Analytic solutions for marginal deformations in open string field theory

Yuji Okawa (DESY)

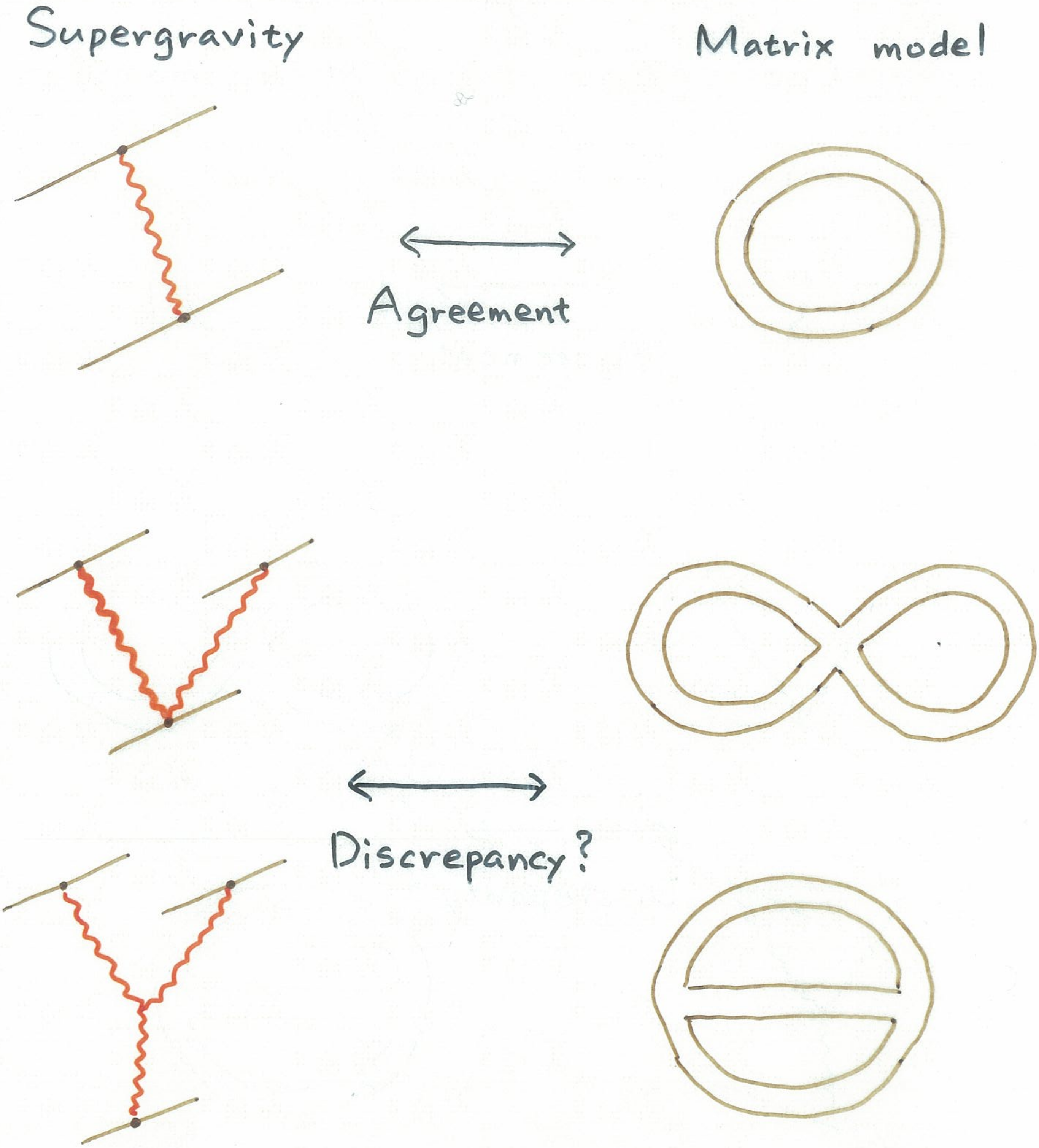
February 11, 2007 at Komaba 2007

Kiermaier, Okawa, Rastelli & Zwiebach, hep-th/0701249

Schnabl, hep-th/0701248

A controversy about the BFSS matrix model

Dine & Rajaraman, hep-th/9710174



Okawa & Yoneya, hep-th/9806108

$$\tilde{\Gamma}_{(2)} = \tilde{\Gamma}_V + \tilde{\Gamma}_Y, \tag{3.41}$$

$$-\tilde{\Gamma}_V = \frac{g^2}{\kappa}\int d\tau_1 d\tau_2 d\sigma_1 d\sigma_2 d\sigma_3 \times \sum_{i,j,k}\left[-\partial_{\sigma_1}\{(P_q+P_3)\Delta_{ij}(\sigma_1,\tau_1,\tau_2)\Delta_{jk}(\sigma_2,\tau_1,\tau_2)\Delta_{ki}(\sigma_3,\tau_1,\tau_2)\}\right], \tag{3.42}$$

$$\begin{aligned}-\tilde{\Gamma}_Y = \frac{g^2}{\kappa}\int & d\tau_1\, d\tau_2\, d\sigma_1\, d\sigma_2\, d\sigma_3 \\ &\times \sum_{i,j,k}\Big[P_1\{\partial_{\tau_1}\partial_{\tau_2}\Delta_{ij}(\sigma_1,\tau_1,\tau_2)\}\Delta_{jk}(\sigma_2,\tau_1,\tau_2)\Delta_{ki}(\sigma_3,\tau_1,\tau_2) \\ &+P_2\{\partial_{\tau_1}\Delta_{ij}(\sigma_1,\tau_1,\tau_2)\}\Delta_{jk}(\sigma_2,\tau_1,\tau_2)\Delta_{ki}(\sigma_3,\tau_1,\tau_2) \\ &-\tfrac{1}{2}P_3(r_{ij}(\tau_1)-r_{ij}(\tau_2))^2\Delta_{ij}(\sigma_1,\tau_1,\tau_2)\Delta_{jk}(\sigma_2,\tau_1,\tau_2)\Delta_{ki}(\sigma_3,\tau_1,\tau_2) \\ &+\tfrac{1}{2}P_3\{(\partial_{\tau_1}^2+\partial_{\tau_2}^2)\Delta_{ij}(\sigma_1,\tau_1,\tau_2)\}\Delta_{jk}(\sigma_2,\tau_1,\tau_2)\Delta_{ki}(\sigma_3,\tau_1,\tau_2) \\ &+(P_4+\partial_{\sigma_1}P_3)\Delta_{ij}(\sigma_1,\tau_1,\tau_2)\Delta_{jk}(\sigma_2,\tau_1,\tau_2)\Delta_{ki}(\sigma_3,\tau_1,\tau_2)\Big].\end{aligned} \tag{3.43}$$

Since the whole dependence on $\{x^2\}$ in $\tilde{\Gamma}_V$ and $\tilde{\Gamma}_Y$ is now contained only in the proper-time propagators, we can expect that the above cancellation could occur before the proper-time integrations, that is, in the prefactors in front of proper-time propagators. This is indeed the case.

3.2.1. The calculation of $\tilde{\Gamma}_V$

Let us begin with $\tilde{\Gamma}_V$. It is easily calculated as follows:

$$\begin{aligned}\tilde{\Gamma}_V = &-\frac{g^2}{\kappa}\sum_{i,j,k}\int_{-\infty}^{\infty} d\tau \int_0^{\infty} d\sigma_2 \int_0^{\infty} d\sigma_3\; 128\sinh^3\frac{\sigma_2\tilde{v}_{jk}}{2}\sinh^3\frac{\sigma_3\tilde{v}_{ki}}{2} \\ &\times\left(\frac{2\tilde{v}_{jk}\cdot\tilde{v}_{ki}}{\tilde{v}_{jk}\tilde{v}_{ki}}\cosh\frac{\sigma_2\tilde{v}_{jk}}{2}\cosh\frac{\sigma_3\tilde{v}_{ki}}{2} - \sinh\frac{\sigma_2\tilde{v}_{jk}}{2}\sinh\frac{\sigma_3\tilde{v}_{ki}}{2}\right) \\ &\times\Delta_{jk}(\sigma_2,\tau,\tau)\Delta_{ki}(\sigma_3,\tau,\tau) \\ = &-\frac{g^2}{\kappa}\sum_{i,j,k}\int_0^{\infty} d\sigma_2 \int_0^{\infty} d\sigma_3\; 128\sinh^3\frac{\sigma_2\tilde{v}_{jk}}{2}\sinh^3\frac{\sigma_3\tilde{v}_{ki}}{2} \\ &\times\left(\frac{2\tilde{v}_{jk}\cdot\tilde{v}_{ki}}{\tilde{v}_{jk}\tilde{v}_{ki}}\cosh\frac{\sigma_2\tilde{v}_{jk}}{2}\cosh\frac{\sigma_3\tilde{v}_{ki}}{2} - \sinh\frac{\sigma_2\tilde{v}_{jk}}{2}\sinh\frac{\sigma_3\tilde{v}_{ki}}{2}\right) \\ &\times\sqrt{\frac{\tilde{v}_{jk}}{2\pi\sinh(2\sigma_2\tilde{v}_{jk})}}\sqrt{\frac{\tilde{v}_{ki}}{2\pi\sinh(2\sigma_3\tilde{v}_{ki})}}\sqrt{\frac{\pi}{\tilde{v}_{jk}\tanh(\sigma_2\tilde{v}_{jk})+\tilde{v}_{ki}\tanh(\sigma_3\tilde{v}_{ki})}} \\ &\times\exp\left[-\sigma_2\left(x_{jk}^2-\frac{(x_{jk}\cdot\tilde{v}_{jk})^2}{\tilde{v}_{jk}^2}\right)-\sigma_3\left(x_{ki}^2-\frac{(x_{ki}\cdot\tilde{v}_{ki})^2}{\tilde{v}_{ki}^2}\right)\right.\end{aligned}$$

as in Ref. [21]. See also Ref. [24]. A computation which is rather close to ours on the supergravity side is presented in [23]. An extension of our work including recoil corrections is given in [25], which should hopefully be read as an accompanying paper to the present work.

Acknowledgements

We would like to thank M. Ikehara for discussions on two-loop computations in matrix models and, especially, to Y. Kazama for collaborative discussions on the role of supersymmetry in the dynamics of D-particles at an early stage of the present work. A part of the present work was done during one of the authors' stay (T.Y.) at ITP, Santa Barbara. T.Y. would like to thank ITP for its hospitality. The research at ITP was supported in part by the National Science Foundation under Grant No. PHY94-07194. The work of T.Y. is supported in part by Grant-in-Aid for Scientific Research (No. 09640337) and Grant-in-Aid for International Scientific Research (Joint Research, No. 10044061) from the Ministry of Education, Science and Culture. The work of Y.O. is supported in part by the Japan Society for the Promotion of Science under the Predoctoral Research Program (No. 08-4158).

Appendix A. Explicit forms of P_q, P_1, P_2, P_3 and P_4

We here present the explicit forms of P_q, P_1, P_2, P_3 and P_4.

$$
\begin{aligned}
P_q =& -45 - 18V_{jk}(\sigma_2)^2 - 18V_{ki}(\sigma_3)^2 + 12C_{jk}(\sigma_2)C_{ki}(\sigma_3)(V_{jk}(\sigma_2)\cdot V_{ki}(\sigma_3)) \\
&-6V_{jk}(\sigma_2)^2V_{ki}(\sigma_3)^2 + 2(V_{jk}(\sigma_2)\cdot V_{ki}(\sigma_3))^2, \qquad (A.1)\\
P_1 =& -(1+2V_{ij}(\sigma_1)^2)\{4+V_{jk}(\sigma_2)^2+V_{ki}(\sigma_3)^2+2(V_{jk}(\sigma_2)\cdot V_{ki}(\sigma_3))^2\} \\
&+(1+2V_{jk}(\sigma_2)^2)\{\tfrac{17}{2}+2V_{ij}(\sigma_1)^2+2V_{ki}(\sigma_3)^2+4(V_{ij}(\sigma_1)\cdot V_{ki}(\sigma_3))^2\} \\
&+(1+2V_{ki}(\sigma_3)^2)\{\tfrac{17}{2}+2V_{ij}(\sigma_1)^2+2V_{jk}(\sigma_2)^2+4(V_{ij}(\sigma_1)\cdot V_{jk}(\sigma_2))^2\} \\
&-10C_{jk}(\sigma_2)C_{ki}(\sigma_3)\{V_{jk}(\sigma_2)\cdot V_{ki}(\sigma_3)+2(V_{ij}(\sigma_1)\cdot V_{jk}(\sigma_2))(V_{ij}(\sigma_1)\cdot V_{ki}(\sigma_3))\} \\
&+2C_{ij}(\sigma_1)C_{jk}(\sigma_2)\{V_{ij}(\sigma_1)\cdot V_{jk}(\sigma_2)+2(V_{ij}(\sigma_1)\cdot V_{ki}(\sigma_3))(V_{jk}(\sigma_2)\cdot V_{ki}(\sigma_3))\} \\
&+2C_{ij}(\sigma_1)C_{ki}(\sigma_3)\{V_{ij}(\sigma_1)\cdot V_{ki}(\sigma_3)+2(V_{ij}(\sigma_1)\cdot V_{jk}(\sigma_2))(V_{jk}(\sigma_2)\cdot V_{ki}(\sigma_3))\} \\
&+16\{-2C_{jk}(\sigma_2)C_{ki}(\sigma_3)-2V_{jk}(\sigma_2)\cdot V_{ki}(\sigma_3)-V_{ij}(\sigma_1)^2(V_{jk}(\sigma_2)\cdot V_{ki}(\sigma_3)) \\
&+(V_{ij}(\sigma_1)\cdot V_{jk}(\sigma_2))(V_{ij}(\sigma_1)\cdot V_{ki}(\sigma_3)) \\
&+2C_{ij}(\sigma_1)C_{ki}(\sigma_3)+2V_{ij}(\sigma_1)\cdot V_{ki}(\sigma_3)+V_{jk}(\sigma_2)^2(V_{ij}(\sigma_1)\cdot V_{ki}(\sigma_3)) \\
&-(V_{ij}(\sigma_1)\cdot V_{jk}(\sigma_2))(V_{jk}(\sigma_2)\cdot V_{ki}(\sigma_3)) \\
&+2C_{ij}(\sigma_1)C_{jk}(\sigma_2)+2V_{ij}(\sigma_1)\cdot V_{jk}(\sigma_2)+V_{ki}(\sigma_3)^2(V_{ij}(\sigma_1)\cdot V_{jk}(\sigma_2)) \\
&-(V_{ij}(\sigma_1)\cdot V_{ki}(\sigma_3))(V_{jk}(\sigma_2)\cdot V_{ki}(\sigma_3))\}, \qquad (A.2)\\
P_2 =& -4C_{ij}(\sigma_1)(V_{ij}(\sigma_1)\cdot r_{ij}(\tau_2))
\end{aligned}
$$

$$
\begin{aligned}
&\times\{4 + V_{jk}(\sigma_2)^2 + V_{ki}(\sigma_3)^2 + 2(V_{jk}(\sigma_2)\cdot V_{ki}(\sigma_3))^2 \\
&-2C_{jk}(\sigma_2)C_{ki}(\sigma_3)(V_{jk}(\sigma_2)\cdot V_{ki}(\sigma_3))\} \\
&+4C_{jk}(\sigma_2)(V_{jk}(\sigma_2)\cdot r_{ij}(\tau_2)) \\
&\times\{\tfrac{17}{2} + 2V_{ij}(\sigma_1)^2 + 2V_{ki}(\sigma_3)^2 + 4(V_{ij}(\sigma_1)\cdot V_{ki}(\sigma_3))^2 \\
&-4C_{ij}(\sigma_1)C_{ki}(\sigma_3)(V_{ij}(\sigma_1)\cdot V_{ki}(\sigma_3))\} \\
&+4C_{ki}(\sigma_3)(V_{ki}(\sigma_3)\cdot r_{ij}(\tau_2)) \\
&\times\{\tfrac{17}{2} + 2V_{ij}(\sigma_1)^2 + 2V_{jk}(\sigma_2)^2 + 4(V_{ij}(\sigma_1)\cdot V_{jk}(\sigma_2))^2 \\
&-4C_{ij}(\sigma_1)C_{jk}(\sigma_2)(V_{ij}(\sigma_1)\cdot V_{jk}(\sigma_2))\} \\
&+4C_{jk}(\sigma_2)V_{jk}(\sigma_2)\cdot(r_{jk}(\tau_2) - r_{ij}(\tau_2)) \\
&+4C_{ki}(\sigma_3)V_{ki}(\sigma_3)\cdot(r_{jk}(\tau_2) - r_{ki}(\tau_2)) \\
&+4\{C_{ij}(\sigma_1) - 2C_{jk}(\sigma_2)(C_{ij}(\sigma_1)C_{jk}(\sigma_2) - V_{ij}(\sigma_1)\cdot V_{jk}(\sigma_2))\} \\
&\times V_{ij}(\sigma_1)\cdot(r_{jk}(\tau_2) - r_{ij}(\tau_2)) \\
&+4\{C_{jk}(\sigma_2) - 2C_{ki}(\sigma_3)(C_{jk}(\sigma_2)C_{ki}(\sigma_3) - V_{jk}(\sigma_2)\cdot V_{ki}(\sigma_3))\} \\
&\times V_{jk}(\sigma_2)\cdot(r_{jk}(\tau_2) - r_{ki}(\tau_2)) \\
&+8\{C_{jk}(\sigma_2)(V_{jk}(\sigma_2)\cdot V_{ki}(\sigma_3)) + C_{ij}(\sigma_1)(V_{ij}(\sigma_1)\cdot V_{ki}(\sigma_3)) \\
&-2C_{jk}(\sigma_2)(V_{ij}(\sigma_1)\cdot V_{ki}(\sigma_3))(C_{ij}(\sigma_1)C_{jk}(\sigma_2) - V_{ij}(\sigma_1)\cdot V_{jk}(\sigma_2))\} \\
&\times V_{ki}(\sigma_3)\cdot(r_{jk}(\tau_2) - r_{ij}(\tau_2)) \\
&+8\{C_{ki}(\sigma_3)(V_{ij}(\sigma_1)\cdot V_{ki}(\sigma_3)) + C_{jk}(\sigma_2)(V_{ij}(\sigma_1)\cdot V_{jk}(\sigma_2)) \\
&-2C_{ki}(\sigma_3)(V_{ij}(\sigma_1)\cdot V_{jk}(\sigma_2))(C_{jk}(\sigma_2)C_{ki}(\sigma_3) - V_{jk}(\sigma_2)\cdot V_{ki}(\sigma_3))\} \\
&\times V_{ij}(\sigma_1)\cdot(r_{jk}(\tau_2) - r_{ki}(\tau_2)) \\
&+64\{-2C_{ij}(\sigma_1)(V_{jk}(\sigma_2)\cdot r_{jk}(\tau_2)) - C_{jk}(\sigma_2)(V_{ij}(\sigma_1)\cdot r_{jk}(\tau_2)) \\
&-C_{ki}(\sigma_3)(C_{jk}(\sigma_2)C_{ki}(\sigma_3) - V_{jk}(\sigma_2)\cdot V_{ki}(\sigma_3))(V_{ij}(\sigma_1)\cdot r_{jk}(\tau_2)) \\
&+C_{jk}(\sigma_2)(V_{ij}(\sigma_1)\cdot V_{ki}(\sigma_3))(V_{ki}(\sigma_3)\cdot r_{jk}(\tau_2)) \\
&-C_{ki}(\sigma_3)(V_{ij}(\sigma_1)\cdot V_{jk}(\sigma_2))(V_{ki}(\sigma_3)\cdot r_{jk}(\tau_2))\}, \qquad \text{(A.3)}
\end{aligned}
$$

$$
\begin{aligned}
P_3 = {}& 7 - 2(C_{jk}(\sigma_2)C_{ki}(\sigma_3) - V_{jk}(\sigma_2)\cdot V_{ki}(\sigma_3))^2 \\
&+4(C_{ij}(\sigma_1)C_{jk}(\sigma_2) - V_{ij}(\sigma_1)\cdot V_{jk}(\sigma_2))^2 \\
&+4(C_{ij}(\sigma_1)C_{ki}(\sigma_3) - V_{ij}(\sigma_1)\cdot V_{ki}(\sigma_3))^2 \\
&+16\{-(C_{jk}(\sigma_2)C_{ki}(\sigma_3) - V_{jk}(\sigma_2)\cdot V_{ki}(\sigma_3)) \\
&-(C_{ij}(\sigma_1)C_{jk}(\sigma_2) - V_{ij}(\sigma_1)\cdot V_{jk}(\sigma_2))(C_{ij}(\sigma_1)C_{ki}(\sigma_3) - V_{ij}(\sigma_1)\cdot V_{ki}(\sigma_3)) \\
&+(C_{ij}(\sigma_1)C_{jk}(\sigma_2) - V_{ij}(\sigma_1)\cdot V_{jk}(\sigma_2)) \\
&+(C_{ij}(\sigma_1)C_{ki}(\sigma_3) - V_{ij}(\sigma_1)\cdot V_{ki}(\sigma_3))(C_{jk}(\sigma_2)C_{ki}(\sigma_3) - V_{jk}(\sigma_2)\cdot V_{ki}(\sigma_3)) \\
&+(C_{ij}(\sigma_1)C_{ki}(\sigma_3) - V_{ij}(\sigma_1)\cdot V_{ki}(\sigma_3))
\end{aligned}
$$

$$+(C_{ij}(\sigma_1)C_{jk}(\sigma_2) - V_{ij}(\sigma_1)\cdot V_{jk}(\sigma_2))(C_{jk}(\sigma_2)C_{ki}(\sigma_3) - V_{jk}(\sigma_2)\cdot V_{ki}(\sigma_3))\}, \tag{A.4}$$

$$\begin{aligned}
P_4 = &-4(V_{ij}(\sigma_1)\cdot r_{ij}(\tau_1))(V_{ij}(\sigma_1)\cdot r_{ij}(\tau_2)) \\
&\times\{2 + (C_{jk}(\sigma_2)C_{ki}(\sigma_3) - V_{jk}(\sigma_2)\cdot V_{ki}(\sigma_3))^2\} \\
&+8(V_{jk}(\sigma_2)\cdot r_{ij}(\tau_1))(V_{jk}(\sigma_2)\cdot r_{ij}(\tau_2)) \\
&\times\{1 + (C_{ij}(\sigma_1)C_{ki}(\sigma_3) - V_{ij}(\sigma_1)\cdot V_{ki}(\sigma_3))^2\} \\
&+8(V_{ki}(\sigma_3)\cdot r_{ij}(\tau_1))(V_{ki}(\sigma_3)\cdot r_{ij}(\tau_2)) \\
&\times\{1 + (C_{ij}(\sigma_1)C_{jk}(\sigma_2) - V_{ij}(\sigma_1)\cdot V_{jk}(\sigma_2))^2\} \\
&+4(V_{ij}(\sigma_1)\cdot V_{ki}(\sigma_3))\{V_{ij}(\sigma_1)\cdot(r_{ij}(\tau_1) - r_{ki}(\tau_1))\} \\
&\times\{V_{ki}(\sigma_3)\cdot(r_{jk}(\tau_2) - r_{ij}(\tau_2))\} \\
&+4(V_{ij}(\sigma_1)\cdot V_{jk}(\sigma_2) - 2C_{ij}(\sigma_1)C_{jk}(\sigma_2)) \\
&\times\{V_{jk}(\sigma_2)\cdot(r_{ij}(\tau_1) - r_{ki}(\tau_1))\}\{V_{ij}(\sigma_1)\cdot(r_{jk}(\tau_2) - r_{ij}(\tau_2))\} \\
&+4\{-C_{jk}(\sigma_2)C_{ki}(\sigma_3) + V_{jk}(\sigma_2)\cdot V_{ki}(\sigma_3) + 2C_{ij}(\sigma_1)^2C_{jk}(\sigma_2)C_{ki}(\sigma_3) \\
&+2(V_{ij}(\sigma_1)\cdot V_{jk}(\sigma_2))(V_{ij}(\sigma_1)\cdot V_{ki}(\sigma_3)) \\
&-4C_{ij}(\sigma_1)C_{jk}(\sigma_2)(V_{ij}(\sigma_1)\cdot V_{ki}(\sigma_3))\} \\
&\times\{V_{jk}(\sigma_2)\cdot(r_{ij}(\tau_1) - r_{ki}(\tau_1))\}\{V_{ki}(\sigma_3)\cdot(r_{jk}(\tau_2) - r_{ij}(\tau_2))\} \\
&+32\{-2(V_{jk}(\sigma_2)\cdot r_{jk}(\tau_1))(V_{ki}(\sigma_3)\cdot r_{ki}(\tau_2)) \\
&-(V_{ki}(\sigma_3)\cdot r_{jk}(\tau_1))(V_{jk}(\sigma_2)\cdot r_{ki}(\tau_2)) \\
&+(C_{jk}(\sigma_2)C_{ki}(\sigma_3) - V_{jk}(\sigma_2)\cdot V_{ki}(\sigma_3))(V_{ij}(\sigma_1)\cdot r_{jk}(\tau_1))(V_{ij}(\sigma_1)\cdot r_{ki}(\tau_2)) \\
&-(C_{ij}(\sigma_1)C_{jk}(\sigma_2) - V_{ij}(\sigma_1)\cdot V_{jk}(\sigma_2))(V_{ki}(\sigma_3)\cdot r_{jk}(\tau_1))(V_{ij}(\sigma_1)\cdot r_{ki}(\tau_2)) \\
&-(C_{ij}(\sigma_1)C_{ki}(\sigma_3) - V_{ij}(\sigma_1)\cdot V_{ki}(\sigma_3))(V_{ij}(\sigma_1)\cdot r_{jk}(\tau_1))(V_{jk}(\sigma_2)\cdot r_{ki}(\tau_2))\}.
\end{aligned} \tag{A.5}$$

Appendix B. Examples of evaluations of P's

In this appendix, we present some examples of evaluations of $\tilde{\Gamma}_{(2)}$ from individual terms in (3.32). For each example below, we first write down the whole expression of the one-particle irreducible, planar contractions, and then present the final form which fits into the form (3.33).

The first two terms in (3.32) involve the quartic vertices and contribute to P_q.

$$\begin{aligned}
&\tfrac{1}{4}g^2\kappa\int d\tau\langle\mathrm{tr}[Y^n(\tau),Y^m(\tau)][Y^n(\tau),Y^m(\tau)]\rangle_{1\,\mathrm{PI,planar}} \\
&= \tfrac{1}{2}g^2\kappa\int d\tau\sum_{i,j,k}(\langle Y^n_{ij}(\tau)Y^m_{ji}(\tau)\rangle_0\ \langle Y^n_{ik}(\tau)Y^m_{ki}(\tau)\rangle_0 \\
&\quad -\langle Y^m_{ij}(\tau)Y^n_{ji}(\tau)\rangle_0\ \langle Y^m_{ik}(\tau)Y^m_{ki}(\tau)\rangle_0)
\end{aligned}$$

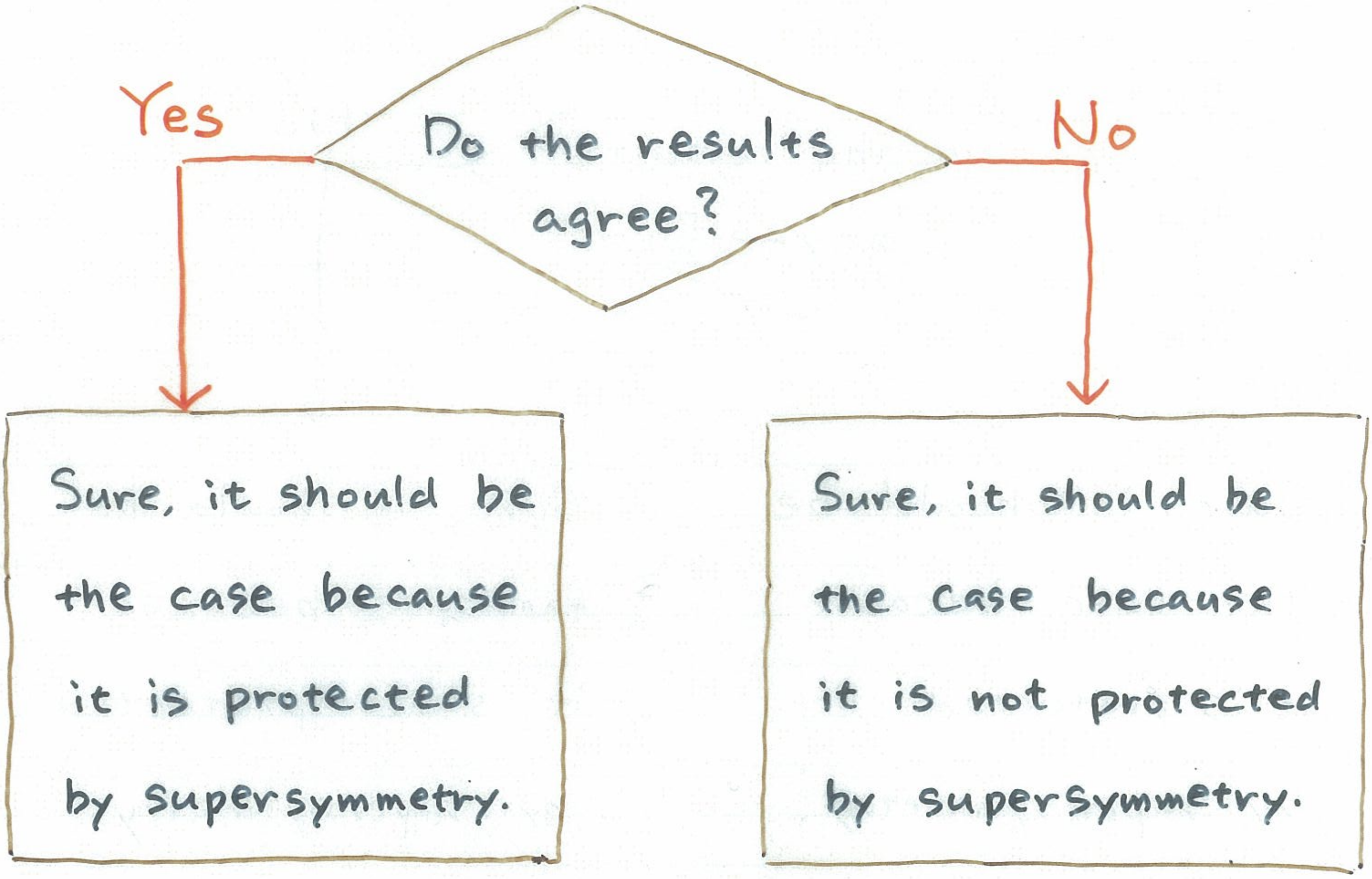
The four-body interactions and the three-loop calculations of the BFSS matrix model

Yuji Okawa (DESY)

February 11, 2007 at Komaba 2007

Based on collaboration with
Kazama & Taylor

What was the lesson?



Open bosonic string field theory

Ground state

tachyonic scalar $T(p)$ in 26 dimensions

First-excited state

massless vector field $A_\mu(p)$

$\vdots$

Degrees of freedom of string field theory

$$\{ T(p),\ A_\mu(p),\ \cdots \}$$
$$\downarrow$$
$$S[\, T(p),\ A_\mu(p),\ \cdots \,]$$

SU(2) gauge fields $\quad$ A single 2×2 matrix field

$$A^a_\mu(x),\ a=1,2,3 \longrightarrow A_\mu(x) = \frac{1}{2}\sum_{a=1}^{3} A^a_\mu(x)\,\sigma^a$$

$\{ T(p),\ A_\mu(p),\ \cdots \} \longrightarrow$ String field Ψ

= a state in a two-dimensional conformal field theory

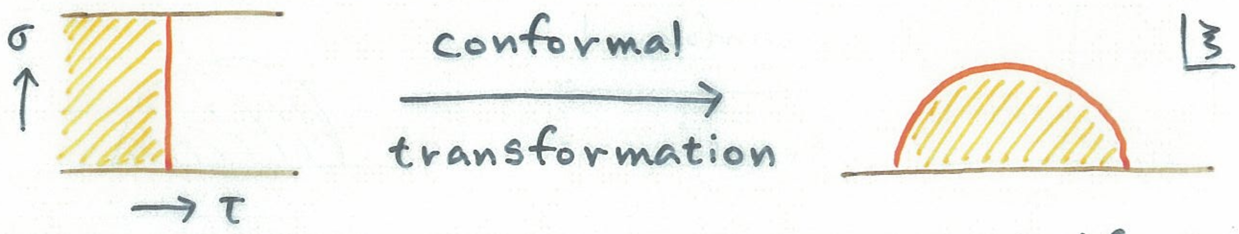
$$\Psi = \int \frac{d^{26}p}{(2\pi)^{26}} \left[\frac{1}{\sqrt{\alpha'}}\, T(p)\, c_1 |0;p\rangle + \frac{1}{\sqrt{\alpha'}}\, A_\mu(p)\, \alpha^\mu_{-1} c_1 |0;p\rangle + \cdots \right]$$

$$A^a_\mu(x) = \operatorname{tr} \sigma^a A_\mu(x)$$

Ψ can be specified by giving $\langle \phi, \Psi \rangle$ for all ϕ in the Fock space.

CFT description

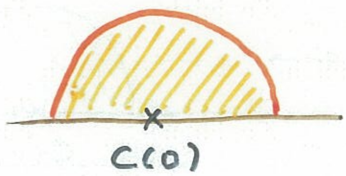
string field = state in the 2D CFT



strip coordinates   upper-half plane

State-operator correspondence

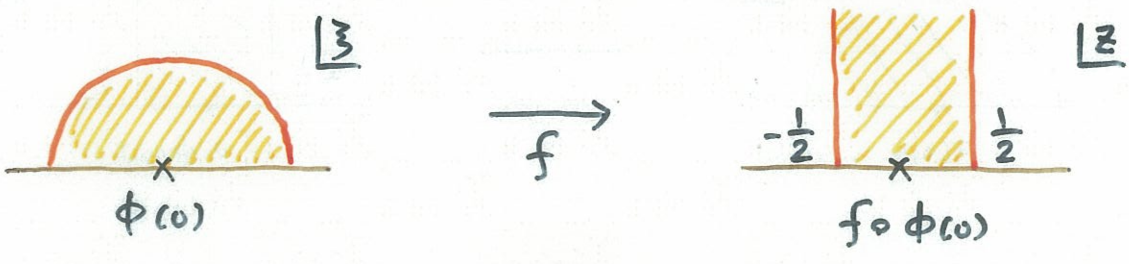
$c_1|0\rangle \longleftrightarrow$



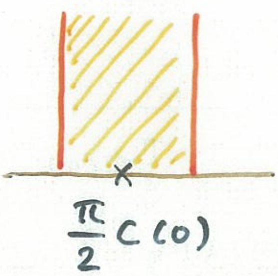
Useful coordinate (sliver frame)

Rastelli & Zwiebach, hep-th/0006240

$$\boxed{z = f(\xi) = \frac{2}{\pi}\arctan \xi}$$



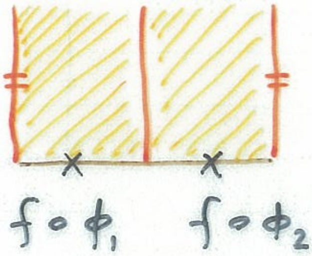
$c_1|0\rangle \longleftrightarrow$



$$\left[\begin{aligned} f\circ c(\xi) &= \left(\frac{df(\xi)}{d\xi}\right)^{-1} c(f(\xi)) \\ f\circ c(0) &= \frac{\pi}{2}\, c(0) \end{aligned}\right]$$

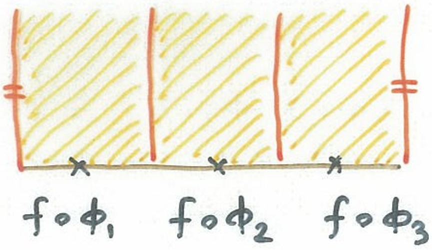
BPZ inner product

$$\langle \phi_1, \phi_2 \rangle =$$



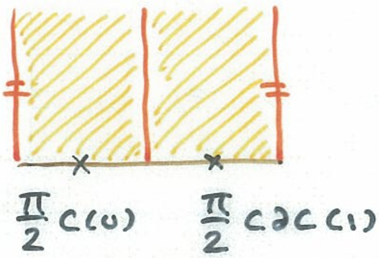
Star product

$$\langle \phi_1, \phi_2 * \phi_3 \rangle =$$



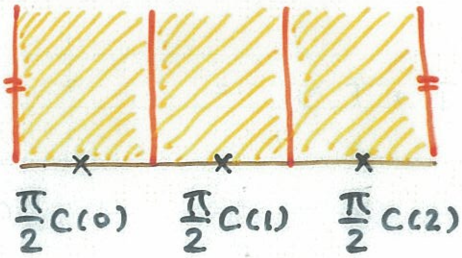
<u>Examples</u> $T = c_1 |0\rangle$

$$\langle T, Q_B T \rangle =$$



$$= -1$$

$$\langle T, T * T \rangle =$$



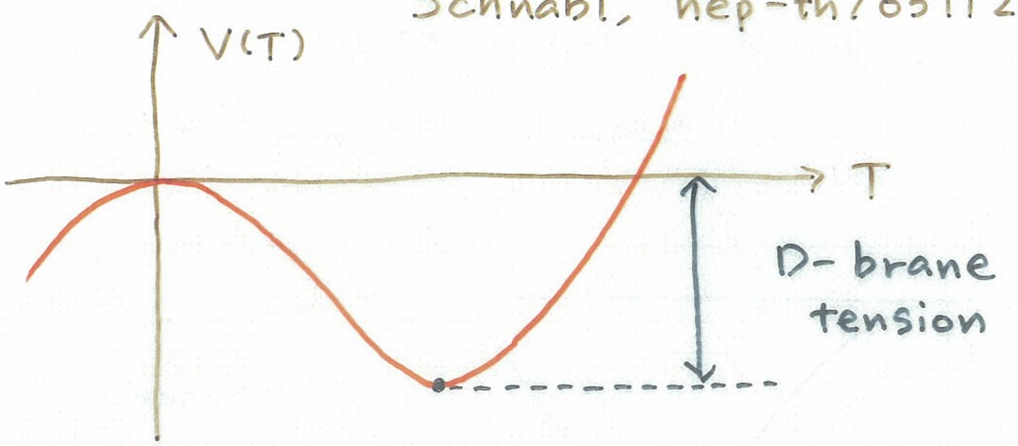
$$= -\frac{81\sqrt{3}}{64}$$

$$\Big[\; \langle c(z_1)\, c(z_2)\, c(z_3) \rangle \text{ on}$$

$$= \left(\frac{n}{\pi}\right)^3 \sin\frac{\pi(z_1 - z_2)}{n} \sin\frac{\pi(z_1 - z_3)}{n} \sin\frac{\pi(z_2 - z_3)}{n} \; \Big]$$

Schnabl's analytic solution for tachyon condensation

Schnabl, hep-th/0511286

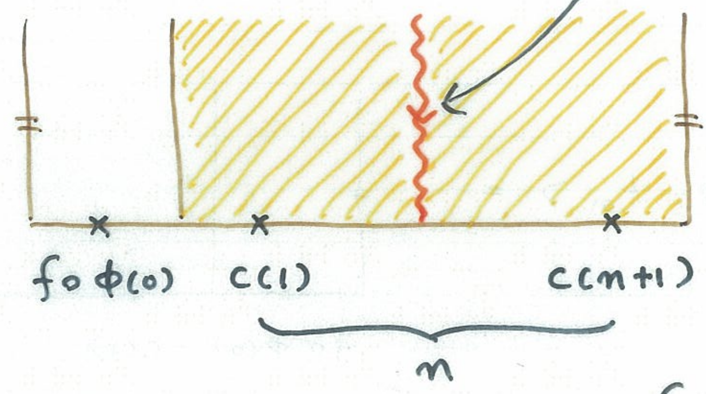


$$\Psi = \lim_{N\to\infty}\left[\sum_{n=0}^{N}\psi_n' - \psi_N\right]$$

$$\psi_n' = \frac{d}{dn}\psi_n$$

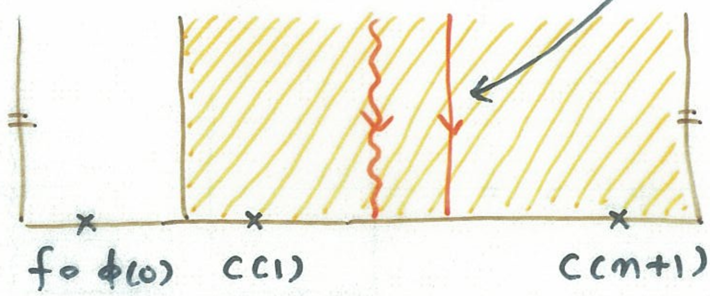
$$\mathcal{B} = \int\frac{dz}{2\pi i}\,b(z)$$

$\langle \phi, \psi_n \rangle =$



$$\mathcal{L} = \int\frac{dz}{2\pi i}\,T(z)$$

$\langle \phi, \psi_n' \rangle =$



$$[\,\mathcal{B}, \mathcal{L}\,] = 0$$

$$(\text{cf. } L_0 e^{-tL_0} = -\partial_t e^{-tL_0})$$

Marginal deformations

For any dimension-one matter primary V, cV is BRST closed.

$$\Psi^{(1)} =$$

$cV(0)$

$$\Longrightarrow \quad Q_B \Psi^{(1)} = 0$$

When the deformation is exactly marginal, we expect a solution of the form

$$\Psi = \sum_{n=1}^{\infty} \lambda^n \Psi^{(n)} \qquad \lambda : \text{deformation parameter}$$

to the nonlinear equation of motion:

$$Q_B \Psi + \Psi * \Psi = 0$$

$$Q_B \Psi^{(1)} = 0$$

$$Q_B \Psi^{(2)} = - \Psi^{(1)} * \Psi^{(1)}$$

$$\vdots$$

$$Q_B \Psi^{(n)} = - \sum_{m=1}^{n-1} \Psi^{(m)} * \Psi^{(n-m)}$$

Formally, $\Psi^{(2)} = - \dfrac{b_0}{L_0} \left[\Psi^{(1)} * \Psi^{(1)} \right]$

$$b_0 = \oint \frac{d\xi}{2\pi i}\, \xi\, b(\xi), \quad L_0 = \oint \frac{d\xi}{2\pi i}\, \xi\, T(\xi)$$

$$\Psi^{(2)} = -\frac{B}{L}[\Psi^{(1)} * \Psi^{(1)}]$$

$$= -\int_0^\infty dT\, B e^{-TL}[\Psi^{(1)} * \Psi^{(1)}]$$

subtle (later)

where

$$B = \oint \frac{dz}{2\pi i} z\, b(z) = \oint \frac{d\xi}{2\pi i} \frac{f(\xi)}{f'(\xi)} b(\xi)$$

$$= b_0 + \frac{2}{3} b_2 - \frac{2}{15} b_4 + \cdots$$

$$L = \{Q_B, B\}$$

$$L(\phi_1 * \phi_2) = L\phi_1 * \phi_2 + \phi_1 * (L - L_L^+)\phi_2$$

$$[L, L_L^+] = L_L^+$$

(L_L^+: left half of $L^+ = L + L^*$)

$$B(\phi_1 * \phi_2) = B\phi_1 * \phi_2 + (-1)^{\phi_1} \phi_1 * (B - B_L^+)\phi_2$$

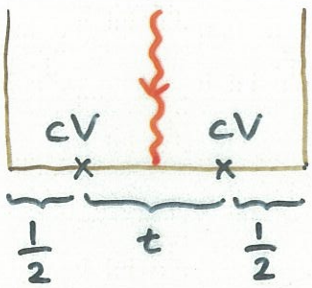
$$[B, L_L^+] = B_L^+$$

(B_L^+: left half of $B^+ = B + B^*$)

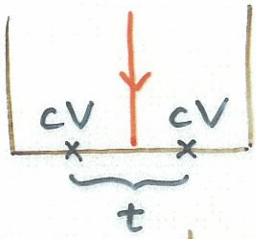
$$L\Psi^{(1)} = 0, \quad B\Psi^{(1)} = 0$$

$$\Psi^{(2)} = -\int_0^1 dt\, \Psi^{(1)} * B_L^+ e^{(1-t)L_L^+} \Psi^{(1)}$$

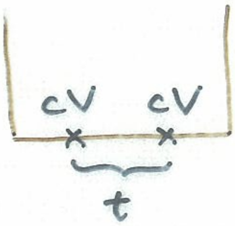
$$\Psi^{(2)} = \int_0^1 dt$$



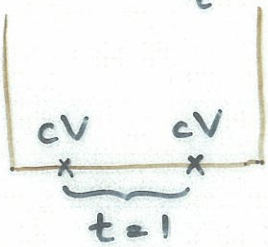
$$Q_B \Psi^{(2)} = -\int_0^1 dt$$



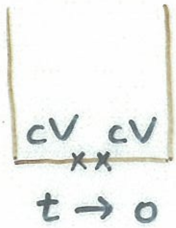
$$= -\int_0^1 dt \frac{\partial}{\partial t}$$



$$= -$$



$$+$$



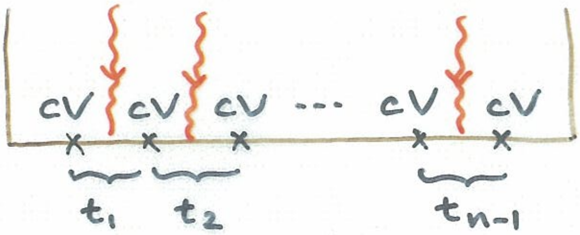
$$\| \quad -\Psi^{(1)} * \Psi^{(1)}$$

$$\lim_{\epsilon \to 0} cV(0)\, cV(\epsilon) = 0 \implies Q_B \Psi^{(2)} = -\Psi^{(1)} * \Psi^{(1)}$$

If $\lim_{\epsilon \to 0} V(0)\, V(\epsilon)$ = finite or vanishing,

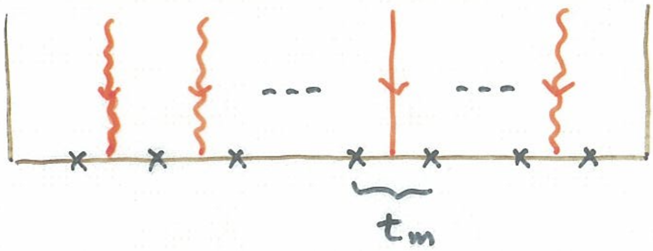
$\Psi^{(2)}$ is finite and $Q_B \Psi^{(2)} = -\Psi^{(1)} * \Psi^{(1)}$.

$$\Psi^{(n)} = \int d^{n-1}t$$

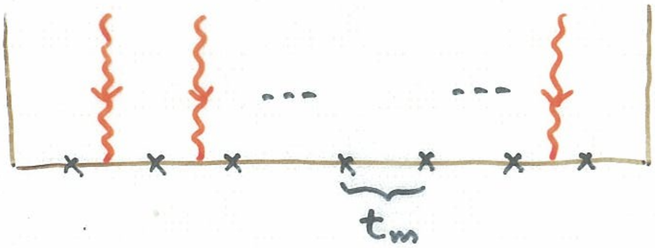


where $\int d^{n-1}t = \int_0^1 dt_1 \int_0^1 dt_2 \cdots \int_0^1 dt_{n-1}$

$$Q_B \Psi^{(n)} = -\sum_m \int d^{n-1}t$$

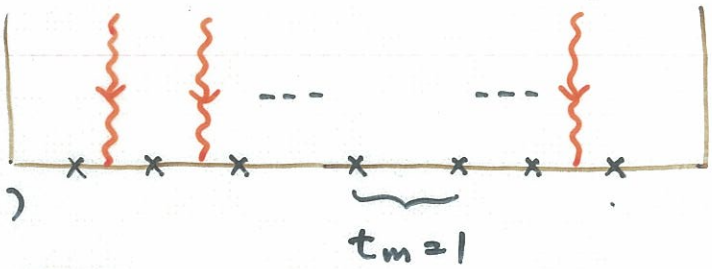


$$= -\sum_m \int d^{n-1}t \frac{\partial}{\partial t_m}$$



$$= -\sum_m \int d^{n-2}t$$

(no dt_m)



$$= -\sum_m \Psi^{(m)} * \Psi^{(n-m)}$$

Example

$$V(t) = \exp\left[\frac{1}{\sqrt{\alpha'}} X^0(t)\right]$$

rolling tachyon

half S-brane decay

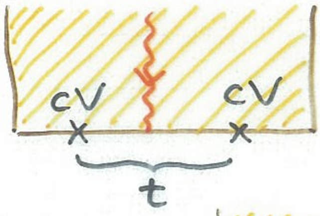
An exact time-dependent solution

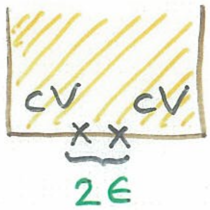
incorporating all α' corrections.

Singular operator product

$$V(z)\,V(w) \sim \frac{1}{(z-w)^2}$$

— Regularization

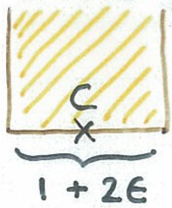
$$\Psi_0^{(2)} = \int_{2\epsilon}^{1} dt$$ 

$$Q_B \Psi_0^{(2)} = -\Psi^{(1)} * \Psi^{(1)} +$$ 

$$cV(-\epsilon)\, cV(\epsilon) = \frac{1}{2\epsilon}\, c\partial c(0) + O(\epsilon)$$

and $c\partial c(0) = Q_B \cdot c(0)$ BRST exact

— Counterterm to satisfy the equation of motion

$$\Psi_1^{(2)} = -\frac{1}{2\epsilon}$$ 

$$\lim_{\epsilon\to 0} \left[Q_B(\Psi_0^{(2)} + \Psi_1^{(2)}) + \Psi^{(1)} * \Psi^{(1)} \right] = 0$$

— Another counterterm to make the solution finite

$$\Psi_2^{(2)} = \ln(2\epsilon)\, \psi_0' \qquad (Q_B \Psi_2^{(2)} = 0)$$

$\Psi^{(2)} \equiv \lim_{\epsilon\to 0} (\Psi_0^{(2)} + \Psi_1^{(2)} + \Psi_2^{(2)})$ is finite

and $Q_B \Psi^{(2)} + \Psi^{(1)} * \Psi^{(1)} = 0.$

— If $V(z)\,V(w) \sim \dfrac{1}{(z-w)^2} + \dfrac{1}{z-w}\,U(w)$,

$\uparrow$ dimension-one primary

$Q_B \Psi_0^{(2)}$ contains 

$\sim O(1)$.

This is not BRST exact $\Longrightarrow$ obstruction

In fact, in this case the deformation is not exactly marginal.

Recknagel & Schomerus, hep-th/9811237

— We have also constructed $\Psi^{(3)}$ for $V(t) = i\sqrt{\dfrac{2}{\alpha'}}\,\partial X(t)$.

A nontrivial condition for the triple operator product $V(t_1)\,V(t_2)\,V(t_3)$ is satisfied.

Future directions

Many potential applications

- rolling tachyon
- lower-dimensional D-branes
- background independence
- worldsheet RG flow and string field theory

  ⋮

Generalization to open superstring field theory