mixing from wrong chirality $B_K$

- **Finite temperature phase transition for early universe**
  - Critical behavior is quite subtle
    - Order of phase transition?
    - Critical exponents?

*In both cases, chiral symmetry plays crucial role.*
Weak matrix elements for flavor physics

Uncertainty in chiral extrapolation

example: decay constant $f_\pi, f_B, \cdots$

- quark mass was far from the chiral regime
- chiral extrapolation gives $10$-$15\%$ error
- We must reproduce chiral log from lattice

Operator mixing with wrong chirality

- Bag parameter $B_K$ for $K - \bar{K}$ mixing

$$\langle K | (\bar{s} \gamma^\mu d)_L (\bar{s} \gamma_\mu d)_L | \bar{K} \rangle = \frac{8}{3} m_K^2 f_K^2 B_K$$

$$\langle \bar{s}_L \gamma^\mu d_L \rangle_L (\bar{s}_L \gamma_\mu d_L)_{MS}^{MS}(\mu) = Z_1 (\bar{s}_L \gamma^\mu d_L) (\bar{s}_L \gamma_\mu d_L)_{lat}(a) + Z_2 (\bar{s}_L d_R) (\bar{s}_R d_L)_{lat}(a) + \cdots$$

Without chiral symmetry, lattice operators can mix with operators having wrong chirality

(huge corrections due to chiral enhancement)

BK from Wilson fermion
PRD60:114504

$B_K$ is uncertain due to huge radiative corrections.
Finite temperature phase transition

Phase diagram for 2+1 flavor (u,d,s)

- Staggered fermion = crossover
- Wilson fermion = 1st order

Scaling test for 2-flavor QCD

- Staggered fermion
  - Exponents do not agree with O(4) or O(2)

<table>
<thead>
<tr>
<th></th>
<th>O(4)</th>
<th>O(2)</th>
<th>QCD</th>
</tr>
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<tbody>
<tr>
<td>$1/\delta$</td>
<td>0.537(7)</td>
<td>0.602(2)</td>
<td>0.64(2)</td>
</tr>
<tr>
<td>$1 - 1/\delta$</td>
<td>0.7939(9)</td>
<td>0.7928(3)</td>
<td>1.03(9)</td>
</tr>
<tr>
<td>$(1 - \beta)/\beta \delta$</td>
<td>0.331(7)</td>
<td>0.394(2)</td>
<td>0.83(12)</td>
</tr>
</tbody>
</table>

Exponents do not agree with O(4) or O(2)

- Wilson fermion
  - consistent with O(4) scaling

Results from two actions do not agree
Chiral symmetry on the lattice

Nielsen-Ninomiya’s theorem

Consider a lattice fermion action $S_F = \bar{\psi} D \psi$ satisfying
- Translational invariance $D\gamma_5 + \gamma_5 D = 0$
- Chiral symmetry:
- Hermiticity
- Locality

Then, doublers must exist

Staggered fermion: 4 spinors $\otimes$ 4 “tastes” (doublers)

to apply QCD (u,d,s) one must take “the fourth root trick” $\det(D) \sim \det(D_{\text{staggered}})^{1/4}$

Very dangerous compromise! Even locality is doubtful.
We develop a method for lattice QCD with dynamical overlap fermion. First large scale simulation.

**Short term goal:**
Derive the chiral dynamics from 1\textsuperscript{st} principle QCD
- Prediction from Chiral Random Matrix theory
- Chiral behavior of light hadrons

**Long term goal:**
Apply our method to flavor physics, finite temperature QCD
Ginsparg-Wilson fermion

Ginsparg-Wilson relation


\[ D\gamma_5 + \gamma_5 D = a D\gamma_5 D \]

Exact chiral symmetry on the lattice (index theorem)


\[ \psi \rightarrow \psi + i\gamma_5 (1 - aD)\psi = \psi + i\gamma_5 \psi \]

\[ \bar{\psi} \rightarrow \bar{\psi} + i\bar{\psi} \gamma_5 \]

Overlap fermion (explicit construction)

\[ D = \frac{1}{a} \left[ 1 + \gamma_5 \text{sign}(H_W) \right], \]

\[ D_W : \text{Wilson Dirac op.}, \quad H_W \equiv \gamma_5 (D_W + M_0) \]

\[ M_0 : \text{negative mass} \]
Pioneering studies on Dynamical Overlap fermion


Dirac operator becomes singular at the zeros of $H_W$

**Conclusion: Large scale simulation is hopeless.**
Problems (all related to the zeros of $H_w$)

- We make rational approximation with completely controlled error except near zero mode.

- Dov makes a discontinous jump when an eigenmode of $H_w$ crosses zero. Hybrid Monte Carlo breaks down.

- A method to cure this problem has been developed. One has to monitor the zero crossing at much higher precision and include correction terms at the exact point of crossing. (Hopelessly huge numerical cost)
Low mode and topology change

- Zeros of $H_w(m)$ arise when the topology changes through localized modes.

$$Tr[\gamma_5] = -Tr[\gamma_5 D_{ov}] = -Tr[\epsilon(H_W)]$$

Our strategy:

use topology conserving gauge action to suppress the low modes
Det($H_w$) term (Fukaya, Vranas)

Fukaya et al. hep-lat/0607020

Introduce negative heavy mass wilson fermion as a UV regulator field, whose mass is exactly the same as that appears in Dov. Infrared physics is unchanged.

\[ Z = \int DU \det(H^2_w) \det(D_{ov})^2 e^{-S} \]

or

\[ \int DU \frac{\det(H^2_w)}{\det(H^2_w + \mu^2)} \det(D_{ov})^2 e^{-S} \]

This term should kill the breakdown of locality topology change, and blow-up of numerical cost simultaneously.
Suppression of low modes by $\frac{\text{det}(H_W^2)}{\text{det}(H_W^2 + \mu^2)}$ is confirmed.

Fukaya et al.
JLQCD GW project

KEK BlueGene (10 racks, 57.3 TFlops)
- Started on March 1, 2006
- 1 rack = 1024 nodes = 2048 CPU

JLQCD collaboration:
KEK: S. Hashimoto, T. Kaneko,
    H. Matsufuru, J. Noaki, M. Okamoto, N. Yamada
RIKEN: H. Fukaya
YITP: T. Onogi
Tsukuba: S. Aoki, K. Kanaya, A. Ukawa, T. Yoshie
Hiroshima: K.-I. Ishikawa, M. Okawa
Taiwan National U: T. W. Chiu collaborators (TWQCD)
Numerical simulation

Dynamical simulation with Nf=2 overlap fermion

Run1 (epsilon-regime)
- 16^3 x 32, 0.11 fm
- quark mass around 3MeV!!
- Fixed topology

Run2 (normal regime)
- 16^3 x 32, a=0.12 fm
- quark mass 6 values in the range of ms/6-ms
- Fixed topology
- At Q=0 accumulated 10,000 trajectories
QCD in epsilon regime


Epsilon-regime: special kinematical situation
\[ \Lambda_{QCD}^{-1} \ll L \leq m_\pi^{-1} \] (usually, \( \Lambda_{QCD}^{-1} \ll m_\pi^{-1} \ll L \))

Different expansion applies in ChPT
\[ L = \frac{f^2 \text{tr}(\partial_\mu U \partial_\mu U^\dagger)}{\text{nonzero mode only}} + \frac{m \text{tr}(U + U^\dagger)}{\text{zero mode}} + \cdots \]

(zero mode integral is described by Bessel function)

**Low eigenvalues of Dirac operator can be evaluated from Chiral Random Matrix Theory (ChRMT)**


Low energy constant \( \sum \) can be determined.
Nonzero mode contribution give finite size corrections.
In principle, \( f_\pi \) can also determined from corrections.
QCD in $\epsilon$ regime (Run1)

- Eigenmode distribution is consistent with Chiral Random Matrix model up to finite volume corrections.

Cumulative distribution of low eigenvalues

\[ \sum_{k=1}^{\infty} \frac{\lambda_k}{\sum \lambda_k} = (2Gev)^3 \]

Low Eigenvalue ratios

Fukaya et al. hep-lat/0702003
QCD in normal regime (Run2)

Preliminary results

the static quark potential

Quark mass dependence of the pion mass

QCD with overlap fermion
Preliminary results

Chiral log visible?

\[
\frac{f}{f_0} = 1 - \frac{N_f}{2} y \log y + \alpha y
\]

\(\alpha\) : fit parameter
\(y\) : \(m_{\pi}^2/(4\pi f_0)\)
\(f_0\) : decay const. in chiral limit

Chiral behavior consistent with chiral log is reproduced.

Quark mass dependence of \(f_\pi\)
How to extract physics from fixed topologies

- In principle, fixing the topology does not affect physics for large enough volume for $\theta = 0$
  (Sum over topologies is needed for $\theta \neq 0$)

- However, there may be finite volume effects which should be estimated.

- Also, it is not obvious the local fluctuation of the topology via instanton anti-instanton pair creation is sufficiently thermalized with topology conserving action.

Measuring topological susceptibility is important.
What is the fixed Q effect?

Brower, Chandrasekaran, Negele, Wiese,

\[
M(\theta) \text{ : hadron mass in } \theta \text{ vacuum} \\
M^Q = M(0) + \frac{1}{2} M''(0) \frac{1}{V \chi t} (1 - \frac{Q^2}{V \chi t})
\]

Example: \( \theta \) dependence from Chiral Perturbation Theory

\[
M_\pi(\theta) = M_\pi(0) \cos(\theta/N_f), \quad M_\pi''(0) = -M_\pi(0)/N_f^2, \quad \chi t = \frac{F_\pi^2}{2N_f} M_\pi^2(0)
\]

\[
M_\pi^Q=0 = M_\pi(0) \left[ 1 - \frac{1}{N_f V M_\pi^2(0) F_\pi^2} \right]
\]

The correction from fixing the topology is 3%-1%

for \( M_\pi(0) = 300\text{MeV} - 500\text{MeV} \) with \((2\text{fm})^4\)
Topological susceptibility

- Measure the topological susceptibility
  - check thermal equilibrium in topology
  - Useful for estimate the finite size effects

Definitions

  
  \[
  \sum_x \langle \langle P(x) \rangle_F \langle P(0) \rangle_F \rangle_A
  \]

  
  n-point function without div.
  
  \[
  \sum_{x_1,x_2,x_3,x_4} \langle \langle P_{12}(x_1)S_{23}(x_2)S_{31}(x_3) \rangle_F \langle P_{45}(x_4)S_{54}(0) \rangle_F \rangle_A
  \]

  
  \[
  -\langle P(x)P(0) \rangle_{Q,V} \rightarrow_{x \rightarrow \infty} 4\left(\frac{\chi_t}{V} - \frac{Q^2}{V^2}\right) / m^2
  \]
(1) Ward-Takahashi identity
\[ \langle P(x) \rangle_{Q,V} = \frac{2Q}{mV} \]

(2) Cluster Property
\[ R(Q,V) \text{ Q distribution} \]
\[ \langle P(x)P(0) \rangle_{Q,V} \quad (|x| \text{ large}) \]
\[ \sim \sum_{Q_1} R(Q_1,V_1) \langle P(x) \rangle_{Q_1,V_1} R(Q - Q_1,V_2) \langle P(0) \rangle_{Q - Q_1,V_2} \]

\[ \langle P(x)P(0) \rangle_{Q,V} \sim \frac{4}{m^2} \left( \frac{\chi_t}{V} - \frac{Q^2}{V^2} \right) \]

Topological susceptibility can be measured indirectly from asymptotic values of Pseudoscalar 2-pt ftn

QCD with overlap fermion
Topological susceptibility

preliminary

QCD with overlap fermion
Summary

- Our proposal for Topology conserving action realized the first large scale simulation with dynamical Ginsparg-Wilson fermion. A new era for lattice QCD.
- We reproduced the ChRMT prediction from 1st principles of QCD and determined the chiral condensate/condensate.
- Chiral behaviour of the light meson is consistent with ChPT.
- Topological susceptibility is also consistent with ChPT.

Future prospects
- 2+1 flavor QCD
- Weak matrix elements \( B_K, \ldots \)
- Finite temperature QCD
Backup slides
Many unquenched simulations are performed or starting now. In addition to rooted staggered by MILC collab., Wilson-type fermions and Ginsparg-Wilson fermions are in progress. Important for cross-check and theoretically clean.

<table>
<thead>
<tr>
<th>Group</th>
<th>Action</th>
<th>$n_f$</th>
<th>$a$ (fm)</th>
<th>$m_\pi$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MILC</td>
<td>Staggered</td>
<td>2+1</td>
<td>0.09, 0.12</td>
<td>$\geq 300$</td>
</tr>
<tr>
<td>Del Debbio et al.</td>
<td>Wilson, O(a)-imp Wilson</td>
<td>2</td>
<td>0.052-0.075</td>
<td>$\geq 300$</td>
</tr>
<tr>
<td>CP-PACS/JLQCD</td>
<td>O(a)-imp Wilson</td>
<td>2+1</td>
<td>0.07, 0.10, 0.12</td>
<td>$\geq 600$</td>
</tr>
<tr>
<td>PACS-CS</td>
<td>O(a)-imp Wilson</td>
<td>2+1</td>
<td>0.07, 0.10, 0.12</td>
<td>$\geq 200$</td>
</tr>
<tr>
<td>ETMC</td>
<td>twisted Wilson</td>
<td>2</td>
<td>0.075, 0.096</td>
<td>$\geq 270$</td>
</tr>
<tr>
<td>JLQCD</td>
<td>Overlap</td>
<td>2 (2+1)</td>
<td>0.11</td>
<td>$\geq 300$</td>
</tr>
<tr>
<td>RBC UKQCD</td>
<td>Domain wall</td>
<td>2+1</td>
<td>0.09-0.13</td>
<td>$\geq 600$</td>
</tr>
</tbody>
</table>
Zolotarev approx to sign(x)
Schwinger model case (fixed topology simulation)

There is indeed a nonzero constant for

\[ \lim_{|x| \to \text{large}} \langle P(x)P(0) \rangle_{Q=0,V} \]

This constant gives topological susceptibility consistent with direct measurement

<table>
<thead>
<tr>
<th>m</th>
<th>$\chi_t$ direct</th>
<th>$\chi_t$ indirect</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.0036(4)</td>
<td>0.0033(3)</td>
</tr>
<tr>
<td>0.2</td>
<td>0.0066(6)</td>
<td>0.0069(13)</td>
</tr>
<tr>
<td>0.3</td>
<td>0.0095(8)</td>
<td>0.0093(22)</td>
</tr>
</tbody>
</table>