Rolling Tachyon

in

2-dim. String Theory

(contribution to conf. in memory of BUNJI SAKITA.)

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based on:

"A time dependent classical solution of $c=1$ string field theory & non-pert. effects"

A. Dhar, G. Mandal & SRW hep-th 9212027

"Rolling Tachyon Sol. of 2-dim. String Theory"

G. Mandal and SRW hep-th: 0312xy

Dynamics of unstable D-branes in ST

Important for various reasons:

1. Offers opportunity to study 'time dependent' solutions in st.

2. Can lead to insights about the 'landscape' of string theory

3. Offers a check on the proposal of Sen that open string theory on the brane would give a complete description of the closed strings into which the brane decays.

Methods to study time dependent Sols.

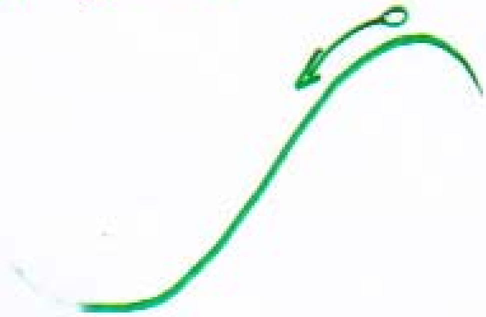
2 methods (Sen, Lambert, Lin, Maldacena, ... Strominger ....

1.) Study time dependent solutions of the tachyon on the D_p - brane :

$T(\vec{x},t)$, with eff. action

$$S = -T_p \int V(T) \sqrt{-\det(\eta_{\mu\nu} + \partial_\mu T \partial_\nu T)}$$

$$V(T) = \frac{1}{\cosh \frac{T}{2}}$$



2.) study classical sol. using BCFT

$$S = \frac{1}{4\pi\alpha'} \int d^2\sigma\, \partial X^\mu \partial X^\nu \eta_{\mu\nu} + \tilde{\lambda} \oint d\tau \cosh X^0(\tau)$$

$$\langle V_c | B \rangle = \langle V_c(0) \rangle_{DISK}$$

Closed st. Vertex op. — Boundary state

2-dim. st. (BCFT)

$$S = \frac{1}{4\pi\alpha'} \int d^2\sigma \left(\eta_{\mu\nu} \partial_a x^\mu \partial_b x^\nu \hat{g}^{ab} + Q \hat{R} \sqrt{\hat{g}}\, x^1 \right.$$
$$\left. + \mu_0 \sqrt{\hat{g}}\, e^{\tilde{\alpha} X^1} \right)$$
$$+ \tilde{\lambda} \oint dt \cosh X^0(t)$$

(Liu, Lambert, Maldacena .... ZZ, Sen)

ZZ : D_0 brane at $X^1 = \infty$

$$V_c = e^{iPX^1 + \sqrt{2} X^1} \cdot e^{iEX^0}, \quad E = |P|$$

closed string vertex operator

Result :

$$\pi^{11/2} (2\pi)^6 \langle V_c(0) \rangle \equiv \mathcal{A} = 2\sqrt{\pi}\, i\, e^{-iE \ln \hat{\lambda} + i\delta(P)}$$

$$e^{i\delta(P)} = \mu^{-\frac{i}{2}P} \frac{\Gamma(iP)}{\Gamma(-iP)} \qquad \text{'leg pole factor'.}$$

$\hat{\lambda} = \sin \pi \tilde{\lambda}$ parameter of BCFT.

6.

$C = 1$ matrix model

New interpretation (Mcgreevy + Verlinde):

Open String field theory associated with N D_0 branes:

$M_{ij}(t)$, $A_{0\,ij}(t)$ $N \times N$ hermitean matrices.

$$S = \frac{1}{g_0} \int dt \left[\frac{1}{2} (D_t M)^2 - V(M) \right]$$

$$D_t = \partial_t - i [A_0, \;]$$

$$V(M) = -\frac{tr}{2} M^2 + \cancel{tr M^3} + \cdots + \overbrace{\mu_0 N}^{\text{fixed}}$$

(Anticipating the fact that this model should also contain a description of closed strings in the continuum limit we need to take the double scaling limit as $N \to \infty$,

$$g_0 - g_c \approx \frac{1}{2\pi} \mu_0 \ln \mu_0 \to 0 \quad \text{as } \mu_0 \to 0$$

$-\mu_0$ is the 'bare' fermi level.

Hold $\mu = \mu_0 N$ = fixed.)

A description of closed strings requires

$$V(M) = -\frac{1}{2} tr M^2 + \mu.$$

1. Classical solution of matrix model and states of D_0 branes:

$$\ddot{M} = M + \cdots \quad , [M, \dot{M}] = 0 \quad \Rightarrow$$

$$M(t) = A \cosh t + B \sinh t \quad , [A, B] = 0$$

$$M_{ij}(t) = \delta_{ij}\left(q_j \cosh t + v_j \sinh t\right)$$

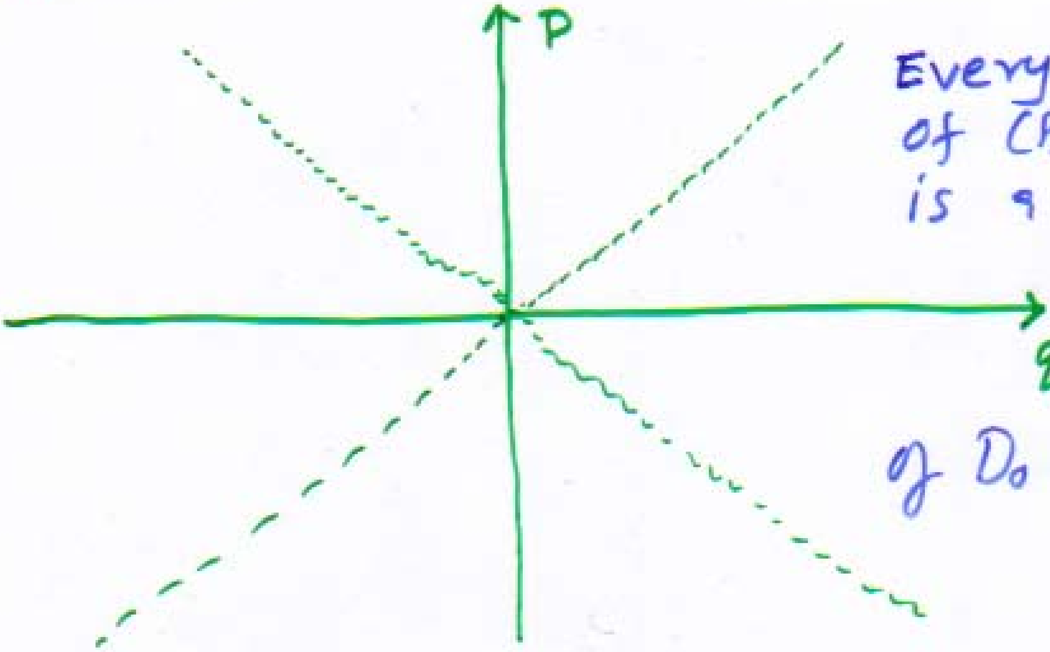
define momentum : $\quad p_j = \dfrac{v_j}{g_0}$

$$H = \sum_{j=1}^{N} \left(\frac{g_0}{2} p_j^2 - \frac{1}{2g_0} q_j^2 \right)$$

rescale $\quad p_j \to g_0 p_j \qquad q_j \to g_0 q_j$

define $\quad u_c(p,q) = 2\pi \sum_{j=1}^{N} \delta(q_j - q)\, \delta(p_j - p)$

$$H = \iint_{-\infty}^{+\infty} \frac{dp\, dq}{2\pi}\, u_c(p,q)\, h(p,q) \, , \quad h(p,q) = \frac{1}{2}(p^2 - q^2)$$



Every point of (p,q) space is a state of D_0 brane.

Conclusion :

We have N decoupled 'rolling' tachyons

There is no lower bound in the energy.

Quantum theory $\Rightarrow$

N D0 branes are interacting !

$$\int \mathcal{D}M(t)\,\mathcal{D}A_0(t)\, e^{iS}$$

$\downarrow$

repulsive interaction among the eigenvalues.

$\downarrow$

Exact map into a decoupled system of

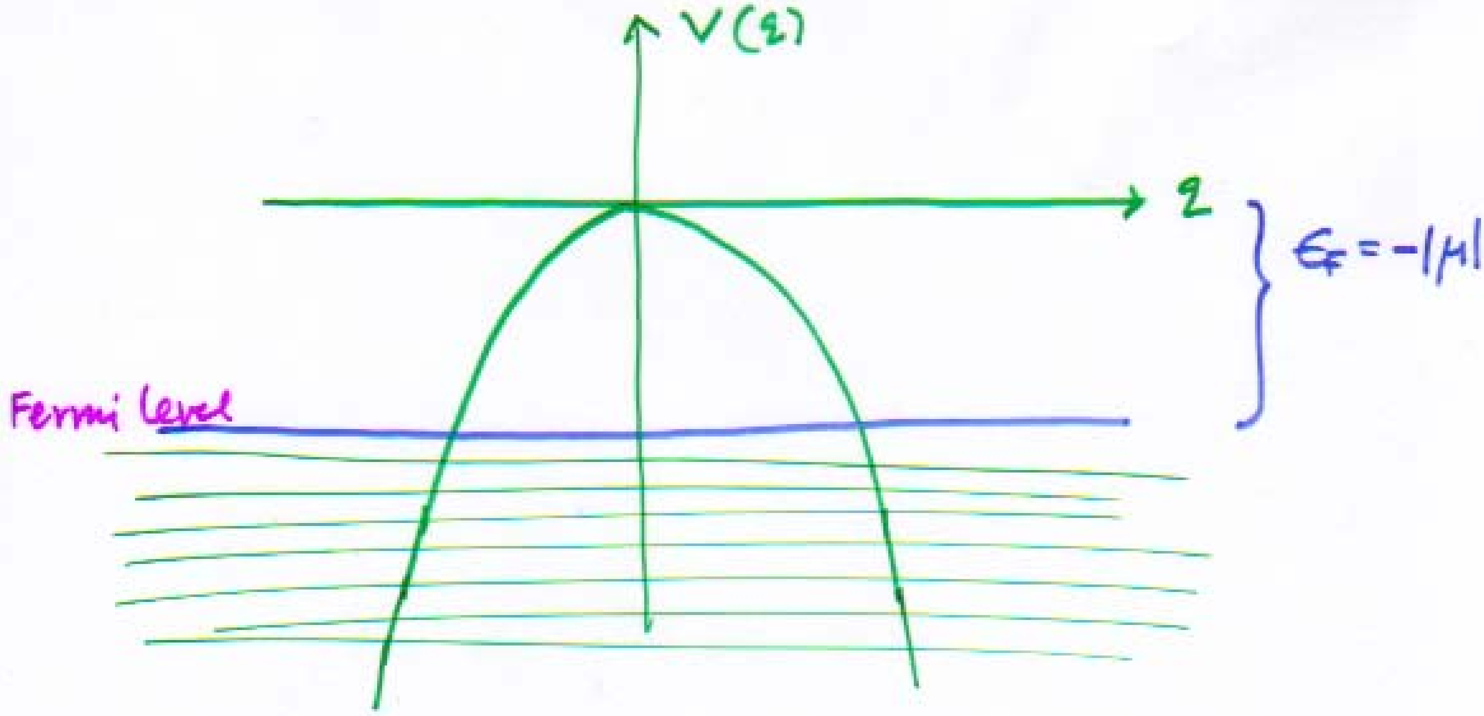
Non-relativistic fermions !

$\downarrow$

Stable gnd. state $\equiv$ filled fermi sea.

10.

Fermi field theory in 1+1 dim.



$$H = \int dq \; \psi^{+}(q) \left[-\frac{1}{2}\frac{d^2}{dq^2} - \frac{1}{2} q^2 + |\mu| \right] \psi(q)$$

$|\mu| = \frac{1}{g_{str}} \equiv \frac{1}{g}$ (comparing scattering amplitudes)

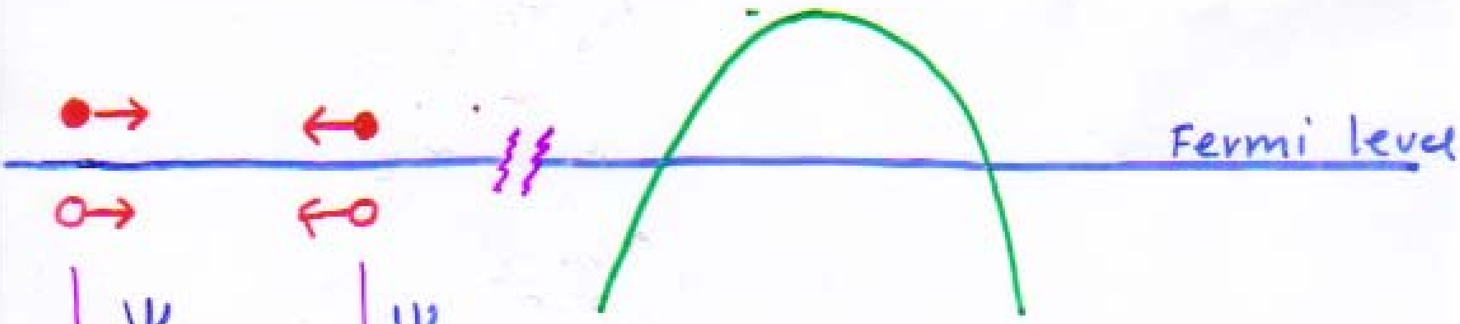
scale $q \to \sqrt{g}\, q$

$$H = \frac{1}{g} \int dq \; \psi^{+}(q) \left[\frac{\hat{p}^2}{2} - \frac{\hat{q}^2}{2} + 1 \right] \psi(q)$$

$$[\hat{p}, \hat{q}] = -ig$$

11/12 What are the 'closed strings'?

Bosonic small fluctuations around fermi sea.



Ψ_R: Right moving pair

Ψ_L: Left moving pair

Far from turning point

$\Psi_{L,R}$ are chiral relativistic fermions.

Standard bosonization

$$\Rightarrow \quad \Psi_L = e^{i 2\sqrt{\pi}\,\phi}, \quad \Psi_R = e^{-i 2\sqrt{\pi}\,\phi}$$

$$\phi(\tau) = \int \frac{dP}{\sqrt{2E}}\, e^{-i\tau P}\, a_P^{\dagger}$$

gives the 'closed string' state

$$|\Psi\rangle_{MM} = e^{i 2\sqrt{\pi} \int \frac{dP}{\sqrt{2E}} a_P^{\dagger} e^{-i\tau P}} |0\rangle$$

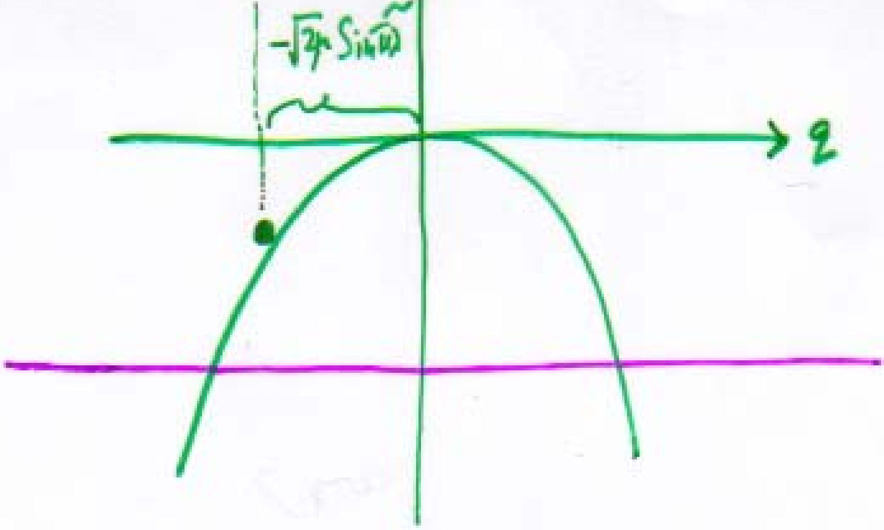
agrees with the closed string state $|\Psi\rangle_c$ (BCFT)

provided $\quad a^{\dagger}(P) \to a^{\dagger}(P)\, e^{i\Phi(P)}$ (leg pole)

$$\tau \equiv -\ln(\sin \pi \tilde{\lambda})$$

Klebanov, Maldacena, Seiberg

13.



$\tau = -\ln(\sin\pi\tilde{\lambda})$ corresponds to

$q = -\sqrt{2\mu}\,\sin\pi\tilde{\lambda}$ is the initial starting point of the fermion, of energy $\mu\cos^2\pi\tilde{\lambda}$.

Hence the source of the divergence is that the fermion is treated as a classical particle. (Klebanov, Maldacena, Seiberg)

The classical fermi fluid picture leads to divergences and is wrong!

The fermion phase space must be treated quantum mechanically : $[\hat{p}, \hat{q}] = -ig$

$\Rightarrow$ boson field theory must be non-commutative.

14.

1+1 dim. non-relativistic fermions

$\Longleftrightarrow$

non-commutative field theory in 2+1 dim. with $\theta = g_{str} \equiv g$.

This corresponds to the quantum fermi fluid and the 2+1 dim. field is the phase space distribution

$$u(p,q,t) = \int dx\, e^{-ipx} \underbrace{\psi^{+}\left(q - \frac{x}{2}g\right) \psi\left(q + \frac{x}{2}g\right)}_{\text{Bilocal field.}}$$

(which carries a specific co-adjoint representation of W_∞ algebra:

$$\left[W(\alpha,\beta),\, W(\alpha',\beta')\right] = 2i \sin \frac{g}{2}(\alpha\beta' - \beta\alpha') \,.\, W(\alpha+\alpha', \beta+\beta')$$)

A. Dhar, G. Mandal + SRW (1992).

Iso Sakita Karabali

15. Summary of results:

The 2+1 dim. NC field theory is the field theory of the phase space density of fermions $\underline{u(p,q,t)}$.

defined on a non-commutative phase space: endowed with a $*$ product

$$A(p,q) * B(p,q) = e^{\frac{ig}{2}(\partial_q \partial_{p'} - \partial_{q'} \partial_p)} A(p,q) B(p',q') \Big|_{\substack{p=p' \\ q=q'}}$$

Dynamics of $u(p,q,t)$:

(1) $$\partial_t u + \left(p \frac{\partial}{\partial q} + q \frac{\partial}{\partial p} \right) u(p,q,t) = 0$$
(Liouville equation)

$$\Leftrightarrow \quad u(p,q,t) = u(p_t, q_t, 0)$$

$$\begin{pmatrix} p_t \\ q_t \end{pmatrix} = \begin{pmatrix} \cosh t & -\sinh t \\ -\sinh t & \cosh t \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix}$$

(2) $u * u = u$, (3) $\int \frac{dp\,dq}{2\pi g}\, u(p,q,t) = N$

Also (Gopakumar, Minwalla + Strominger ~ 2000)

Solution (Rolling Tachyon) :

$$u(p,q,t) = u_0(p,q) + u_1(p,q,t)$$

$u_0(p,q)$ corresponds to the gnd. state

and

$$u_1(p,q,t) = 2\, e^{-\frac{1}{g}\left[(p_t - p_0)^2 + (q_t - q_0)^2\right]}$$

$$\int \frac{dp\,dq}{2\pi g}\, u_0 = (N-1) \quad , \quad \int u_1 \frac{dp\,dq}{2\pi g} = 1.$$

One can show that $(u_0+u_1) * (u_0+u_1) \approx (u_0+u_1)$

note that $u_1(p,q,0)$ corresponds to the

single fermion wave function:

$$\psi_1 = e^{-\frac{(x-q_0)^2}{2}} \cdot e^{i p_0 x}$$

which has expotentially small overlap with the levels at and below the fermi level.

17.

Position space density:

$$\rho(q,t) = \int \frac{dP}{2\pi g}\, u(P,q,t)$$

$$\Rightarrow \rho = \rho_0 + \rho_1$$

$$\rho_0(q) \xrightarrow[g\to 0]{} \frac{1}{g}\sqrt{q^2-1}$$

$$\rho_1(q,t) = \frac{e^{-\frac{(q-\bar{q}(t))^2}{2\sigma(t)}}}{\sqrt{2\pi}\sqrt{\sigma(t)}}$$

$$\sigma(t) = \frac{g}{2}\cosh 2t \qquad \text{(Dispersion)}$$

$\sigma(t) \sim o(1)$ for $T_c = -\ln g^{1/2}$ (characteristic time)

- for $t \ll T_c$, $u(P,q,t)$ represents the fermi sea + 1 D_0 brane

- (in this region the solution does not satisfy the equations describing small fluctuations around the fermi sea)

18.

$$t \gg T_c$$

$u(p,q,t)$ satisfies the equations of motion of 'closed' strings $\Leftrightarrow$ small oscillations around the fermi sea.

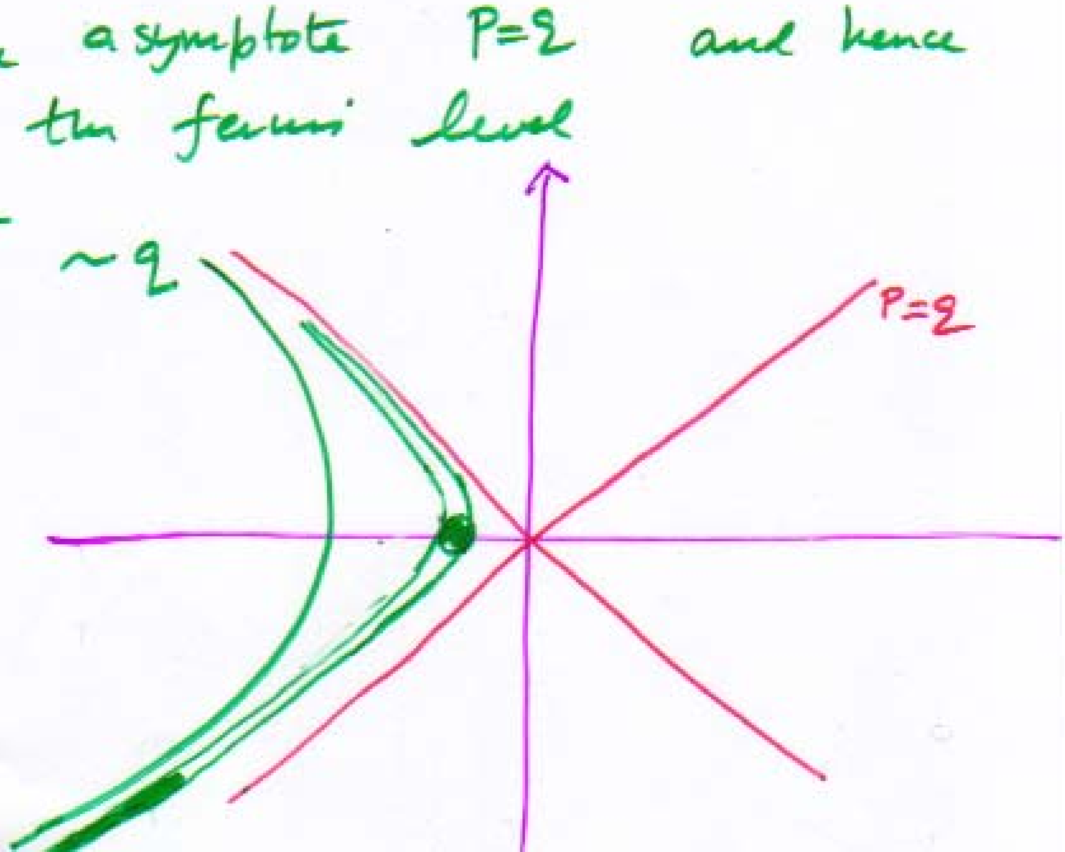
($\simeq$ collective field theory)

In fact for $t \gg T_c$

$$u_1(p,q,t) \sim 2 \exp\left[-e^{2t}\left((p-q-p_0e^{-t})^2 + (-p+q-q_0e^{-t})^2\right]\right.$$

describes a phase space density exponentially close to the asymptote $p=q$ and hence is close to the fermi level

$$p = \sqrt{q^2-\mu} \sim q$$



Conclusion :

- Our exact bosonization of non-relativistic fermions gives a complete description of both the closed string states and the sources which are the D0 branes

- The essential ingredient of the field theory is that it is non-commutative with $\theta = g$ (the string coupling)

This fact may have implications for the general formulation of open string field theory.