Gauge Theories, D-Branes and Strings

Igor Klebanov

Department of Physics

Princeton University

Talk at the Symposium in Honor of T. Yoneya
University of Tokyo, Komaba
February 10, 2007

Introduction

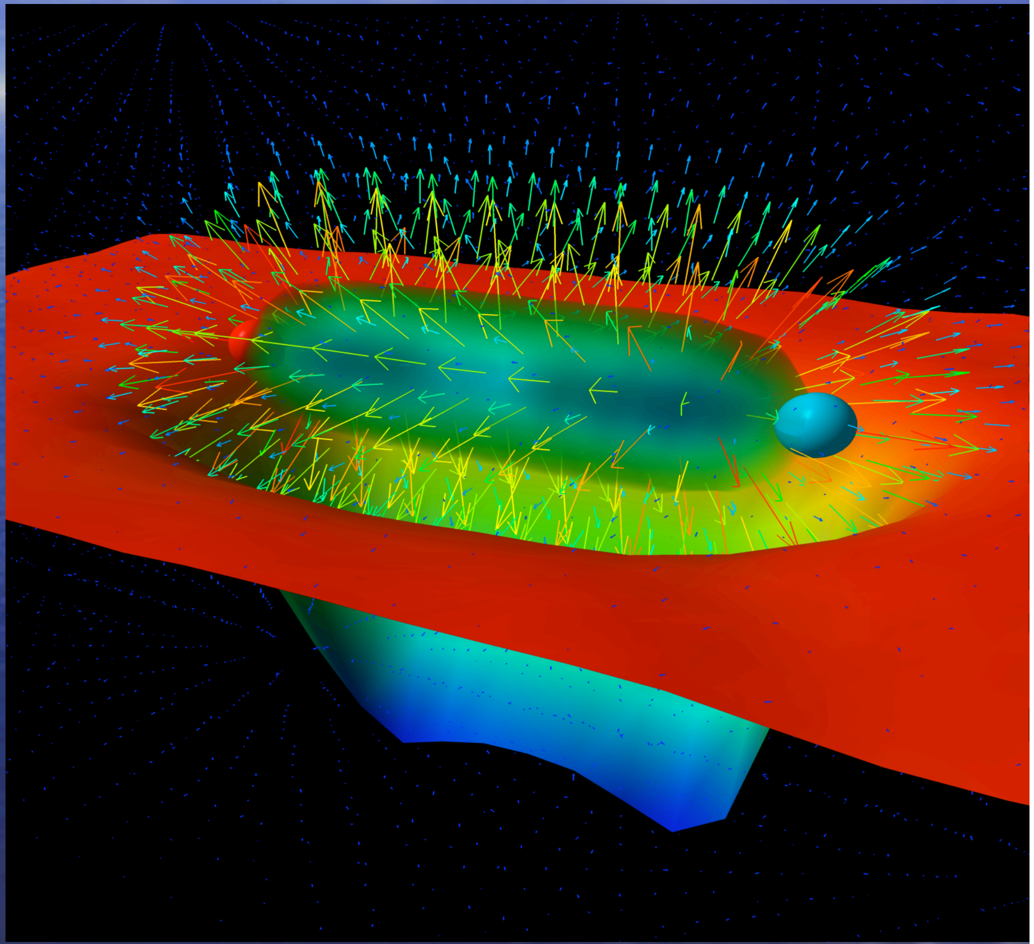
- String theory was invented to describe strong interactions, but in 1973-74 two important developments changed its course.
- Due to discovery of the Asymptotic Freedom, QCD emerged as the field theory for the strong interactions.

  Gross & Wilczek, Politzer 

- Yoneya, and Scherk and Schwarz showed that closed string theory describes quantum gravity.
- Some 25 years later these two seemingly different faces of string theory, the gauge theoretic and the gravitational, have merged in the context of the AdS/CFT correspondence and its extensions. In this talk I review both the basic concepts and some of the recent progress in this field.

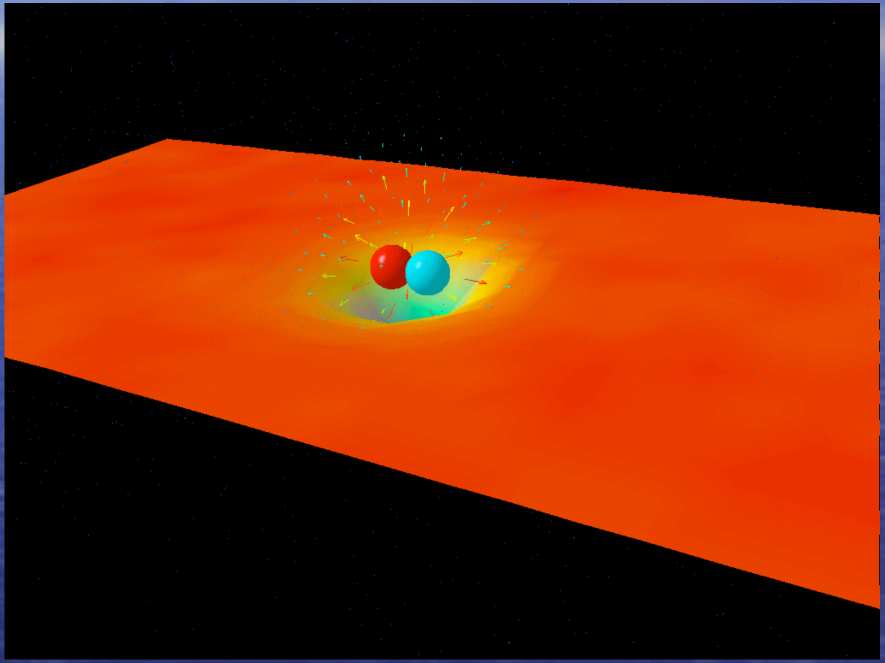
QCD and String Theory

- At short distances, must smaller than 1 fermi, the quark-antiquark potential is approximately Coulombic, due to the Asymptotic Freedom.
- At large distances the potential should be linear (Wilson) due to formation of confining flux tubes.



Flux Tubes in QCD

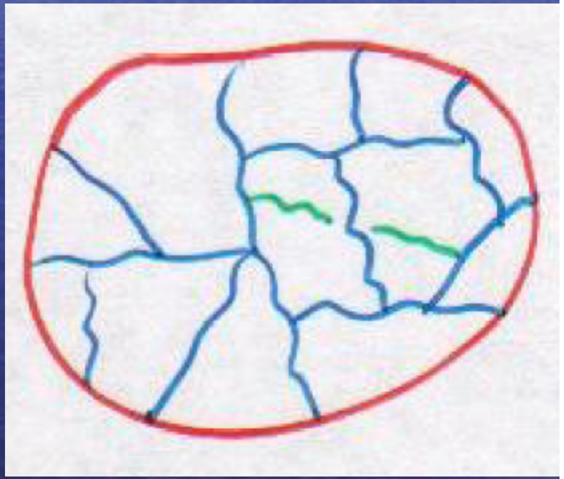
- When these objects are a lot longer than their diameter (which is around a fermi), their dynamics is approximately described by the Nambu-Goto area action. So, strings have been observed, at least in numerical simulations of gauge theory (animation from lattice work by D. Leinweber et al, Univ. of Adelaide)



- While the success of this `effective string' description is very nice, one could ask a deeper, more ambitious question:
- Are there examples of EXACT duality between gauge theory and string theory? (In QCD it would not be limited to a long string approximation, and would incorporate ALL the features of gauge theory including the nearly Coulombic short distance potential).

Large N Gauge Theories

- Connection of gauge theory with string theory is strengthened in `t Hooft's generalization from 3 colors (SU(3) gauge group) to N colors (SU(N) gauge group).
- Make N large, while keeping the `t Hooft coupling fixed: $\lambda = g_{\rm YM}^2 N$
- The probability of snapping a flux tube by quark-antiquark creation (meson decay) is 1/N. The string coupling is 1/N.
- In the large N limit only planar diagrams contribute, but 4-d gauge theory is still very difficult.



Breaking the Ice

- Dirichlet branes (Polchinski) led string theory back to gauge theory in the mid-90's.
- A stack of N Dirichlet 3-branes realizes N=4 supersymmetric SU(N) gauge theory in 4 dimensions. It also creates a curved background of 10-d theory of closed superstrings (artwork by E.Imeroni)

$$ds^2 = \left(1 + \frac{L^4}{r^4}\right)^{-1/2} \left(-(dx^0)^2 + (dx^i)^2\right) + \left(1 + \frac{L^4}{r^4}\right)^{1/2} \left(dr^2 + r^2 d\Omega_5^2\right)$$

which for small r approaches $AdS_5 \times \mathbf{S}^5$

- Successful matching of graviton absorption by D3-branes, related to 2-point function of stress-energy tensor in the SYM theory, with a gravity calculation in the 3-brane metric (IK; Gubser, IK, Tseytlin) was a precursor of the AdS/CFT correspondence.

Conformal Invariance

- In the N=4 SYM theory there are 6 scalar fields (it is useful to comine them into 3 complex scalars: Z, W, V) and 4 gluinos interacting with the gluons. All the fields are in the adjoint representation of the SU(N) gauge group.
- The Asymptotic Freedom is canceled by the extra fields; the beta function is exactly zero! Hence, the theory is invariant under scale transformations $x^{\mu} \to \lambda x^{\mu}$. It is also invariant under space-time inversions.
- Such a theory is called a Conformal Field Theory (CFT).

The AdS/CFT duality

Maldacena; Gubser, IK, Polyakov; Witten

- Relates conformal gauge theory in 4 dimensions to string theory on 5-d Anti-de Sitter space times a 5-d compact space. For the N=4 SYM theory this compact space is a 5-d sphere.
- When a gauge theory is strongly coupled, the radius of curvature of the dual AdS_5 and of the 5-d compact space becomes large: $$\frac{L^2}{\alpha'} \sim \sqrt{g^2_{\rm YM} N}$$
- String theory in such a weakly curved background can be studied in the effective (super)-gravity approximation, which allows for a host of explicit calculations. Corrections to it proceed in powers of $$\frac{\alpha'}{L^2} \sim \lambda^{-1/2}$$
- Feynman graphs instead develop a weak coupling expansion in powers of λ. At weak coupling the dual string theory becomes difficult.

- Gauge invariant operators in the CFT_4 are in one-to-one correspondence with fields (or extended objects) in AdS_5
- Operator dimension is determined by the mass of the dual field; e.g. for scalar operators GKPW

$$\Delta_\pm = 2 \pm \sqrt{4 + m^2 L^2}$$

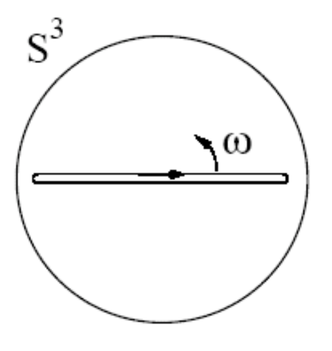
- The BPS protected operators are dual to SUGRA fields of m~1/L. Their dimensions are independent of λ.
- The unprotected operators (Konishi operator is the simplest) are dual to massive string states. AdS/CFT predicts that at strong coupling their dimensions grow as $\lambda^{1/4}$.

Spinning Strings vs. Highly Charged Operators

- Vibrating closed strings with large angular momentum on the 5-sphere are dual to SYM operators with large R-charge (the number of fields Z)
  Berenstein, Maldacena, Nastase
- Generally, semi-classical spinning strings are dual to long operators, e.g. the dual of a high-spin operator

$$\mathrm{Tr}\ F_{+\mu} D_+^{J-2} F_+{}^{\mu}$$

is a folded string spinning around the center of AdS_5. Gubser, IK, Polyakov



Exact Integrability

- Perturbative calculations of anomalous dimensions are mapped to integrable spin chains, suggesting exact integrability of the N=4 SYM theory. Minahan, Zarembo; Beisert, Staudacher
- For example, for the `SU(2) sector' operators Tr (ZZZWZW…ZW) , where Z and W are two complex scalars, the Heisenberg spin chain emerges at 1 loop. Higher loops correct the Hamiltonian but seem to preserve its integrability.
- This meshes nicely with earlier findings of integrability in certain subsectors of QCD. Lipatov; Faddeev, Korchemsky; Braun, Derkachov, Manashov
- The dual string theory approach indicates that in the SYM theory the exact integrability is present at very strong coupling (Bena, Polchinski, Roiban). Hence it is likely to exist for all values of the coupling.

- The structure of dimensions of high-spin operators is $$\Delta - S = f(g)\ln S + O(S^0), \qquad g = \frac{\sqrt{g_{YM}^2 N}}{4\pi}$$
- The function f(g) is independent of the twist; it is universal in the planar limit.
- At strong coupling, the AdS/CFT corresponds predicts via the spinning string energy calculations Gubser, IK, Polyakov; Frolov, Tseytlin

$$f(g) = 4g - \frac{3\ln 2}{\pi} + \ldots$$

- At weak coupling the expansion of the universal function f(g) up to 3 loops is

$$f(g) = 8g^2 - \frac{8}{3}\pi^2 g^4 + \frac{88}{45}\pi^4 g^6 + O(g^8)$$

- The coefficients in f(g) are related to the corresponding coefficients in QCD through selecting at each order the term with the highest transcendentality. Kotikov, Lipatov, Onishchenko, Velizhanin
- Recently, great progress has been achieved on finding f(g) at 4 loops and beyond!
- Using the integrability in AdS/CFT, the `dressing phase' in the magnon S-matrix was determined first in the large g expansion (Beisert, Hernandez, Lopez), and then via appropriate resummation in the small g expansion (Beisert, Eden, Staudacher).
- f(g) is determined through solving an integral equation, which corrects an earlier similar equation derived by Eden and Staudacher

$$f(g) = 16g^2\hat{\sigma}(0)$$

$$\hat{\sigma}(t) = \frac{t}{e^t - 1}\left(K(2gt,0) - 4g^2\int_0^\infty dt' K(2gt,2gt')\hat{\sigma}(t')\right)$$

- Perturbative solution of the BES equation gives the 4-loop term in f(g)

$$-16\left(\frac{73}{630}\pi^6+4\zeta(3)^2\right)g^8$$

(it differs by relative sign from the ES prediction which did not include the `dressing phase')

- Remarkably, an independent 4-loop calculation by Bern, Dixon, Czakon, Kosower and Smirnov yielded a numerical value that prompted them to conjecture exactly the same analytical result.
- This has led the two groups to the same conjecture for the complete structure of the perturbative expansion of f(g): it is the one yielded by the BES integral equation.

- This approach yields analytic predictions for all planar perturbative coefficients

$$
\begin{aligned}
f(g) = 8g^2 - \frac{8}{3}\pi^2 g^4 + \frac{88}{45}\pi^4 g^6 - 16\left(\frac{73}{630}\pi^6 + 4\,\zeta(3)^2\right) g^8 \\
+ 32\left(\frac{887}{14175}\pi^8 + \frac{4}{3}\pi^2\zeta(3)^2 + 40\,\zeta(3)\,\zeta(5)\right) g^{10} \\
- 64\left(\frac{136883}{3742200}\pi^{10} + \frac{8}{15}\pi^4\zeta(3)^2 + \frac{40}{3}\pi^2\zeta(3)\,\zeta(5)\right. \\
\left. + 210\,\zeta(3)\,\zeta(7) + 102\,\zeta(5)^2\right) g^{12} \\
+ 128\left(\frac{7680089}{340540200}\pi^{12} + \frac{47}{189}\pi^6\zeta(3)^2 + \frac{82}{15}\pi^4\zeta(3)\,\zeta(5) + 70\pi^2\zeta(3)\,\zeta(7)\right. \\
\left. + 34\pi^2\zeta(5)^2 + 1176\,\zeta(3)\,\zeta(9) + 1092\,\zeta(5)\,\zeta(7) + 4\,\zeta(3)^4\right) g^{14}
\end{aligned}
$$

- So far the 4-loop answer is only known numerically. Recently, a new method yielded improved numerical precision and agrees with the analytical prediction to around 0.001%.

Cachazo, Spradlin, Volovich

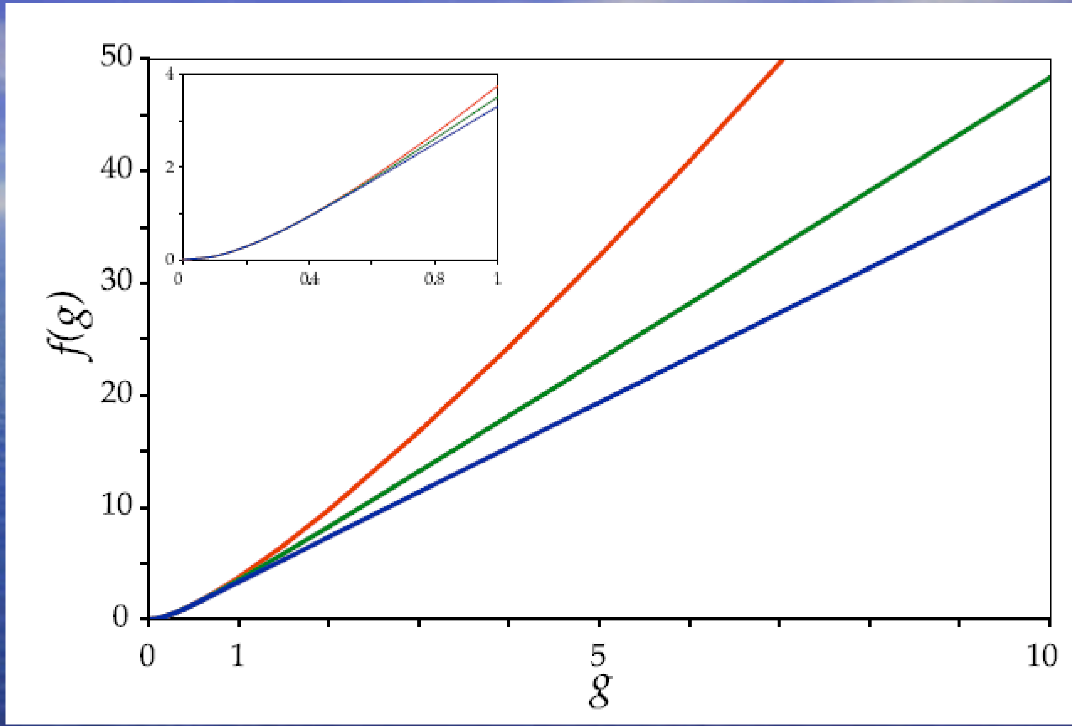
- To solve the equation at finite coupling, we use a basis of linearly independent functions $s(t) = \sum_{n\geq 1} s_n \frac{J_n(2gt)}{2gt}$
- Determination of $f(g) = 8g^2 s_1$ is tantamount to inverting an infinite matrix.

$$s_n \frac{J_n(2gt)}{2gt} = \frac{J_1(2gt)}{2gt} + 8nZ_{2n,1}\frac{J_{2n}(2gt)}{2gt} - 2nZ_{nm}s_m\frac{J_n(2gt)}{2gt}$$
$$-16n(2m-1)Z_{2n,2m-1}Z_{2m-1,r}s_r\frac{J_{2n}(2gt)}{2gt} \; ,$$

$$Z_{mn} = \int_0^\infty \frac{J_m(2gt)J_n(2gt)}{t(e^t-1)}$$

- Truncation to finite matrices converges very rapidly. Benna, Benvenuti, IK, Scardicchio

- The blue line refers to the BES equation, red line to the ES, green line to the equation where the dressing kernel is divided by 2.
- The first two terms of the BES large g asymptotics are in very precise agreement with the AdS/CFT spinning string predictions.



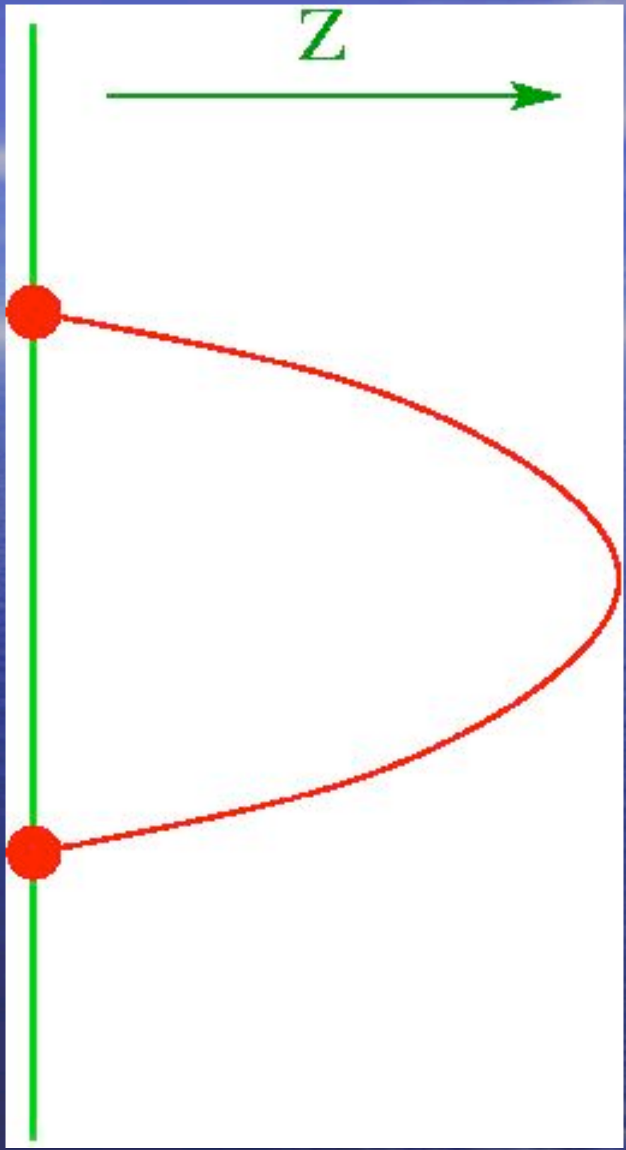
$$f(g) = (4.000000 \pm 0.000001)g - (0.661907 \pm 0.000002)$$

- Recently, the exact analytic solution was obtained for the leading order BES equation. Alday, Arutyunov, Benna, Eden, IK
- Expanding at strong coupling, $$s_k = \frac{1}{g} s_k^{\ell} + \frac{1}{g^2} s_k^{s\ell} + \dots$$

  The solution is $$s_{2n-1}^{\ell} = s_{2n}^{\ell} = \frac{(-1)^{n-1}(2n-1)!!}{2^n (n-1)!}$$

- Since $s_1 = 1/2$, this proves that $f(g) = 4g + O(1)$

- The z-direction of AdS is dual to the energy scale of the gauge theory: small z is the UV; large z is the IR.
- In a pleasant surprise, because of the 5-th dimension z, the string picture applies even to theories that are conformal (not confining!). The quark and anti-quark are placed at the boundary of Anti-de Sitter space (z=0), but the string connecting them bends into the interior (z>0). Due to the scaling symmetry of the AdS space, this gives Coulomb potential (Maldacena; Rey, Yee)

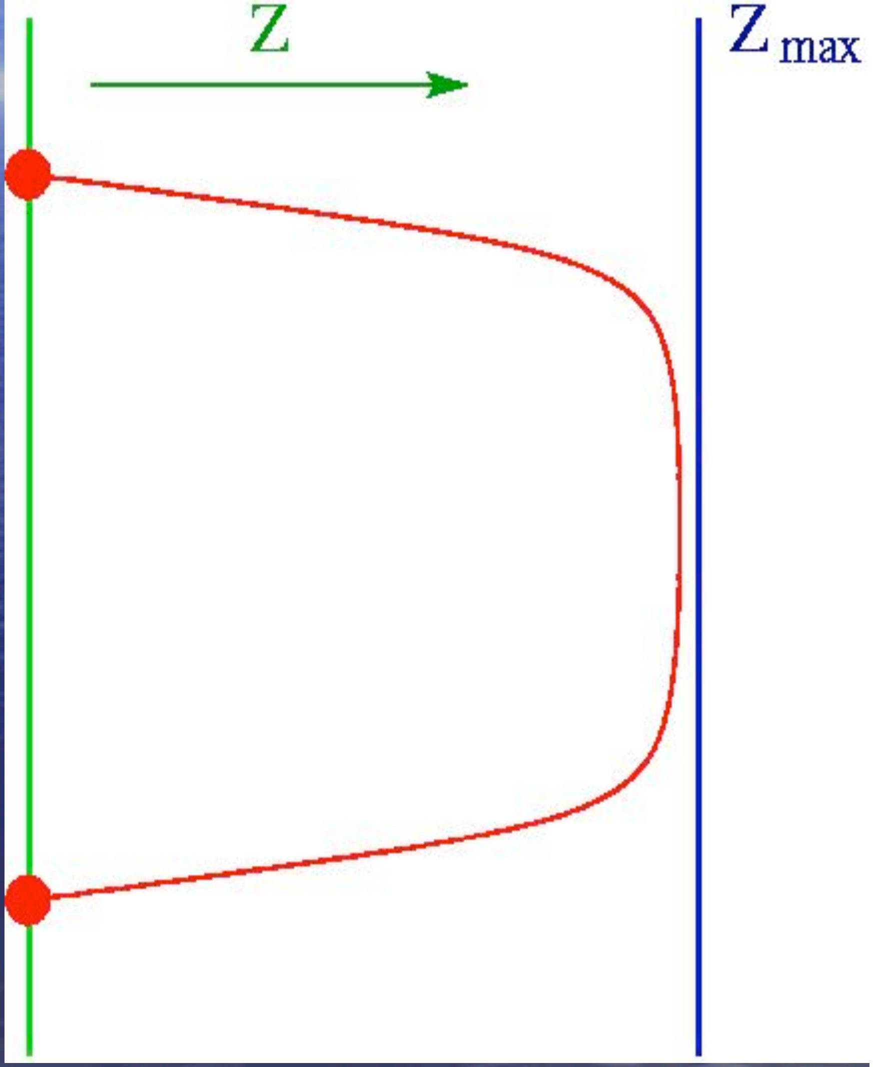


String Theoretic Approaches to Confinement

- It is possible to generalize the AdS/CFT correspondence in such a way that the quark-antiquark potential is linear at large distance.
- The 5-d metric should have a warped form (Polyakov):

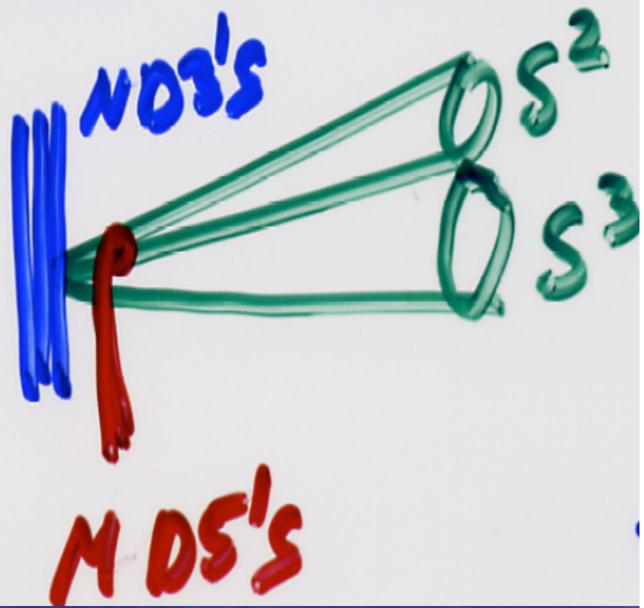
$$ds^2 = \frac{dz^2}{z^2} + a^2(z)\big(-(dx^0)^2 + (dx^i)^2\big)$$

- The space ends at a maximum value of z where the warp factor is finite. Then the confining string tension is $\frac{a^2(z_{\max})}{2\pi\alpha'}$



Confinement in SYM theories

- Introduction of minimal supersymmetry (N=1) facilitates construction of gauge/string dualities.
- A useful tool is to place D3-branes and wrapped D5-branes at the tip of a 6-d cone, e.g. the conifold.
- The 10-d geometry dual to the gauge theory on these branes is the warped deformed conifold (IK, Strassler)



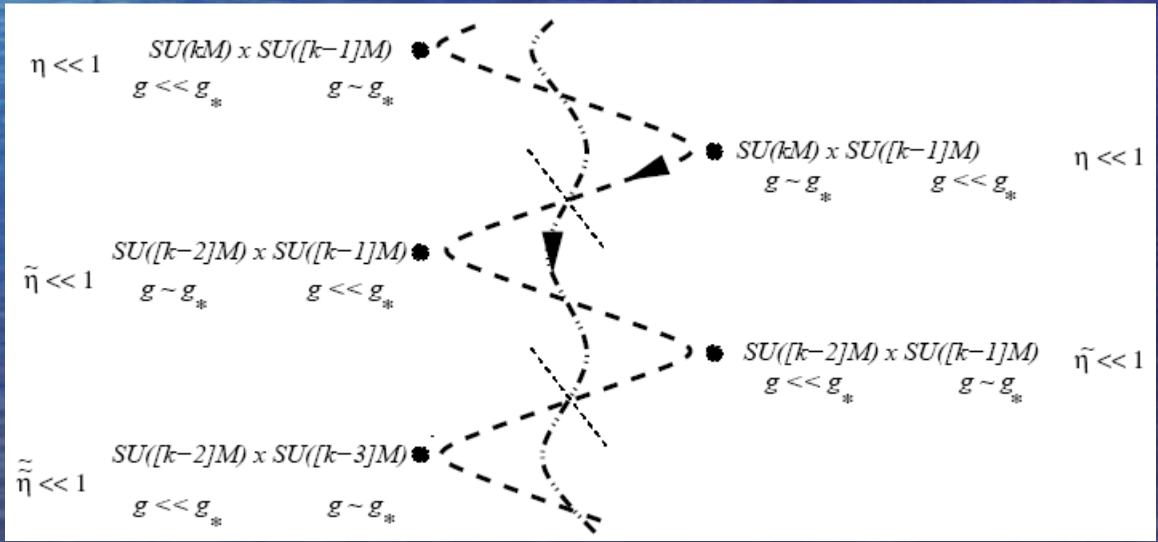
$$ds_{10}^2 = h^{-1/2}(\tau)\big(-(dx^0)^2 + (dx^i)^2\big) + h^{1/2}(\tau)ds_6^2$$

- ds_6^2 is the metric of the deformed conifold, a simple Calabi-Yau space defined by the following constraint on 4 complex variables:

$$\sum_{i=1}^{4} z_i^2 = \varepsilon^2$$

- In the UV the background differs from AdS only through logarithmic effects indicating a logarithmic running of the gauge couplings. Surprisingly, the 5-form flux, dual to N, also changes logarithmically with the RG scale. IK, Tseytlin

- What is the explanation in the dual SU(kM)xSU((k-1)M) SYM theory coupled to bifundamental chiral superfields A_1, A_2, B_1, B_2 ? A novel phenomenon, called a duality cascade, takes place: k repeatedly changes by 1 as a result of the Seiberg duality IK, Strassler
(diagram of RG flows from a review by M. Strassler)



- Dimensional transmutation in the IR. The dynamically generated confinement scale is

$$\sim \varepsilon^{2/3}$$

- The pattern of R-symmetry breaking is the same as in the SU(M) SYM theory: Z_{2M} -> $Z_{2.}$
- In the IR the gauge theory cascades down to SU(2M) x SU(M). The SU(2M) gauge group effectively has $N_f=N_c$.
- The baryon and anti-baryon operators Seiberg

$$\mathcal{A} = \epsilon^{i_1 \ldots i_{N_c}} A^{a_1}_{\alpha_1 i_1} \ldots A^{a_{N_c}}_{\alpha_{N_c} i_{N_c}}$$

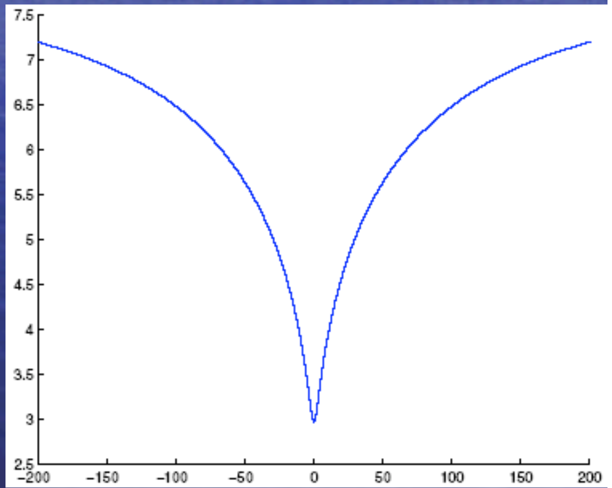
$$\mathcal{B} = \epsilon_{i_1 \ldots i_{N_c}} B^{i_1}_{\dot{\alpha}_1 a_1} \ldots B^{i_{N_c}}_{\dot{\alpha}_{N_c} a_{N_c}}$$

acquire expectation values and break the U(1) symmetry under which A_k -> e^{ia} A_k; B_l -> e^{-ia} B_l. Hence, we observe confinement without a mass gap: due to $U(1)_{baryon}$ chiral symmetry breaking there exist a Goldstone boson and its massless scalar superpartner.

- The KS solution is part of a moduli space of confining SUGRA backgrounds, resolved warped deformed conifolds.
  Gubser, Herzog, IK; Butti, Grana, Minasian, Petrini, Zaffaroni
- This family of solutions is dual to the \`baryonic branch' in the gauge theory:

$$\mathcal{A} = i\Lambda_1^{2M}\zeta\ , \qquad \mathcal{B} = i\Lambda_1^{2M}/\zeta$$

- Using the SUGRA solutions, various quantities have been calculated as a function of the modulus U=ln |ζ|.
- Here is a plot of the string tension: a fundamental string at the bottom of resolved warped deformed conifold is dual to an \`emergent' chromo-electric flux tube.   Dymarsky, IK, Seiberg



- All of this provides us with an exact solution of a class of 4-d large N confining supersymmetric gauge theories.
- This should be a good playground for testing various ideas about strongly coupled gauge theory.
- Some results on glueball spectra are already available, and further calculations are ongoing. Krasnitz; Caceres, Hernandez; Dymarsky, Melnikov; Berg, Haack, Muck

Conclusions

- Throughout its history, string theory has been intertwined with the theory of strong interactions
- The AdS/CFT correspondence makes this connection precise. It makes a multitude of dynamical statements about strongly coupled conformal (non-confining) gauge theories.
- Integrability has become a crucial tool for solving the gauge theory as a function of coupling. The initial worries about `3-loop discrepancies' have recently been replaced by interpolations from weak to strong coupling that successfully test AdS/CFT.
- Its extensions to confining theories provide a new geometrical view of such important phenomena as dimensional transmutation and chiral symmetry breaking. They also exhibit a new type of RG flows: the duality cascades.

- Happy Birthday, Yoneya-san!