Anomalies and marginal deformations in brane tilings

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Based on
U(1) anomaly cancellation
Exactly marginal deformations

**N=1 quiver gauge theories**

Gauge groups
Matter contents
Superpotential

**This talk**

**D5-NS5 system**

Structure of the system

**Brane tilings**

Hanany et al., 2005

Toric data

**Toric Calabi-Yau**
**Brane configuration**

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<th>2</th>
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<td>N D5-branes</td>
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This configuration preserves N=1 SUSY

89-rotation = R-symmetry

Slant NS5s are also used

projection to 46 $\rightarrow$ (p,q) web

projection to 57 $\rightarrow$ **brane tiling**

(Feng et al. hep-th/0511287)
**Brane tiling**

Boundaries of NS5 divide the torus into faces

Due to the NS5 charge conservation, some of faces become \((N,+1)\) or \((N,-1)\) branes

**SU(N) can live only on \((N,0)\)**

Open strings stretched between two \((N,0)\) faces

**bi-fundamental chiral multiplets**

\((p,q)\) brane = bound state of \(p\) D5 and \(q\) NS5
**Bipartite graph**

Brane tilings are usually represented as Bipartite Graphs drawn on tori.

A bipartite graph is a graph consisting of two kinds of vertices and edges connecting different kinds of vertices.
The superpotential can be read off from a brane tiling.

The superpotential

\[ W = \sum_{k} \pm h_k \mathcal{O}_k, \quad \mathcal{O}_k = \text{tr} \prod_{I \in k} \Phi_I \]

\( k \) : label of \((N, \pm 1)\) faces \\
\( I \) : label of intersection (chiral multiplets)

\( I \in k \) means \( I \) is a corner of \( k \)

If \( h_k = 1 \), the moduli space is toric CY

(hep-th/0601063 S.Franco and D.Vegh)
In this talk, I discuss anomalies and marginal deformations in gauge theories.

**Three kinds of anomalies**

<table>
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<th>Type</th>
<th>Group</th>
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<td>gauge anomalies</td>
<td>SU($N^3$)</td>
</tr>
<tr>
<td>global U(1) anomalies</td>
<td>U(1)SU($N^2$)</td>
</tr>
<tr>
<td>’t Hooft anomalies</td>
<td>U($1^3$)</td>
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All these anomalies (or anomaly cancellation conditions) should be reproduced by brane configuration.

In what follows I focus on the anomaly free global U(1) symmetries

Global symmetries of $N=1$ quiver SQCD

- $U(1)_R \times 1$ R-symmetry
  - 89-rotation
  - supercharge

- $U(1)_M \times 2$ Mesonic symmetries
  - $B$-field $B_{\mu 5}$, $B_{\mu 7}$
  - $\text{tr}(\Phi_1 \Phi_2 \Phi_3)$

- $U(1)_B \times (d-3)$ Baryonic symmetries
  - $\text{det}(\Phi_1)$
  - $U(1)$ gauge fields on branes

$d$: the number of NS5-branes.

We here discuss only the baryonic symmetries.
Anomaly free $U(1)_B$

Let us consider baryonic $U(1)$ symmetries

They are realized as gauge symmetries on branes

A $U(1)_B$ is specified by giving charge assignment $Q_I$

Charge assignment $Q_I$  
Anomaly cancellation

Certain constraints

What in the brane configuration correspond to these constraints?
**Charge assignment and gauge fields on branes**

Gauge fields on branes

\[ A^a_\mu (x^\mu, y^i) = S_a(y^i)V_\mu(x^\mu) \]

\( a: \) label of faces

\( A^a_\mu: \) gauge fields on branes

\( V_\mu: \) gauge field in 4-dim

Masslessness \( \rightarrow \) \( S_a \) is constant on the face \( a \)

The charges are given as differences of \( S_a \)

\[ Q = S_a - S_b \]

This guarantees that the symmetry does not rotate mesonic operators. Especially the superpotential is invariant under the U(1).

\( \rightarrow \) This U(1) is a classical symmetry.

Anomaly cancellation \( \rightarrow \) Constraints on \( S_a \)
Boundary condition

\[(A^1_\mu \pm A^2_\mu \pm A^3_\mu)\big|_{\text{junc}} = 0\]

is imposed on gauge fields on branes

\[\rightarrow S_1 \pm S_2 \pm S_3 = 0\]

This is equivalent to the anomaly cancellation condition

\[A^1_\mu = S_1 V_\mu\]
\[A^2_\mu = S_2 V_\mu\]
\[A^3_\mu = S_3 V_\mu\]

\[U(1)_B\] anomaly cancellation conditions
= boundary condition imposed on gauge fields on branes
+ constantness of gauge fields on each face.

We refer to number assignments \(S_A\) to faces satisfying the above condition as baryonic charges.

It will turn out later that baryonic charges are useful in the analysis of exactly marginal deformations.
Exactly marginal deformations

(hep-th/0702049  Y.I., H.Isono, K.Kimura, M.Yamazaki)

In the low energy limit, the gauge theories flow into IR fixed points.

Conformal manifold \( \subset \) parameter space

\[ \beta_a = \beta_k = 0 \]

\[ (g_a, h_k) \]

# of conditions = # of couplings

\( g_a \): gauge couplings
\( h_k \): superpotential couplings

Generically we have isolated IR fixed points in non-supersymmetric gauge theories

This is, however, not the case in supersymmetric gauge theories.
The $\beta$-functions are given with the anomalous dimensions $\gamma_I$

**NSVZ exact $\beta$ functions for gauge couplings $g_a$**

$$\beta_a \propto 3 - \frac{1}{2} \sum_{I \in a} (1 - \gamma_I)$$

**$\beta$ functions for superpotential couplings $h_k$**

$$\beta_k \propto -3 + \sum_{I \in k} \left(1 + \frac{1}{2} \gamma_I \right)$$

There are identically vanishing $d-1$ linear combinations

$$\beta[S_A] \equiv \sum_a S_a \beta_a + \sum_k S_k \beta_k = 0$$

($S_A$ are the baryonic charges)

These $d-1$ relations effectively decreases the number of conditions.

$# \text{ of conditions} = # \text{ of couplings} - (d-1)$

**The dimension of the conformal manifold is $d-1$.**
The conformal submanifold is parameterized by RG invariant parameters associated with vanishing linear combinations $\beta[S_A]$

\[ \begin{align*}
\text{d} - 1 \text{ deformations} & \quad \begin{cases} 
1 \text{ diagonal gauge coupling} \\
1 \beta\text{-like deformation} \\
d - 3 \text{ deformations}
\end{cases} 
\end{align*} \]

What are the corresponding deformations in the brane system?
Deformations of the brane system

For the analysis of deformations, you should take account of equations of motion.

NS5-branes $\rightarrow$ holomorphic curve in 4567
D5-branes $\rightarrow$ discs with boundaries on the NS5.

(I still use the *incorrect* figure for the simplicity)

You can move these

We have $d$ parameters
2 of them are redundant
1 constraint from EOM

Only $d-3$ degrees of freedom
We also have $d-3$ degrees of freedom associated with the Wilson lines on the NS5-branes.

$d-3$ complex parameters for brane configuration.

We also have background supergravity fields.

More detailed analysis shows the following relations

\[
\begin{align*}
\text{diagonal gauge coupling} & \sim e^{-\phi} + iC_{57} \\
\beta\text{-like deformation} & \sim \chi + iB_{57} \\
\text{The other } d-3 & \sim \text{deformation of branes}
\end{align*}
\]
Summary

Gauge theory

\[ \text{U}(1)_B \text{ anomaly cancellation} \]

\[ \text{Exactly marginal deformations} \]

Brane configuration

\[ \text{Boundary condition of vector fields} \]

\[ \text{Deformation of branes \\ \\ \\ \\ & background fields} \]
open questions

How to get superconformal $U(1)_R$ ?

How to realize the a-maximization ?

Solitons and BPS operators

Breaking conformal sym. and cascading

SUSY breaking

Relation to the dimer model

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