

RECONSTRUCTING ADS

ANTAL JELICKI

CONTRIBUTION TO A SYMPOSIUM
CELEBRATING T. YONEYA'S
60th BIRTHDAY

FEB 10, 2007, KOMABA

BACKGROUND:

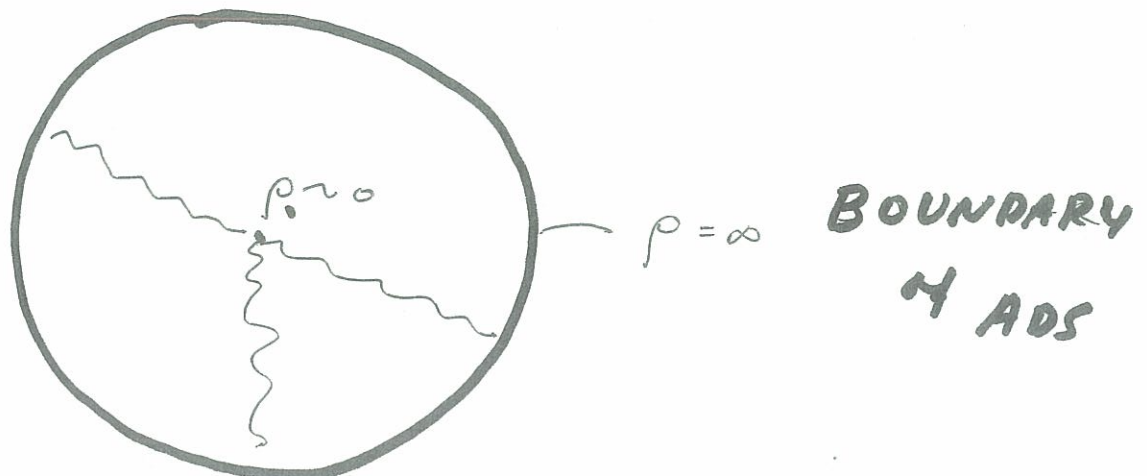
1. MATRIX MODEL MAPS IN ADS/CFT
(with A. DONUS and J. P. RODRIGUES)
2. $1/2$ BPS CORRELATORS AS COI S-MATRIX
(with TAMIAKI YONEYA)
3. RECONSTRUCTING ADS INTERACTIONS
(with SERA CREMONINI AND R. DE MELLO-KOCH)
to appear.

THE QUESTION: RECONSTRUCTING INTERACTIONS IN ADS SPACETIME

- SUPERGRAVITY $\leftarrow \leftarrow$
- STRING THEORY

EARLY INSIGHT FROM THE CLASSIC
PAPER BY GABER-KLEBANOV-POLYAKOV
WITTEN (GKPW)

HOLOGRAPHIC RELATION CONNECTING
YANG-MILLS CORRELATORS WITH
BOUNDARY AMPLITUDES IN SUPRA



IN PARTICULAR : 3-POINT CORRELATORS

$$\langle O_{\Delta_1}(x_1) O_{\Delta_2}(x_2) O_{\Delta_3}(x_3) \rangle = \frac{C(1,2,3)}{|x_1-x_2|^{\alpha_3} |x_2-x_3|^{\alpha_1} |x_3-x_1|^{\alpha_2}}$$

$C(1,2,3)$ - RELATES TO CUBIC INTERACTIONS

• QUESTIONS OF HOLOGRAPHY

BERENSTEIN - MALDACENA - NASTASE limit

$J \rightarrow$ LARGE R - CHARGE

• RESOLVED BY YONENAGA AND

COLLABORATORS

STUDENTS

S. DOBASHI

H. SHIMADA

M. ASANO, Y. SEKINO

A. MIWA

• EUCLIDEAN CONTINUATION

TUNELLING PROCESS!

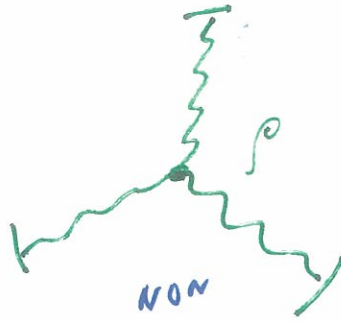
RETURNING TO ADS: THE HOLOGRAPHIC FORMULA OF GKP-W

ALWAYS APPEARED ANALOGOUS TO

- S-MATRIX RELATION
- INVOLVES A CORRESPONDENCE 5 - 4d DATA

BUT : PROJECTION

BULK - BOUNDARY

$$\int d\rho \sqrt{g} \quad \overset{\text{NON}}{\Psi_1(\rho, x_1)} \overset{\text{NON}}{\Psi_2(\rho, x_2)} \overset{\text{NON}}{\Psi_3(\rho, x_3)}$$


NON NORMALIZABLE WAVEFUNCTIONS.

IN PRACTICE : PROBLEM IN OBTAINING
FLAT S-MATRIX FROM THE Y-D
CORRELATORS

H. NASTASE, A.J.

GIDDINGS, POLCHINSKI (?)

RECENTLY SOME LIGHT WAS SHED ON
THIS QUESTION: $\frac{1}{2}$ BPS

- CORRELATORS of CHIRAL PRIMARY OPERATORS

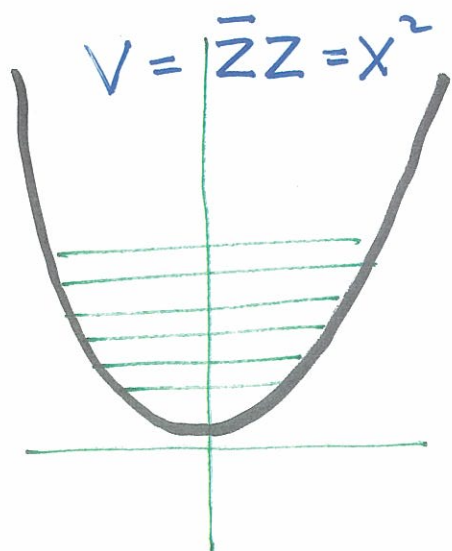
$$Z(x) = \phi_1 + i \phi_2(x)$$

$$\langle O_{J_1} O_{J_2} O_{J_3} \rangle \quad O_J = \text{Tr}(Z^J)$$

GIVEN BY A MATRIX MODEL $\dot{\bar{Z}}Z + \bar{Z}\dot{Z}$

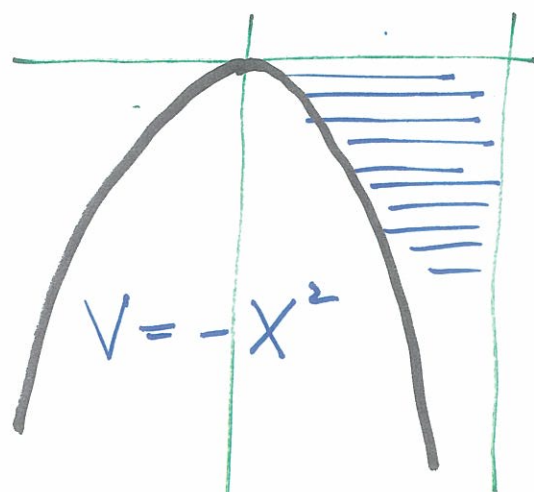
- AGREE WITH S-MATRIX of $C=1$

$D=2$ NONCRITICAL STRINGS



EUCLIDEAN
TIME
→
YONEDA
et al.

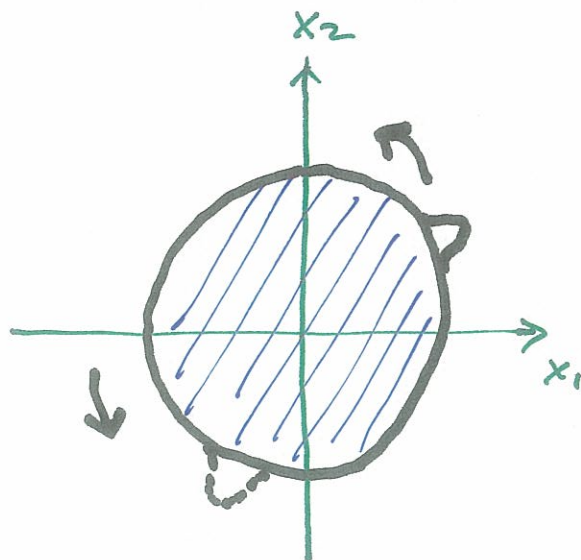
OKUYAMA



$C=1$ SCATTERING
YONEDA + A.J.

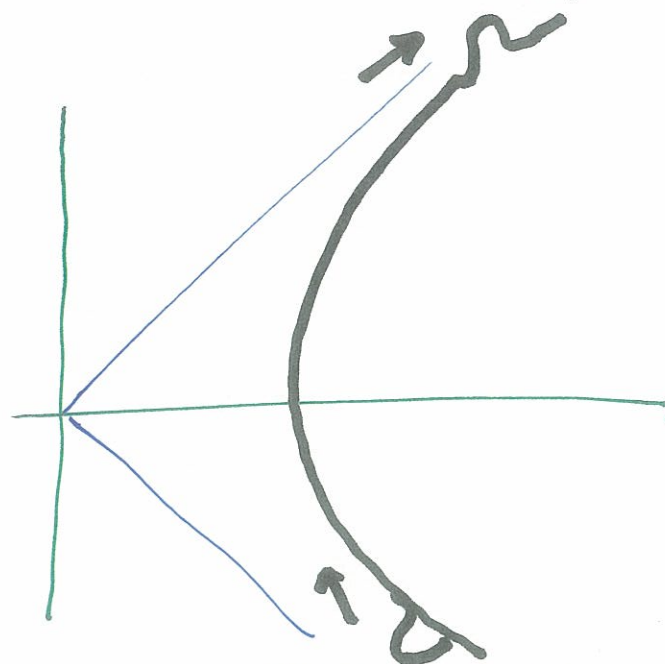
T-S. TAI

THE PHASE SPACE (FERMI DROPLET) PICTURE:



FLUCTUATIONS : ROTATIONS

$$x_1 + ix_2 = r e^{i\phi} \quad \phi \rightarrow \phi - t$$



EUCLIDEAN
TIME

• THE MATRIX MODEL INTEGRAL

$$C(j, h, j') = \int d\bar{Z} dZ \text{Tr} Z^{j_1} \text{Tr} Z^{j_2} \dots e^{-\text{Tr} \bar{Z} Z}$$

CORRESPONDS IN THE C=1 TRANSLATION

$$\langle 0 | \text{Tr} (P-X)^{i k_1'} \dots \text{Tr} (P+X)^{i k_2} \text{Tr} (P+X)^{i k_1} | 0 \rangle$$

$$\langle \text{out}; k_1', k_2' \dots \mid k_1, k_2, \dots; \text{in} \rangle$$

$$= S_{(k)(k')} \quad S\text{-MATRIX}$$

FLAT S-MATRIX(?)

- THE QUESTION (OF RECONSTRUCTING
ADS GRAVITY FROM YM
CORRELATORS)

- CONTAINS SOME OF THE (NONTRIVIAL,
ISSUES INVOLVED IN AN OFF
SHELL EXTENSION / ? / ? /

- A DIRECT WAY TO APPROACH IT
IS TO WORK IN THE HAMILTONIAN
LANGUAGE AND ESTABLISH THE
RELATIONSHIP AT THAT LEVEL.

- ALSO DIFFICULT : MULTIMATRIX
LOOP SPACE
BUT ..

- RECENTLY IT WAS DONE (IN THE LIMITED) SECTOR of $\frac{1}{2}$ BPS

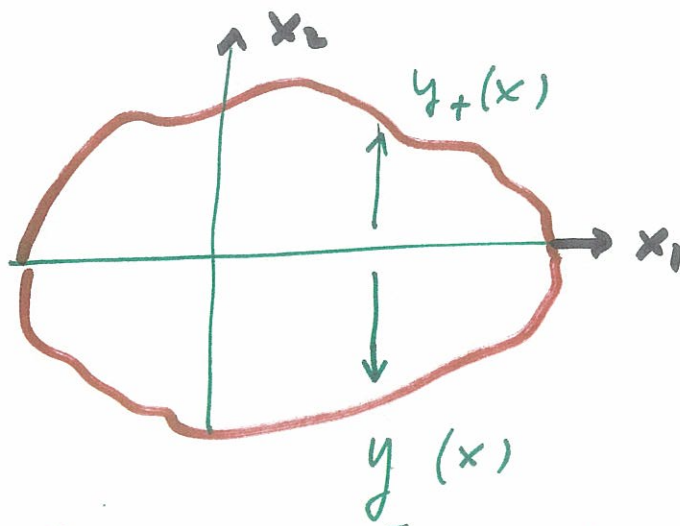
YANG-MILLS:
(MATRIX MODEL)
HAMILTONIAN
S. CORLEY, A.J., RANGSOLAN

REDUCTION
 $M=100$
SUGRA
LLM

LIN, LUNIN AND MALDACENA:

$$E = \int_D dx_1 \int dx_2 (x_1^2 + x_2^2) U(x_1, x_2)$$

FERMION
DROPLET



COLL. HAMILT.

$$H = \int dx \left(\frac{y_+^3}{3} - \frac{y_-^3}{3} + x^2 (y_+ - y_-) \right)$$

OF A 1-MATRIX MODEL

$$\left. \begin{array}{l} x_1 = x \rightarrow X \\ x_2 = y \rightarrow P \end{array} \right\} H = \frac{1}{2} \text{Tr} (P^2 + X^2)$$

+ S. YAMAGUCHI / For Wilson Loops

● STRATEGY (with S. CREMONINI + R. De Mello) ⁽⁹⁾

WE HAVE A HAMILTONIAN RELATIONSHIP
FOR $1/2$ BPS STATES

IN SU_4RA $ADS_5 \times S^5$

$$ds^2 = (-ch^2 p dt^2 + dp^2 + sh^2 p dn^2) + (\sin^2 \theta d\theta^2 + d\phi^2)$$

THIS CORRESPONDS TO SPECIAL FLUCTUATIONS
OF CHIRAL PRIMARY FIELDS

$$S_g \sim \left[\frac{\cos \theta}{ch p} e^{i\phi} \right]^j \quad a$$

Highest weight

USE THAT AS THE STARTING
POINT TO GENERATE THE FULL
THEORY.

1. RECONSTRUCT FIRST

$$\psi_{jnm}(t, \rho, \theta, \phi) \sim \text{NONTRIVIAL } \rho, \theta \text{ DEP.}$$

$$\sim L_+^n I_-^{j-m} \psi_j\left(\frac{\cos\theta}{\sin\rho} e^{i\phi}\right)$$

- DONE IN DONOS, A.J. + RODRIGUES
hep-th / 0507124

2. FROM INTERACTION +

CPU'S DEVELOP ANALOGOUS
RAISING - LOWERING WARD ID.

$$V_{j_1 j_2 j_3} \rightarrow V_{j_1 n_1 j_2 n_2 j_3 n_3}$$

BUT. THERE ARE PROBLEMS
IN THIS LOGIC

• STRUCTURE of HIGHEST WEIGHT INTERACTION (in SUSRA)

$$V_{j_1 j_2 j_3}^{h.w} \sim (\underbrace{\epsilon_3 - \epsilon_2 - \epsilon_1}_{\text{Prefactor}}) \underbrace{C(j_1 j_2 j_3)}_{\text{From Correlator}}$$

For $\frac{1}{2}$ BPS $C(j_1 j_2 j_3) \propto \delta_{j_1 + j_2, j_3}$
 $\rightarrow$ R-charge conserv.
 $\rightarrow \epsilon = j$ as BPS

CONSEQUENTLY $V^{h.w} = 0$

CANNOT BE A STARTING POINT FOR AN ITERATIVE SCHEME!

ON THE OTHER HAND

(12)

IF WE LOOK AT THE INTERACTION
OF $1/2$ BPS STATES IN
COLLECTIVE FIELD THEORY / MATRIX MODEL

$$V^{\text{col}} = \int d\tau \frac{1}{\cos^2 \frac{\tau}{2}} \alpha(\tau)^3$$

$$d\tau = \frac{dx}{\sqrt{1-x^2}}$$

$$\alpha(\tau) = \sum e^{ik} \alpha_k$$

CHIRAL BOSON

$$V = \sum_{k_1, k_2, k_3 \leq 0} f(k_1 + k_2 + k_3) \alpha_{k_1} \alpha_{k_2} \alpha_{k_3}$$

$$f(k) \sim \frac{k}{\sin \pi k} \rightarrow \left(\frac{1}{\pi} \right) \quad k = j_3 - j_1 - j_2 = 1$$

DOES NOT VANISH!

IN SEEMING DISAGREEMENT
WITH A DIRECT HAMILTONIAN
COMPARISON WITH SYMRA.

THE RESOLUTION OF THIS LITTLE
'PARADOX' IS SIMPLE:

CONSIDER ADDING THE FOLLOWING
SURFACE TERM TO THE LAGRANGIAN

$$\Delta S = \int d\tau \partial_\tau \left(-t g_\tau \alpha(\tau)^3 \right) \quad \left| \begin{array}{l} \text{YONEYA} \\ + \\ \text{WADIA} \\ \text{76'} \\ \text{IN Y.M} \end{array} \right.$$

THEN

$$\tilde{V}_{\omega_L} = \int d\tau t g_\tau \partial_\tau \left(\alpha(\tau)^3 \right)$$

$$\rightarrow (k_3 - k_1 - k_2) \cdot 2\pi \alpha(-k_3) \alpha(k_1) \alpha(k_2)$$

RIGHT PREFACTOR ✓

HOLOGRAPHY FRIENDLY ✓

WANTIES ON SHELL

SUGRA

MATRIX / COLL

$$a = \left(\frac{\cos\theta}{\text{Chp}} e^{i\phi} \right) \longleftrightarrow d\tau = \frac{dx}{\phi_0(x)}$$

TRANSFORM

THERE IS A MAP

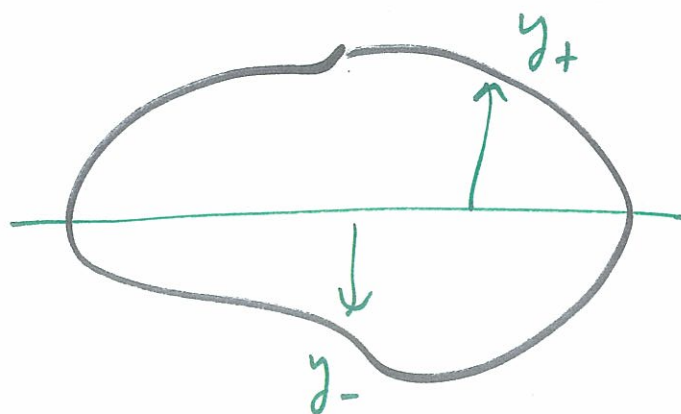
MATRIX MODEL MAP:

- START WITH THE FORMULA of LLM

$$U(x_1, x_2, y) = \frac{y^2}{\pi} \int_D dx_1' dx_2' \frac{U(x_1', x_2')}{[x_1'^2 + x_2'^2 + y^2]}$$

D - DOMAIN OF NONZERO $U + \frac{1}{2}$
FERMI DROPLET
(ON THE RIGHT)

$U(x_1, x_2, y)$
SPECIFIES THE
METRIC..
of SUGRA



NONLINEAR MAP:

$$\int dx_1 \int dx_2 \begin{matrix} y_+(x, t) \\ y_-(x, t) \end{matrix}$$

$$\begin{aligned} y_+ - y_- &= \phi(x) \\ &= \text{Tr } \delta(x - M) \end{aligned}$$

LINEARIZE BOTH SIDES
ABOUT THE ADS - GROUND STATE

⇒ LINEAR MAP (BETWEEN SMALL FLUCTUATIONS)

(15)

$$\delta g(t, \rho, \theta, \phi) = \int d\tau K(a; \tau) \underline{\underline{d(\tau)}}$$

↑
SUGRA

$$a = \frac{\cos \theta}{\partial \rho} e^{i(\phi - t)}$$

↑
MATRIX
 $\text{Tr} \delta(x - X)$

⇒ EXTENSION (WITH DONOS, RODRIGUES)

$$\begin{matrix} X \\ Y \\ \vdots \end{matrix} \left\{ \begin{array}{l} \text{TWO MATRICES} \\ \rightarrow B + B^+ \end{array} \right.$$

$$\tilde{\Psi} = \text{Tr} (\delta(x - X) B^s)$$

DEVELOPED A LINEARIZED (EIGEN)

EQUATION :

$$K \Psi = \omega \Psi$$

$$\left\{ \begin{array}{l} \omega_{jn} = j + 2\pi \\ \Psi_{jn} = \sin(\omega_{jn} \tau) e^{in\phi} \end{array} \right.$$

SOLUTION

- These wavefunctions are governed by the SL_2 raising ops

$$\tilde{L}_0 = \hat{H} = \left| \frac{\partial}{\partial \tau} \right| \quad \tilde{L}_{\pm} = \sqrt{\left| \frac{\partial}{\partial \alpha} \right|} \sqrt{\left| \frac{\partial}{\partial \beta} \right|} e^{i(\alpha \pm \beta)}$$

ACTION-ANGLE
VARIABLES FOR $h = a^\dagger a + b^\dagger b$.

- MAP TO SUGRA WAVEFUNCTIONS

$$\psi_{in}(p, \phi) = \underbrace{\int d\tau \int d\sigma}_{2 \text{ dim.}} K_2(p, \phi; \tau, \sigma) \underbrace{\begin{bmatrix} e^{i\omega\tau} & e^{i\phi\sigma} \end{bmatrix}}_{\tilde{\psi}_{in}}$$

$\Rightarrow$ IN THE SPECIAL CASE

$$n=0 \quad \int d\sigma K_2 = K_{LLn}(pe^{i\phi}; \tau)$$

REDUCES TO THE LLN KERNEL. $\uparrow$
ONE MATRIX

THE DUAL SUGRA - WAVEFUNCTIONS
OBEY A TRUNCATED ADS

EQUATION : ONLY TWO COORDINATES

$$\left[\omega_{jn}^2 + \underbrace{\partial_\rho^2 + \frac{2}{\sin^2 \rho} \partial_\rho + \frac{1}{\sin^2 \rho} \partial_\phi^2}_{\square} \right] \psi_{jn}$$

BY DOUBLE EUCLIDEAN CONTINUATION

THE TWO DIMENSIONAL LAPLACIAN

IS SEEN TO BE IDENTICAL TO

THE 2D BLACK HOLE Witten
Wadia et al

MAP :

$$\psi(\rho, \phi) = \int d\tau \int d\tilde{\phi} \left[\sin \rho \sin(\tilde{\phi} - \phi) - \cos \tau \right] \tilde{\psi}_{jn}(\tau, \tilde{\phi})$$

A NONLOCAL MAP : T. YONEYA + A.J.
DAS, DHAR, WADIA (1992)

⇒ NONLINEAR EXTENSION

- TO ADDRESS THE ISSUE OF (CUBIC) INTERACTION WE HAVE TO EXTEND THE MATILIX MODEL MAP FROM LINEAR TO NONLINEAR CASE
- IT IS HELPFULL TO UNDERSTAND ITS ORIGIN THROU CANONICAL TR.

$$\begin{pmatrix} P_1 & X_1 \\ P_2 & X_2 \end{pmatrix}$$

 $\Leftrightarrow$

$$\begin{aligned} X &= \frac{X_1 + P_2}{\sqrt{2}} & P_X \\ Y &= \frac{X_1 - P_2}{\sqrt{2}} & P_Y \end{aligned}$$

$$Z = r e^{i\phi} = X_1 + iX_2$$

THE LINEAR MAP

$$\tilde{\Psi}(x, y) = \int dx_1 \int dx_2 \underbrace{\delta\left[x_1 - \frac{x+y}{\sqrt{2}}\right]}_{K_2(x, y; r e^{i\phi})} e^{i x_2 \frac{x-y}{\sqrt{2}}} \Psi(x_1, x_2)$$

RELATES

$$\tilde{\Psi}_{jn} = H_{j+n}(x) H_n(y) \Leftrightarrow e^{i j \phi} L_n^j(r) = \Psi_{jn}$$

THE ASSOCIATED OVERLAP

VERTICES ARE DIFFERENT:

$$V_{j_1 n_1 j_2 n_2 j_3 n_3} = \int d(r^2) \int d\phi \mu(r) \psi_{j_1 n_1}(r) \psi_{j_2 n_2}(r, \phi) \psi_{j_3 n_3}(r, \phi)$$

IS CONSERVING $\propto \delta_{j_3, j_1 + j_2}$

ASSOCIATED WITH ϕ -INTEGRAL

$$\tilde{V}_{j_1 n_1 j_2 n_2 j_3 n_3} = \int dx \int dy \mu(x, y) \tilde{\psi}_{j_1 n_1}(x, y) \tilde{\psi}_{j_2 n_2}(x, y) \tilde{\psi}_{j_3 n_3}(x, y)$$

IS NOT CONSERVING

R-CHARGE ACTS AS $\tilde{J} = \frac{1}{2}(P_x^2 + X^2) - \frac{1}{2}(p_y^2 + Y^2)$

DYNAMICAL SYMMETRY

NOT AS A COORDINATE SYMMETRY

THE HAMILTONIANS IN THE TWO PICTURES

$$H = \omega_{jn} A_{jn}^+ A_{jn} + (\epsilon_3 - \epsilon_2 - \epsilon_1) V_{123} A_1^+ A_2 A_3$$

$$\tilde{H} = \omega_{jn} \tilde{A}_{jn}^+ A_{jn} + (\epsilon_3 - \epsilon_1 - \epsilon_2) \tilde{V} \tilde{A} \tilde{A} \tilde{A}$$

BUT, WE CAN PROOVE A THEOREM:

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THERE EXISTS A NONLINEAR FIELD
REDEFINITION

$$\begin{aligned}\tilde{A}_{jn} = & A_{jn} + c_{jn, j_1 n_1 j_2 n_2} A_{j_1 n_1} A_{j_2 n_2} \\ & + d_{jn, j_1 n_1 j_2 n_2} A_{j_1 n_1}^+ A_{j_2 n_2} \\ & + f_{jn, j_1 n_1 j_2 n_2} A_{j_1 n_1}^+ A_{j_2 n_2}^+\end{aligned}$$

SUCH THAT AT CUBIC
LEVEL

$$\underline{\underline{H = \tilde{H}}}$$

- BRUTE FORCE CONTRACTION.
- EMERGENCE OF A SCHEME: CANONICAL TRANSFORMATIONS IN (p, q) SPACE AND FIELD TRANSF (LINEAR + NONLIN).
- MORE ELEGANT CONST.

WARD IDENTITIES

(21)

NOW WE COME TO THE DISCUSSION OF
WARD IDENTITIES WHICH WILL BE
USED TO DETERMINE THE VERTEX

$$V_{\substack{\text{highest w.} \\ j_1 j_2 j_3}} \Rightarrow V_{j_1 n_1 j_2 n_2 j_3 n_3}$$

FROM THE LINEAR MAP

$$\psi_{j,n}(p, \phi) = \int d\tau d\sigma K_2(p, \phi; \tau, \sigma) \underbrace{e^{i\omega\tau + i n\phi}}_{\text{matrix model}}$$

ON THE MATRIX SIDE:

$$\tilde{L}_+ = a^\dagger b^\dagger \rightarrow e^{i\alpha} e^{i\beta} |\partial_\alpha| |\partial_\beta|$$

$$\tilde{L}_- = a b$$

$$\tilde{L}_0 = a^\dagger a + b^\dagger b \rightarrow \left| \frac{\partial}{\partial \tau} \right|$$

$$\tau = \alpha + \beta$$

$$\sigma = \alpha - \beta$$

These induce the associated
generators on the space-time
side:

$$L_{\pm} = [i \sin \rho \partial_{\rho} \mp \hat{\omega} \sin \rho]$$

$$L_0 = \hat{\omega} = \sqrt{-\partial_{\rho}^2 + \frac{-1}{\sin^2 \rho} \frac{\partial^2}{\partial \phi^2}}$$

which obey

$$[L_{\pm}, L_0] = \pm 2 L_{\pm}$$

and we have

$$\psi_{jn}(\rho, \phi) = N (L_+)^n \psi_{j0}(\rho, \phi)$$

This is consistent with the
property showed earlier that
the map generates solutions

$$\square_{\text{AdS}} \psi = 0$$

FROM THE CORRESPONDENCE IN
THE CASE OF $1/2$ BPS STATES
WE WERE ABLE TO ESTABLISH

$$\tilde{V}_{j_1 j_2 j_3}^{\omega_L} \rightarrow V_{j_1 j_2, j_3 = j_1 + j_2} \delta_{j_1 + j_2, j_3}$$

WHICH CONSERVES

THIS WAY ONE ESTABLISHES THE
MEASURE OF (ADS) INTEGRATION

$$V_{j_1 j_2 j_3} = \int_0^{2\pi} d\phi \int d\rho \left[\frac{1}{\sin \rho} \right] \bar{\Psi}_{j_1}(\rho, \phi) \Psi_{j_2}(\rho, \phi) \Psi_{j_3}(\rho, \phi)$$

$$\Psi_j(\rho, \phi) = (\cos \rho e^{i\phi})^j$$

NOW WE USE THE SAME MEASURE
TO CONSIDER

$$V_{j_1 n_1 j_2 n_2 j_3 n_3} = \int d\phi \int dp \frac{1}{f_{\text{inp}}} \left. \begin{array}{l} L_+^{n_1} \psi_{j_1} \\ L_+^{n_2} \psi_{j_2} \\ L_+^{n_3} \psi_{j_3} \end{array} \right\}^*$$

TO DERIVE

THE 'WARD' IDENTITY:

$$\begin{aligned} & \int \frac{dp}{f_{\text{inp}}} [L_-^{(2)} + L_-^{(3)} - L_+^{(1)}] \bar{\psi}_{n_1}^{j_1} \psi_{n_2}^{j_2} \psi_{n_3}^{j_3} \\ & + \int dp [L_0^{(2)} + L_0^{(3)} - L_0^{(1)}] \psi_{n_1}^{j_1} \psi_{n_2}^{j_2} \psi_{n_3}^{j_3} = 0 \end{aligned}$$

DETAILED ANALYSIS OF RELATIONS
OF THIS KIND SHOWS THAT THEY
ARE SUFFICIENT TO DETERMINE
THE ANSWER * FROM $V_{j_1 j_2 j_3}$
highest weight.

CONCLUSION :

THERE IS A LINEAR AND THEN A NONLINEAR
MATRIX MODEL MAP:

LINEAR : GIVES A BRIDGE BETWEEN THE
SOLUTION OF EIGEN EQUATIONS ON M.M.
SIDE $\leftrightarrow$ LAPLACIAN IN ADS (5)

NONLINEAR : RELATES THE (CUBIC)
INTERACTIONS + HIGHER

THERE IS PROBABLY A MORE
ELEGANT WAY TO RELATE THE
TWO PICTURES ...

HAPPY BIRTHDAY

TAIYAKI

AND MANY MORE!
