# Anomalies and marginal deformations in brane tilings

**@Komaba2007** (2007/2/11(Sun))

The Univ. of Tokyo (Hongo)
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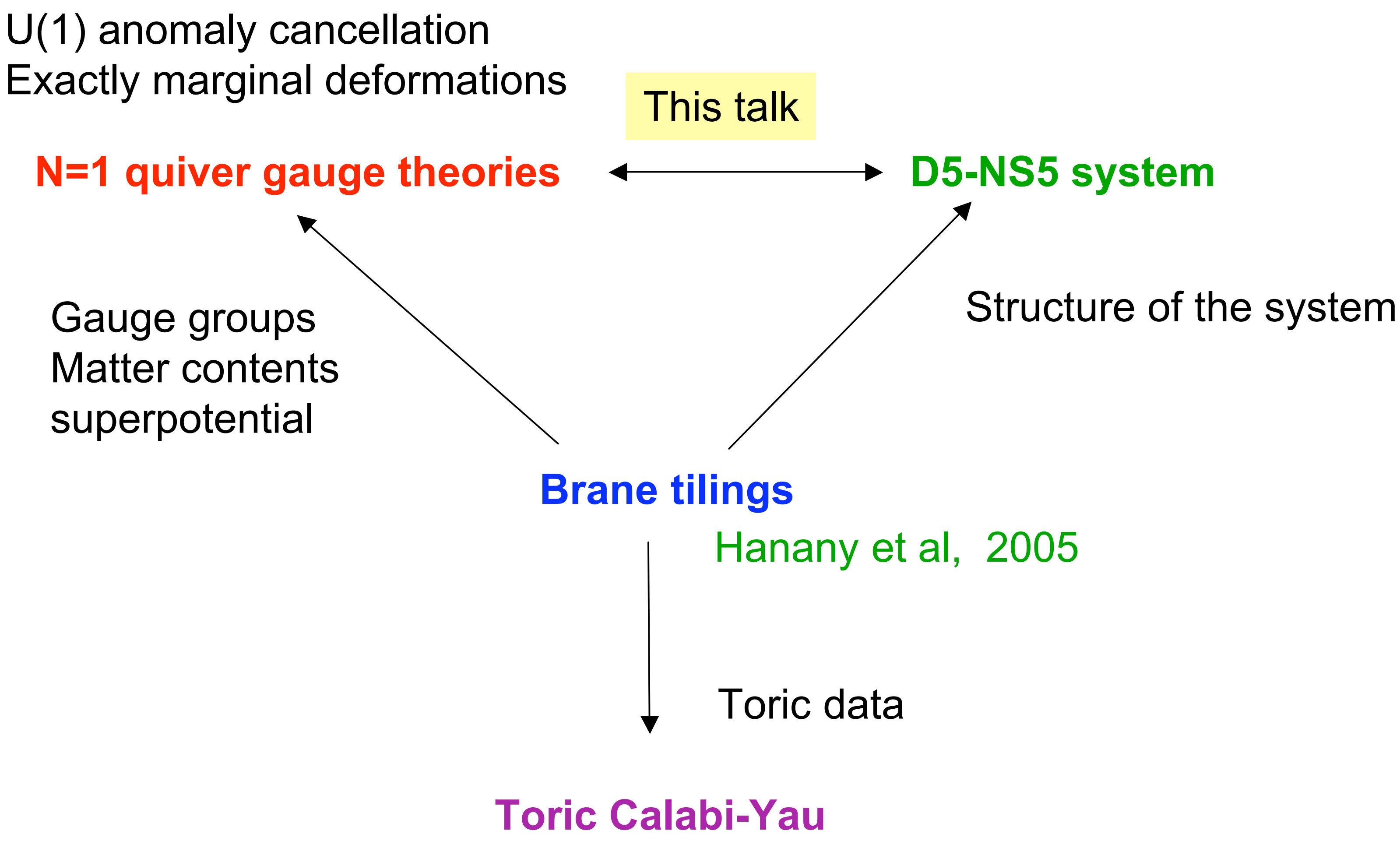
Based on
Y.I., JHEP06(2006)011 (hep-th/0605097)
Y.I., JHEP12(2006)041 (hep-th/0609163)
Y.I., H. Isono, K. Kimura, M. Yamazaki, hep-th/0702049


U(1) anomaly cancellation
Exactly marginal deformations
This talk
N=1 quiver gauge theories
D5-NS5 system
Gauge groups
Matter contents
superpotential
Structure of the system
Brane tilings
Hanany et al, 2005
Toric data
Toric Calabi-Yau

# Brane configuration

Compact ($T^2$) → 5, 7

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| N D5-branes | o | o | o | o | | o | | o | | |
| NS5-branes | o | o | o | o | o | o | | | | |
| | o | o | o | o | | | o | o | | |

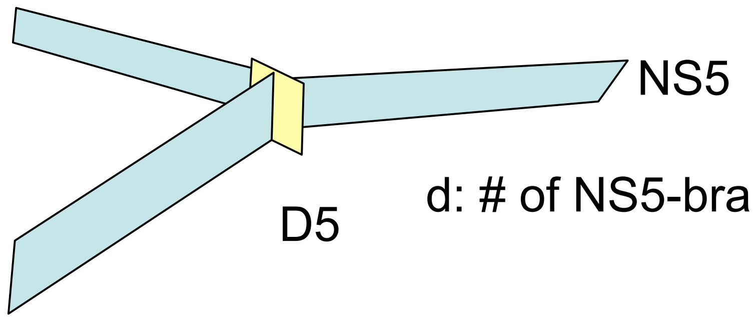
89-rotation
= R-symmetry

Slant NS5s
are also used

This configuration preserves N=1 SUSY

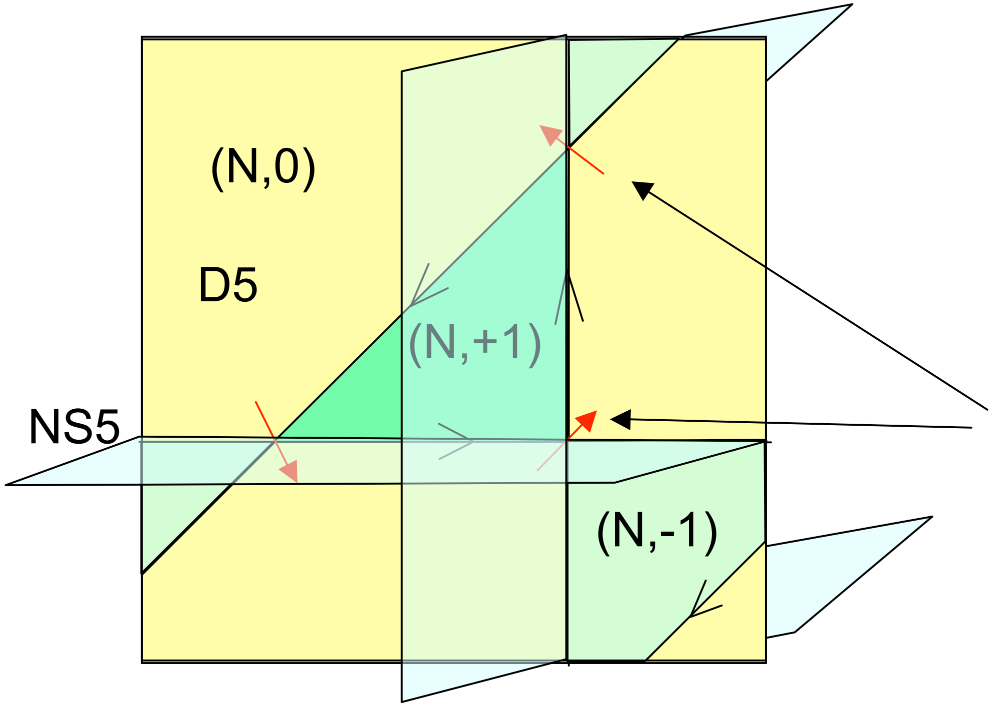
projection to 46 → (p,q) web

projection to 57 → **brane tiling**
(Feng et al. hep-th/0511287)




d: # of NS5-branes

## Brane tiling




(p,q) brane = bound state of p D5 and q NS5

Boundaries of NS5 divide the torus into faces

Due to the NS5 charge conservation, some of faces become (N,+1) or (N,-1) branes

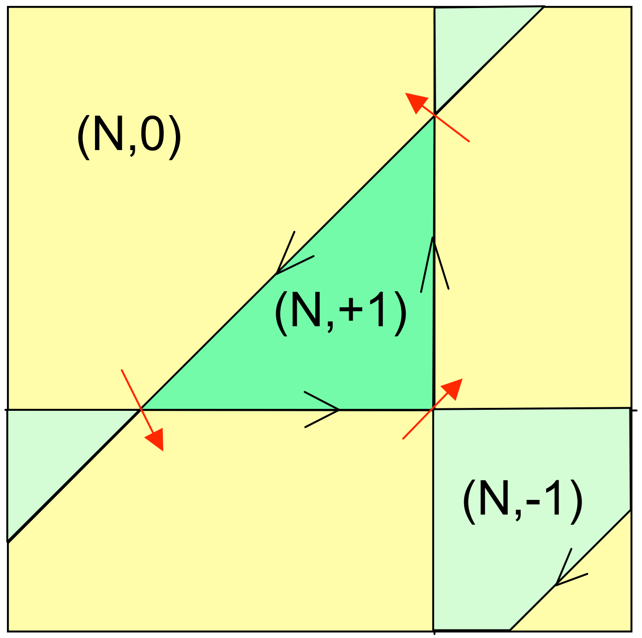
**SU(N) can live only on (N,0)**

Open strings stretched between two (N,0) faces

**bi-fundamental chiral multiplets**

# Bipartite graph

Brane tilings are usually represented as Bipartite Graphs drawn on tori

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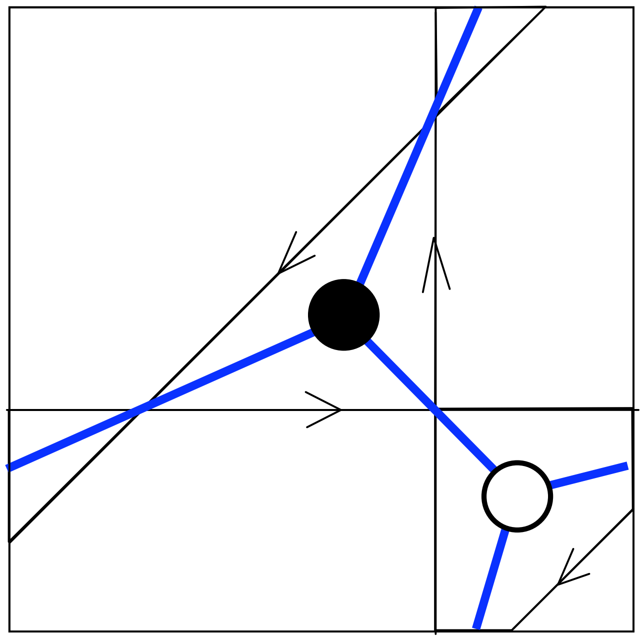
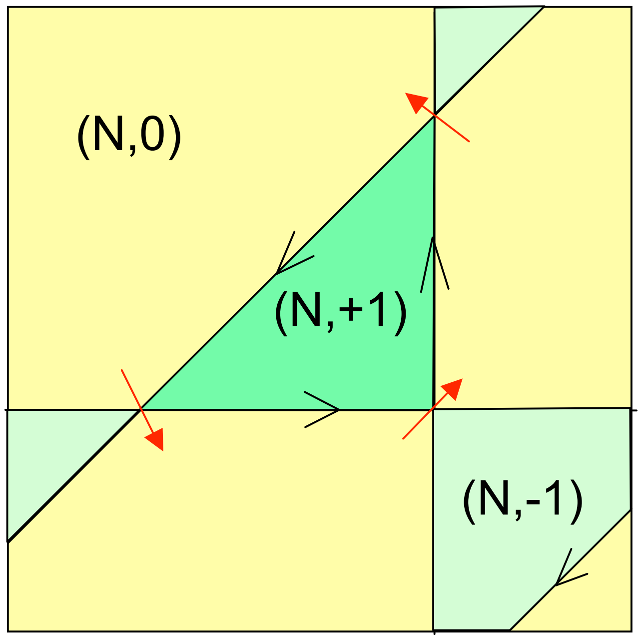


A bipartite graph is a graph consisting of two kinds of vertices and edges connecting different kinds of vertices.

# Superpotential




The superpotential can be read off from a brane tiling.

The superpotential

$$W = \sum_k \pm h_k \mathcal{O}_k, \quad \mathcal{O}_k = \mathrm{tr} \prod_{I \in k} \Phi_I$$

k : label of (N,±1) faces
I : label of intersection (chiral multiplets)

$I \in k$ means I is a corner of k

If $h_k$=1, the moduli space is toric CY

(hep-th/0601063 S.Franco and D.Vegh)

In this talk, I discuss anomalies and marginal deformations in gauge theories.

## Three kinds of anomalies

**gauge anomalies** **$SU(N)^3$**

**global U(1) anomalies** **$U(1)SU(N)^2$**

**’t Hooft anomalies** **$U(1)^3$**

All these anomalies (or anomaly cancellation conditions) should be reproduced by brane configuration.

In what follows I focus on the anomaly free global U(1) symmetries

Y.I., JHEP06(2006)011 (hep-th/0605097)
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# Global symmetries of N=1 quiver SQCD

$U(1)_R$ ×1 — R-symmetry (89-rotation)
- → supercharge
- → $\mathrm{tr}(\Phi_1\Phi_2\Phi_3)$
- → $\det(\Phi_I)$

$U(1)_M$ ×2 — Mesonic symmetries (B-field $B_{\mu5}$, $B_{\mu7}$)
- → $\mathrm{tr}(\Phi_1\Phi_2\Phi_3)$
- → $\det(\Phi_I)$

$U(1)_B$ ×(d−3) — Baryonic symmetries (U(1) gauge fields on branes)
- → $\det(\Phi_I)$

d: the number of NS5-branes.

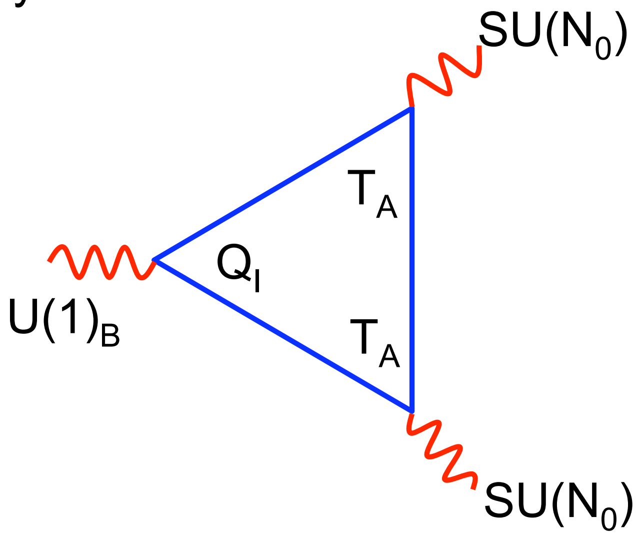
We here discuss only the baryonic symmetries.

# Anomaly free $U(1)_B$

Let us consider baryonic U(1) symmetries

They are realized as gauge symmetries on branes

A $U(1)_B$ is specified by giving charge assignment $Q_I$




Charge assignment $Q_I$ ⟵ Anomaly cancellation

Certain constraints

What in the brane configuration correspond to these constraints?

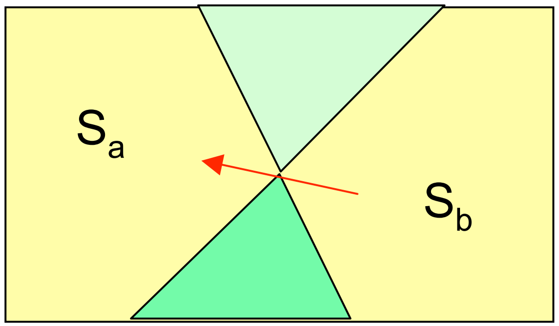
# Charge assignment and gauge fields on branes

Gauge fields on branes

$$A_\mu^a(x^\mu, y^i) = S_a(y^i) V_\mu(x^\mu)$$

a: label of faces
$A_\mu{}^a$: gauge fields on branes
$V_\mu$: gauge field in 4-dim

Masslessness → $S_a$ is constant on the face a




The charges are given as differences of $S_a$

$$Q = S_a - S_b$$

This guarantees that the symmetry does not rotate mesonic operators.
Especially the superpotential is invariant under the U(1).

⇨ This U(1) is a classical symmetry.

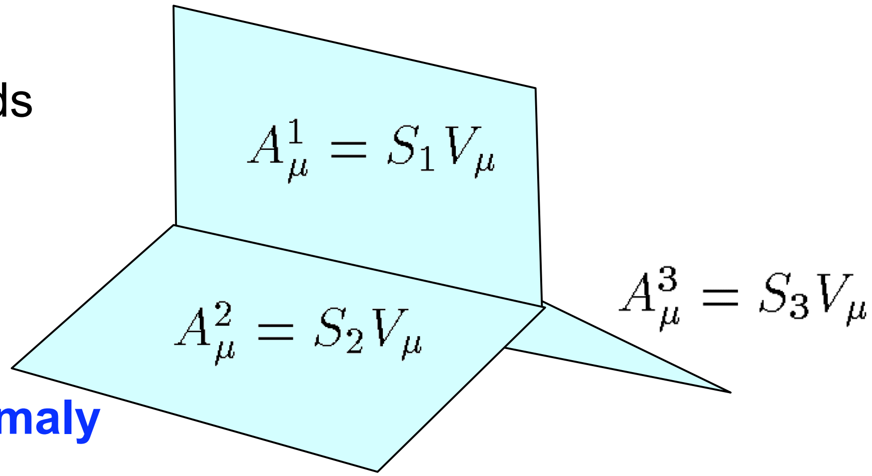
Anomaly cancellation ⟶ Constraints on $S_a$

Boundary condition

$$(A^1_\mu \pm A^2_\mu \pm A^3_\mu)|_{\rm junc} = 0$$

is imposed on gauge fields on branes

⇨ $S_1 \pm S_2 \pm S_3 = 0$




**This is equivalent to the anomaly cancellation condition**

> $U(1)_B$ anomaly cancellation conditions
> = boundary condition imposed on gauge fields on branes
> + constantness of gauge fields on each face.

We refer to number assignments $S_A$ to faces satisfying the above condition as baryonic charges.

It will turn out later that baryonic charges are useful in the analysis Of exactly marginal deformations.

## Exactly marginal deformations

(hep-th/0702049 Y.I., H.Isono, K.Kimura, M.Yamazaki)

In the low energy limit, the gauge theories flow into IR fixed points.

Conformal manifold $\subset$ parameter space

$$\beta_a = \beta_k = 0 \qquad\qquad (g_a, h_k)$$

# of conditions = # of couplings

$g_a$: gauge couplings
$h_k$: superpotential couplings

Generically we have isolated IR fixed points in non-supersymmetric gauge theories

This is, however, not the case in supersymmetric gauge theories.

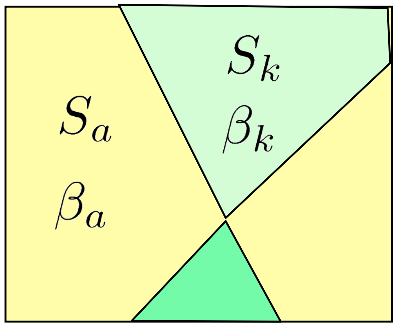
The β-functions are given with the anomalous dimensions $\gamma_I$

NSVZ exact β functions for gauge couplings $g_a$

$$\beta_a \propto 3 - \frac{1}{2}\sum_{I\in a}(1-\gamma_I)$$

β functions for superpotential couplings $h_k$

$$\beta_k \propto -3 + \sum_{I\in k}\left(1+\frac{1}{2}\gamma_I\right)$$




There are identically vanishing d−1 linear combinations

$$\beta[S_A] \equiv \sum_a S_a\beta_a + \sum_k S_k\beta_k = 0$$

a: (N,0) faces
k: (N,±1) faces

($S_A$ are the baryonic charges)

These d−1 relations effectively decreases the number of conditions.

# of conditions = # of couplings – (d−1)

**The dimension of the conformal manifold is d−1.**

The conformal submanifold is parameterized by RG invariant parameters associated with vanishing linear combinations $\beta[S_A]$

$d-1$ deformations
- 1 diagonal gauge coupling
- 1 β-like deformation
- $d-3$ deformations

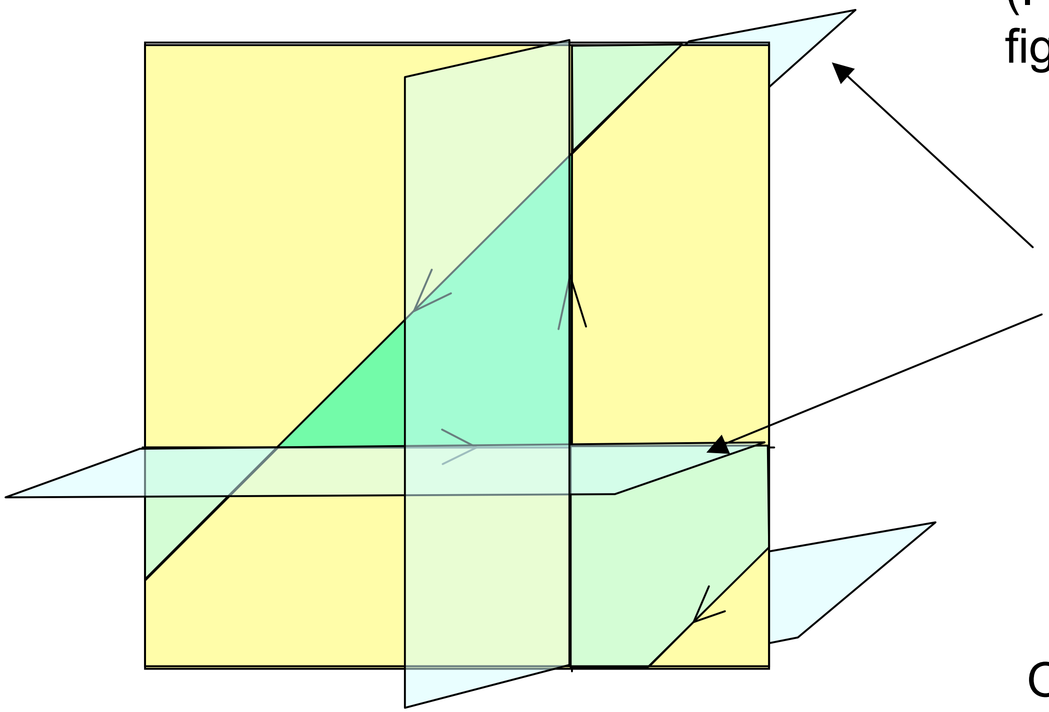
What are the corresponding deformations in the brane system?

## Deformations of the brane system

For the analysis of deformations, you should take account of equations of motion.

NS5-branes → holomorphic curve in 4567
D5-branes → discs with boundaries on the NS5.

(I still use the *incorrect* figure for the simplicity)



You can move these

We have d parameters
2 of them are redundant
1 constraint from EOM

Only $d-3$ degrees of freedom

We also have d − 3 degrees of freedom associated with the Wilson lines on the NS5-branes.

→ d − 3 complex parameters for brane configuration.

We also have background supergravity fields.

More detailed analysis shows the following relations

d − 1 deformations:

- diagonal gauge coupling ∼ $e^{-\phi} + iC_{57}$
- β-like deformation ∼ $\chi + iB_{57}$
- The other d − 3 ∼ deformation of branes

# Summary

| Gauge theory | | Brane configuration |
| --- | --- | --- |
| **$U(1)_B$ anomaly cancellation** | ⟷ | **Boundary condition of vector fields** |
| **Exactly marginal deformations** | ⟷ | **Deformation of branes & background fields** |

**open questions**

How to get superconformal $U(1)_R$ ?

How to realize the a-maximization？

Solitons and BPS operators

Breaking conformal sym. and cascading

SUSY breaking

Relation to the dimer model

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