November 2003
@ Komaba 2003

Algebra of boundary states in closed string field theory

Univ. Tokyo
Y. Matsuo

based on
hep-th/0306189 , 0311xxx
[KMW1] [KMW2]

with
I. Kishimoto & E. Watanabe

§1 Introduction

Analysis of nonperturbative phenomena

Soliton solutions are essential

Magnetic monopole
Instanton
} for Yang-Mills

Blackhole for Einstein gravity

They are solutions of

nonlinear equation of motion

↳ Possible to carry nontrivial topological charge

Soliton solutions in string theory

Magnetic 5 brane

D-brane ⟸ completely stringy description is possible by

$\boxed{\text{Boundary state}}$

Boundary state

Implimentation of boundary conditions of
open string in closed string Hilbert space

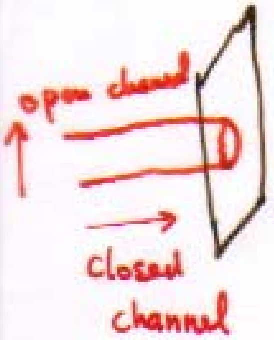
Example) D p-brane

$$\partial_\sigma X^\mu \Big|_{\sigma=0,\pi} = 0 \qquad \mu = 0, \cdots, p$$

$$\partial_\tau X^i \Big|_{\sigma=0,\pi} = 0 \qquad i = p+1, \cdots, d-1$$

$\uparrow^{i}$ (transverse)

$\longrightarrow \mu$ (longitudinal)



$\Downarrow$ Modular transformation $(\sigma \leftrightarrow \tau)$

$$\begin{cases} \partial_\tau X^\mu \big|_{\tau=0} |B\rangle = 0 \\ \partial_\sigma X^i \big|_{\tau=0} |B\rangle = 0 \end{cases}$$

relations between left/right movers

$\Downarrow$

$$|B\rangle = \exp\left(\sum_{n=1}^{\infty} \frac{1}{n} S_{\mu\nu}\, \alpha_{-n}^{\mu}\, \tilde{\alpha}_{-n}^{\nu} \right) |0; x^i\rangle$$

p=0 along Neumann direction

$X^i = x^i$ along Dirichlet direction

$$S_{\mu\nu} = \begin{pmatrix} -1 & & & & \\ & \ddots & & & \\ & & -1 & & \\ & & & +1 & \\ & & & & \ddots & \\ & & & & & +1 \end{pmatrix}$$

Note: Background dependent description
Does not look like a solution of field theory

Background independent characterization

$$(L_n - \tilde{L}_{-n})|B\rangle = 0 \qquad \cdots\cdots (*)$$

This **linear** condition is not enough!

(e.g. $Q_{BRS}|B\rangle = 0$)

Cardy condition = integrability in open string channel

$$\langle B| q^{(L_0+\tilde{L}_0)/2} |B'\rangle = \chi(q)$$

$$\underset{\substack{\uparrow \\ \text{modular} \\ \text{transformation}}}{=} \sum_i n_{BB'}{}^i \, \chi_i(\tilde{q})$$

$\chi_i(\tilde{q})$ = Character of irreducible representation in open string channel

$$\boxed{n_{BB'}{}^i : \text{ must be positive integer}}$$

$\Updownarrow$

nonlinear constraint which must be imposed with $(*)$

Can we translate it as the nonlinear equation in closed string field theory?

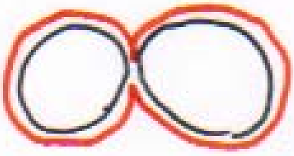
Main claim

Many (all ?) boundary states satisfy
universal nonlinear equation
(idempotency relation)

$$\boxed{|B\rangle * |B\rangle = (\text{constant})\, |B\rangle} \qquad \cdots\cdots (*)$$

$*$: star product of closed string field theory

HIKKO type



Witten type

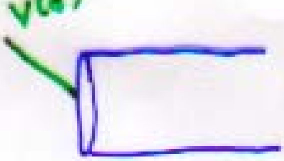
- Analogue of VSFT eq. of motion for open string

$$\mathcal{Q}\Psi + \Psi * \Psi = 0$$

$\mathcal{Q}$: pure ghost BRST operator

$$\left.\begin{array}{l}\text{Sliver}\\ \text{butterfly}\\ \text{identity}\end{array}\right\} \longleftrightarrow \text{Boundary state}$$

Variation of (*)



$$\delta|B\rangle = \oint \frac{d\sigma}{2\pi}\, V(\sigma)\,|B\rangle$$

Insertion of operators at the boundary

Note: Boundary state gives identification

$$X^L(\sigma) \Longleftrightarrow X^R(\sigma)$$

same as open string oscillator

Open string = Infinitesimal deformation of D-brane

example $\phi'(x)$ collective mode to describe transverse variation

We show that $\delta|B\rangle$ satisfies

$$|B\rangle * \delta|B\rangle + \delta|B\rangle * |B\rangle = (\text{const})\,\delta|B\rangle$$

iff

$\oint \frac{d\sigma}{2\pi} V(\sigma)$ is marginal deformation

$\Rightarrow$ idempotency relation (*) knows on-shell condition for open string

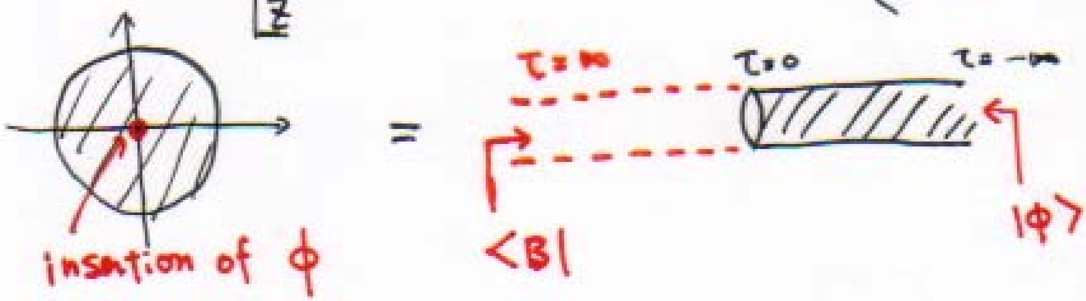
§2. Proof of idempotency relation

§2-1 Geometrical (Intuitive) proof

Boundary state

$\langle B|\phi\rangle$: 1 point function on Disk

$(z = e^{\tau + i\theta})$

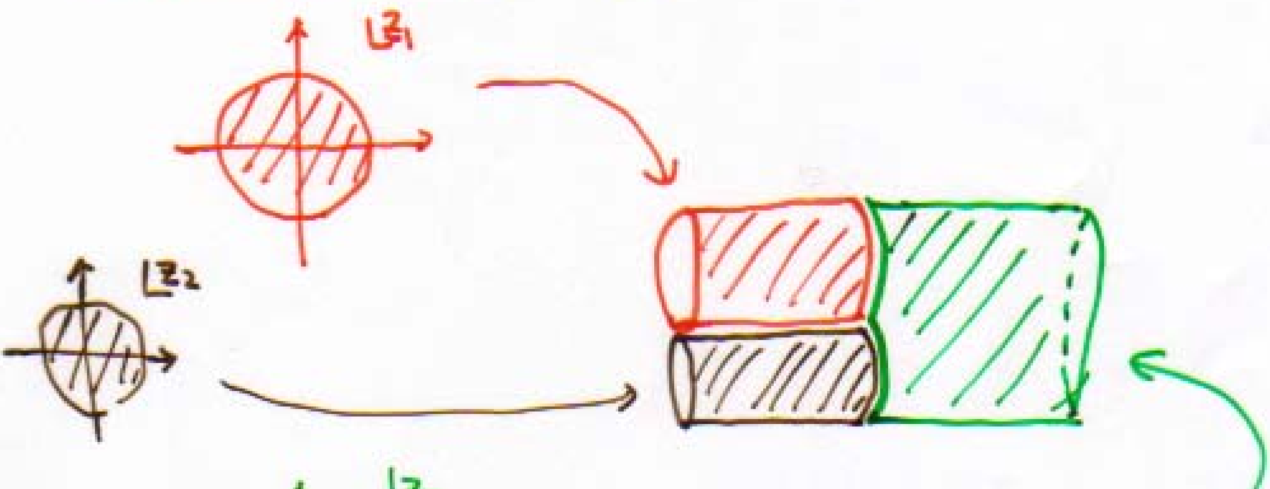


Role of boundary state

① Cut the cylinder at $\tau = 0$

② Set the boundary condition

3-string vertex (Lightcone type)



Patching together 3 world sheet

$|B_a\rangle * |B_b\rangle$

When $a = b$

$$|B_a\rangle * |B_a\rangle \propto |B_a\rangle \qquad (*)$$

- Boundary states are only states that have this property (stripping half cylinder)

$\Updownarrow$

$\boxed{\text{(*) idempotency relation is satisfied only by Boundary state ?}}$

§2.2 Proof by operator formalism [KMW1]

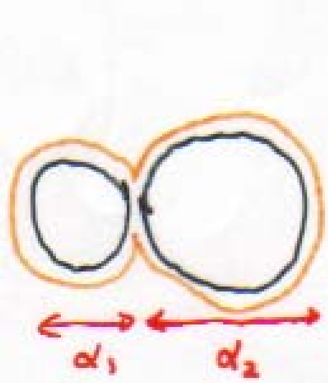
- Boundary state for D p-brane (with flux F)

$$|B(x^\perp)\rangle = \exp\Big(-\sum_n \frac{1}{n}\alpha^\mu_{-n} \mathcal{O}_{\mu\nu} \tilde{\alpha}^\nu_{-n} + \sum_n (c_{-n}\tilde{b}_{-n} + \tilde{c}_{-n} b_{-n})\Big)$$
$$\times\, c_0^+ c_1 \tilde{c}_1 \,|\, p^\parallel = 0,\, x^\perp\rangle \otimes |0\rangle_{gh}$$

$$\mathcal{O}^\mu{}_\nu = \begin{cases} \left((1+F)^{-1}(1-F)\right)^\mu{}_\nu & \mu,\nu = 0 \cdots p \\ -\delta^\mu{}_\nu & \mu,\nu = p+1, \cdots, d-1 \end{cases}$$

- F : constant electro/magnetic flux on D-brane
- $x^\perp$: location of D-brane in transverse direction

- To take HIKKO type star product
  - Introduce α-parameter



  - Adjust ghost zero mode

$$|B(x^\perp)\rangle \Rightarrow \Phi(x^\perp, \alpha) \equiv c_0^- b_0^+ \,|B(x^\perp)\rangle \otimes |\alpha\rangle_{gh}$$

2 HIKKO's star product

$$|V(1,2,3)\rangle = \int \delta(1,2,3)\ [\mu(1,2,3)]^2\ \wp^{(1)}\wp^{(2)}\wp^{(3)}$$

$$\times \prod_{r=1}^{3} \left(1+\frac{1}{\sqrt{2}} w_I^{(r)} \bar{c}_0^{(r)}\right) e^{F(1,2,3)}\ |p_1,\alpha_1\rangle |p_2,\alpha_2\rangle |p_3,\alpha_3\rangle$$

$$F(1,2,3) = \sum_{\pm}\sum_{r,s=1}^{3}\sum_{m,n\geq 1} \tilde{N}^{rs}_{mn}\left(\frac{1}{2} a_m^{(\pm)(r)\dagger}\ a_n^{(\mp)(s)\dagger} + \sqrt{m}\,\alpha_r\, c_m^{(\pm)(r)\dagger} (\sqrt{n}\,\alpha_s)^{-1} \bar{c}_n^{(\mp)(s)\dagger}\right)$$

$$+\frac{1}{2}\sum_{\pm}\sum_{r}\sum_{n} \tilde{N}^{r}_{n}\, a_n^{(\pm)(r)\dagger}\, \mathbb{P} - \frac{\tau_0}{4\alpha_1\alpha_2\alpha_3}\mathbb{P}^2$$

$$\mathbb{P} = \alpha_1 p_2 - \alpha_2 p_1$$

$$w_I^{(r)} = \cdots$$

$$\mu(1,2,3) = \exp(-2g(\beta)) \quad : \quad g(\beta) = (\beta^2+\beta+1)\left(\frac{\log|\beta|}{\beta+1} - \frac{\log|\beta+1|}{\beta}\right)$$

$$\tau_0 = \sum_{r=1}^{3} \alpha_r \log|\alpha_r| \qquad\qquad \beta = -\frac{\alpha_1}{\alpha_1+\alpha_2}$$

$$\int \delta(1,2,3) = \int d\alpha\, dp\ (2\pi)^d\, \delta^d(p_1+p_2+p_3)\ 2\pi\,\delta(\alpha_1+\alpha_2+\alpha_3)$$

$$\wp^{(i)} = \oint \frac{d\theta}{2\pi}\ e^{-i\theta(N_+^{(i)} - N_-^{(i)})} \quad : \quad N_\pm := \sum n\left(a_n^{(\pm)\dagger} a_n^{(\pm)} + c^\dagger \bar{c} + \bar{c}^\dagger c\right)$$

$$|\Phi_1 * \Phi_2\rangle_3 = \int d\bar{c}_0^{(1)} d\bar{c}_0^{(2)}\ \langle \Phi_1|\ {}_2\langle\Phi_2|\ V(1,2,3)\rangle$$

$${}_2\langle\Phi| = \int d\bar{c}_0^{(1)} \langle R(1,2)|\Phi\rangle_1$$

$\langle R(1,2)|$: Reflector

Some detail of computation (matter part)

boundary state

$$|\Phi_1\rangle \otimes |\Phi_2\rangle = e^{\frac{1}{2}a^\dagger M a^\dagger} \underline{|p_1, \alpha_1\rangle \otimes |p_2, \alpha_2\rangle}$$

↑ before integrating along transverse direction

$$a^\dagger = \begin{pmatrix} a^{(+)\dagger} \\ a^{(-)\dagger} \end{pmatrix} \qquad a^{(\pm)\dagger} = \begin{pmatrix} a_n^{\omega(\pm)\mu\dagger} \\ a_n^{\omega(\pm)\mu\dagger} \end{pmatrix}$$

$$M = \begin{pmatrix} 0 & -\mathcal{O}_{MN}\delta_{mn}\delta_{rs} \\ -\mathcal{O}^T_{MN}\delta_{mn}\delta_{rs} & 0 \end{pmatrix}$$

Exponential factor in V_3

$$F(1,2,3) = \frac{1}{2}a^\dagger N a^\dagger + a^\dagger \mu + a^{(3)\dagger}\tilde{N}^{33}a^{(3)\dagger} - \frac{\tau_0}{4\alpha_1\alpha_2\alpha_3}\mathbb{P}^2$$

$$N = \eta_{MN}\begin{pmatrix} n & 0 \\ 0 & n \end{pmatrix} \qquad n = \begin{pmatrix} \tilde{N}^{11} & \tilde{N}^{12} \\ \tilde{N}^{21} & \tilde{N}^{22} \end{pmatrix}$$

$$\mu = \begin{pmatrix} \mu^{(+)} \\ \mu^{(-)} \end{pmatrix} \qquad \mu^{(\pm)} = \begin{pmatrix} \tilde{N}^{13}a^{(3)(\pm)\dagger} + \frac{1}{2}\tilde{N}^1\mathbb{P} \\ \tilde{N}^{23}a^{(3)(\pm)\dagger} + \frac{1}{2}\tilde{N}^2\mathbb{P} \end{pmatrix}$$

$$\begin{cases} |\Phi_1 * \Phi_2\rangle = \det^{-1/2}(1-MN)\, e^H\, |p_1+p_2, \alpha_1+\alpha_2\rangle \\ H = \frac{1}{2}a^\dagger N^{33}a^\dagger + \frac{1}{2}\tilde{N}^3(a^{(+)\dagger}+a^{(-)\dagger})\mathbb{P} - \frac{\tau_0}{4\alpha_1\alpha_2\alpha_3}\mathbb{P}^2 \end{cases}$$

$$+\frac{1}{2}\mu^T \underline{M(1-NM)^{-1}}\mu$$

↑

How to manage this factor?

Neumann matrix

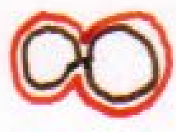
$$\tilde{N}^{rs}_{mn} = \delta_{rs}\delta_{mn} - 2\left(A^{(r)T}\Gamma^{-1}A^{(s)}\right)_{mn}$$

$$\tilde{N}^{r}_{m} = -\left(A^{(r)}\Gamma^{-1}B\right)_{m}$$

$$A^{(1)}_{mn} = -\frac{2}{\pi}\sqrt{mn}\,(-1)^{m+n}\frac{\beta\sin(m\pi\beta)}{n^2-m^2\beta^2}, \quad A^{(2)}_{mn} = \text{similar}, \quad A^{(3)}_{mn} = \delta_{m,n}$$

$$B_m = -\frac{2\alpha_3}{\pi\alpha_1\alpha_2}\, m^{-3/2}(-1)^m \sin(m\pi\beta)$$

$$\Gamma_{mn} = \delta_{mn} + \sum_{r=1,2}\left(A^{(r)}A^{(r)T}\right)_{mn}$$



$A^{(r)}$, B : overlap of Fourier basis

Essential property of $A^{(r)}$

HAVE INVERSE $D^{(r)}$

$$\sum_{r=1,2}\left(A^{(r)}D^{(r)}\right)_{mn} = \delta_{mn} \qquad \left(D^{(r)}A^{(s)}\right)_{mn} = \delta_{rs}\delta_{mn}$$

$$D^{(r)} \equiv -\frac{\alpha_3}{\alpha_r}\, C\, A^{(r)T} C^{-1} \qquad C_{mn} := m\,\delta_{mn}$$

In a sense, they are "rectangular" matrices

$$\begin{pmatrix} A^{(1)} & A^{(2)} \end{pmatrix}\begin{pmatrix} D^{(1)} \\ D^{(2)} \end{pmatrix} = 1$$

$$\begin{pmatrix} D^{(1)} \\ D^{(2)} \end{pmatrix}\begin{pmatrix} A^{(1)} & A^{(2)} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

One can compute the inverse now

$$(1-NM)^{-1} = \begin{pmatrix} (1-n^2)^{-1} & -D\, n\, (1-n^2)^{-1} \\ -D^T n\, (1-n^2)^{-1} & (1-n^2)^{-1} \end{pmatrix}$$

$$(1-n^2)^{rs} = 4 A^{(r)T} \Gamma^{-2} A^{(s)}$$

$$((1-n^2)^{-1})^{rs} = \frac{1}{4} D^{(r)} \Gamma^{2} D^{(s)T}$$

Some computations (after using this result) give

$$H_m = \frac{1}{2} a^+ M a^+ \qquad (= \text{quadratic part of } \Phi_F)$$

Determinant factor

$$\det(1-MN)^{1/2} \rightarrow C(\beta)$$

We arrive at

$$\boxed{\Phi_F(p_1^\perp, \alpha_1) * \Phi_F(p_2^\perp, \alpha_2) = C(\beta) \frac{\partial}{\partial C_0} \Phi_F(p_1^\perp + p_2^\perp, \alpha_1 + \alpha_2)}$$

$\Downarrow$ Fourier transformation for $p^\perp$

(Ishibashi state $\Rightarrow$ Cardy state)

$$\boxed{\Phi_F(x_1^\perp, \alpha_1) * \Phi_F(x_2^\perp, \alpha_2) = C(\beta)\, \delta(x_1^\perp - x_2^\perp) \frac{\partial}{\partial C_0} \Phi(x_1^\perp, \alpha_1 + \alpha_2)}$$

$C(\beta) \frac{\partial}{\partial C_0}$: universal factor for ANY Boundary state

We proved (for $\alpha_1, \alpha_2 > 0$) [KMW1, KMW2]

$$\delta^{d-p-1}(x^\perp - y^\perp)\, \mathcal{Q}\, \tilde{\tilde{\Phi}}(x^\perp, \alpha_1+\alpha_2) + g_{open}\, \tilde{\tilde{\Phi}}(\overset{x^\perp}{\alpha_1}) \star \tilde{\tilde{\Phi}}(y^\perp, \alpha_2) = 0$$

- $\mathcal{Q} \equiv K^3 \hat{\alpha}^2 c_0^+$: pure ghost BRST operator
- K: cut-off parameter ($K \to \infty$) ~ matrix size
- $\tilde{\tilde{\Phi}}(x^\perp, \alpha) \equiv -\frac{1}{g_{open}\,\alpha} \Phi(x^\perp, \alpha)$

Note: constant factor is determined in [KMW2] by using identity by Cremmer-Gervais ('75)
simplification occurs only when $\underline{d=26}$

- Superposition along transverse direction

$$\Phi_f(\alpha) \equiv \int d^{d-p-1} x^\perp\, f(x^\perp)\, \tilde{\tilde{\Phi}}(x^\perp, \alpha)$$

with

$$f(x^\perp)^2 = f(x^\perp)$$ Analogue of noncommutative soliton

$\Downarrow$

$$\boxed{\mathcal{Q}\, \Phi_f(\alpha_1+\alpha_2) + g_{open}\, \Phi_f(\alpha_1) \star \Phi_f(\alpha_2) = 0}$$

Universal relation for <u>any</u> boundary states

(in flat background)

VSFT-like scenario

Analogy is striking!

Action: $S = \frac{1}{2}\Phi\cdot\mathcal{Q}\Phi + \frac{g_{open}}{3}\Phi\cdot(\Phi\star\Phi)$

$$\mathcal{Q}^2 = 0,\quad (\mathcal{Q}\Phi_1)\cdot\Phi_2 + (-1)^{|\Phi_1|}\Phi_1\cdot(\mathcal{Q}\Phi_2) = 0$$

$$\mathcal{Q}(\Phi_1\star\Phi_2) = (\mathcal{Q}\Phi_1)\star\Phi_2 + (-1)^{|\Phi_1|}\Phi_1\star(\mathcal{Q}\Phi_2)$$

- No physical state at perturbative vacuum $\Phi = 0$
- Gauge symmetry

$$\delta_\Lambda\Phi = \mathcal{Q}\Lambda + g_{open}(\Phi\star\Lambda - \Lambda\star\Phi)$$

- E.O.M.

$$\mathcal{Q}\Phi + g_{open}\Phi\star\Phi = 0$$

- Solutions = Boundary states

$$\Phi_f(\tilde{\alpha}) = \lim_{M\to\infty}\frac{1}{2M}\int_{-M}^{M} d\alpha\, e^{i\alpha\tilde{\alpha}}\,\Phi_f(\alpha)$$

- Vanishing coefficient $\frac{1}{2M}$ can be renormalized to coupling constant
- Re-expansion from new vacuum

$$S' = \frac{1}{2}\Phi\cdot\mathcal{Q}_0\Phi + \frac{g_{open}}{3}\Phi\cdot(\Phi\star\Phi) + S(\Phi_0)$$

$$\mathcal{Q}_0\Phi = \mathcal{Q}\Phi + g_{open}\left(\Phi_0(\tilde{\alpha})\star\Phi - (-1)^{|\Phi|}\Phi\star\Phi_0(\tilde{\alpha})\right)$$

Physical state for $\mathcal{Q}_0$
= open string on D-brane

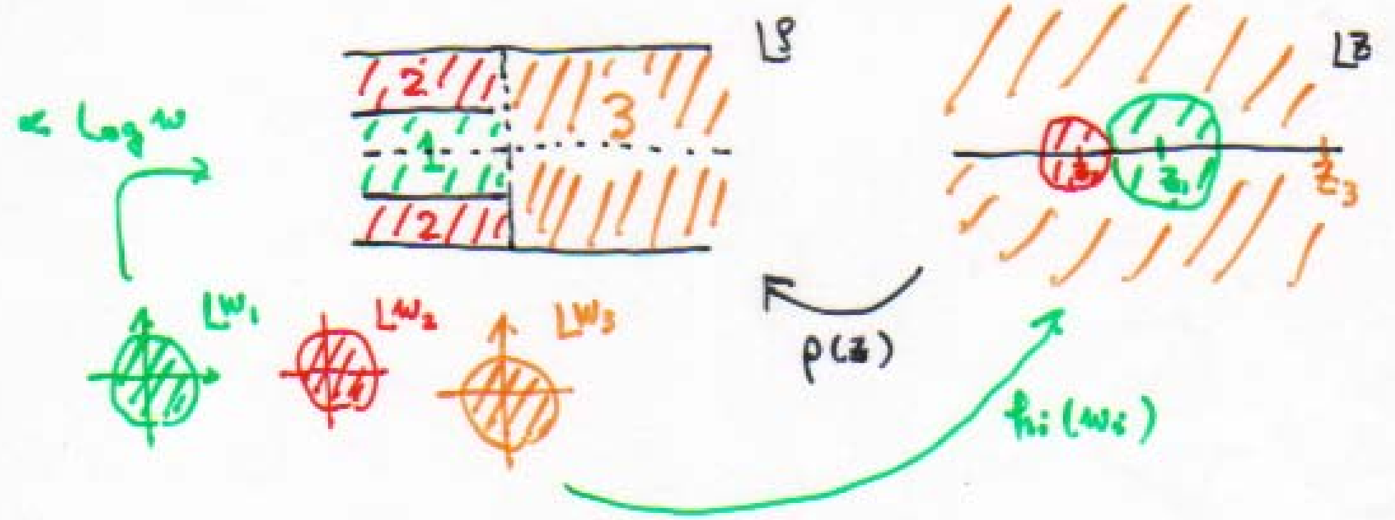
Calculation by CFT (KMW2)

LPP's definition of n-string vertex

$$\langle v_N | A_1\rangle |A_2\rangle \cdots |A_N\rangle = \langle h_1[\mathcal{O}_{A_1}] \cdots h_N[\mathcal{O}_{A_N}]\rangle$$

$$(|A_i\rangle \equiv \mathcal{O}_{A_i}(0)|0\rangle)$$

Construction of mapping



Mandelstam map

$$\rho(z) = \alpha_1 \log(z-1) + \alpha_2 \log z$$

h_r: w_r-plane $\longrightarrow$ z-plane

$$\rho(h_r(w_r)) = \alpha_r \log w_r + \tau_0 + i\pi \sum_{s=1}^{r} \alpha_s$$

HIKKO vertex

$$(\Phi_1(\alpha_1) \star \Phi_2(\alpha_2)) \cdot \Phi_3(\alpha_3) \propto \langle h_1[b_0^- \mathcal{P} \Phi_1]\, h_2[b_0^- \mathcal{P} \Phi_2]\, h_3[b_0^- \mathcal{P} \Phi_3]\rangle$$

$$\mathcal{P} \equiv \int_{-\pi}^{\pi} \frac{d\theta}{2\pi}\, e^{i\theta(L_0 - \tilde{L}_0)}$$: projector for level matching

Derivation of idempotency relation

- Use of CFT $\Rightarrow$ background independent

- We prove only weak condition

$$(L_n - \tilde{L}_{-n})|B_i\rangle = 0 \quad (i=1,2)$$
$$\Rightarrow (L_n - \tilde{L}_{-n})\left(|B_1\rangle \star |B_2\rangle\right) = 0$$

- Check of Cardy condition
  - not yet ...
  - Proved only for toroidal compactification

    Check of orbifold seems to be possible

Idea of proof

We show

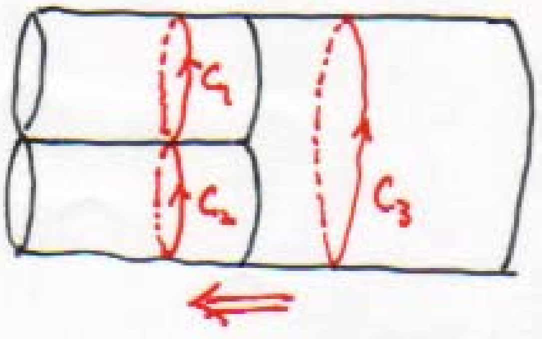
$$((L_n - \tilde{L}_{-n})\Phi)\cdot(\Phi_{B_1} \star \Phi_{B_2}) = 0 \quad \text{for } \forall \Phi$$

$$\downarrow \text{LPP}$$

$$\langle h_3[b_0^- \varphi L_n \Phi]\, h_1[b_0^+ O_{B_1}]\, h_2[b_0^+ O_{B_2}]\rangle + \cdots$$

We write $L_n \Phi = \oint_{C_3} w_3^{n+1}\, T(w_3)\, \Phi(0)\, \frac{dw_3}{2\pi i}$

in the correlator & deform the contour



- Transformation of $T(z)$

$$f\circ T(w) = \left(\frac{dz}{dw}\right)^2 T(z) + \frac{c}{12} S(z,w)$$

- Treatment of interaction point

§3. Deformation of boundary state

Question: Does idempotency relation remaining true for curved D-brane?

Infinitesimal deformation of boundary state = open string mode

$$\begin{cases} \delta\Phi_{\bullet} = \oint \frac{d\sigma}{2\pi} V(\sigma)\cdot\Phi \\ \delta\Phi(x^\perp,\alpha_1) \star \Phi(y^\perp,\alpha_2) + \Phi(x^\perp,\alpha_1) \star \delta\Phi(y^\perp,\alpha_2) \\ \quad = \delta^{d-p-1}(x^\perp - y^\perp)\, C\cdot c_0^+\, \delta\Phi(x^\perp,\alpha_1+\alpha_2) \\ \qquad \cdots\cdots (*) \end{cases}$$

[KMW1] Check of this relation for

① Scalar $V_S(\sigma) = e^{ikX(\sigma)}$

② Vector $V_V(\sigma) = (\zeta\cdot\partial X)\, e^{ikX(\sigma)}$

(*) implies

$$\begin{cases} k_\mu G^{\mu\nu} k_\nu = 2 \quad \text{for ①} \\ k_\mu G^{\mu\nu} k_\nu = 0 \quad \text{for ②} \end{cases}$$

$G^{\mu\nu} \equiv ((1+F)^{-1}(1-F))^{\mu\nu}$ open string metric

Correct open string spectrum.

Computation is rather tricky!

$$\delta_S \Phi(\alpha_1) * \Phi(\alpha_2) = \cdots = (-\beta)^{\frac{1}{2}k^2}\, C\cdot C_0^+\, \delta_S \Phi(\alpha_1+\alpha_2) \qquad \beta = \frac{\alpha_1}{\alpha_3}$$

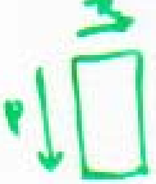
$$\Phi(\alpha_1) * \delta_S \Phi(\alpha_2) = \cdots = (1+\beta)^{\frac{1}{2}k^2}\, C\cdot C_0^+\, \delta_S \Phi(\alpha_1+\alpha_2)$$

On-shell condition

$$(-\beta)^{\frac{1}{2}k^2} + (1+\beta)^{\frac{1}{2}k^2} = 1 \iff k^2 = 2$$

How $\log|\beta|$ appears?

$$\sum_{k=1}^{\infty} \frac{1}{k} - \sum_{p=1}^{\infty} \frac{1}{p} \quad : \text{ indefinite}$$



Neumann matrix size $1 : |\beta|$ rectangular

We need subtle cut-off

"string bit"

Transversality of vector mode

Does (*) imply $\zeta\cdot k = 0$ (transversality)?

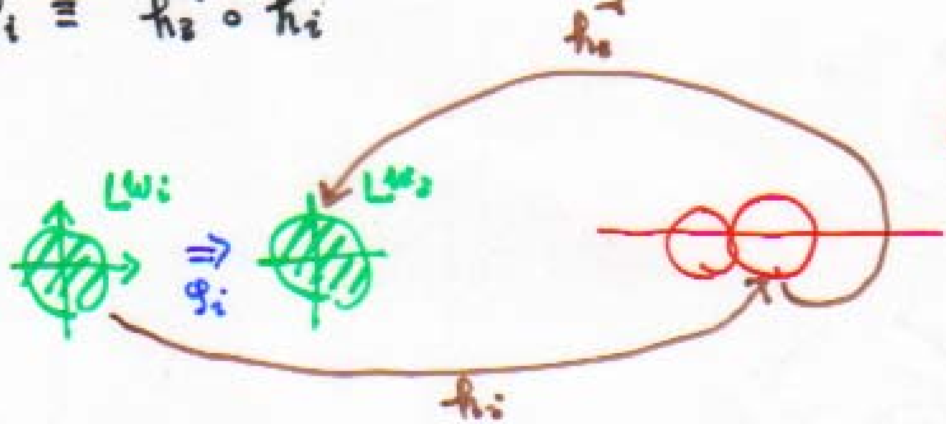
$$\delta_V \Phi * \Phi + \Phi * \delta_V \Phi - \delta_V \Phi$$
$$= \cdots + \zeta\cdot k\,(\infty \times 0)$$

We could not confirm $\zeta\cdot k = 0$ is needed.

LPP formulation

$$(*) \Longleftrightarrow \left[\oint \frac{d\sigma_1}{2\pi} \varphi_1 [V(\sigma_1)] + \oint \frac{d\sigma_2}{2\pi} \varphi_2 [V(\sigma_2)] + \oint \frac{d\sigma_3}{2\pi} V(\sigma_3) \right] |B\rangle_3 = 0$$

$$\varphi_i \equiv h_3^{-1} \circ h_i$$



Sufficient condition

= $V(\sigma)$ is marginal deformation

= open string physical state

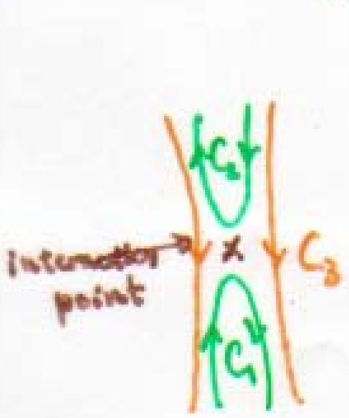
Is it necessary condition?

Example $\quad V_v(z) = :\zeta\partial X e^{ikX}: \qquad V_s(z) = e^{ikX}$

$$T(z) V_v(w) \sim \frac{-ik\cdot\zeta}{(z-w)^3} V_s(w) + \frac{\frac{k^2}{2}+1}{(z-w)^2} V_v(w) + \frac{1}{z-w} V_v(w)$$

$$\Downarrow$$

$$V_v(w)(dw)^{\frac{k^2}{2}+1} = V_v(z)(dz)^{\frac{k^2}{2}+1} + \frac{ik\cdot\zeta}{2} \left(\frac{\partial_z^2 w}{\partial_z w}\right) V_s(z)(dz)^{\frac{k^2}{2}+1}$$



interaction pt: curvature singularity

"correct coordinate" $\quad y = (z-z_0)^{1/2}$

$$\int_{C_1+C_2-C_3} V_v(z)\,dz = \frac{k\cdot\zeta}{2} V_s(z_0)$$

$\rightarrow$ $k\cdot\zeta = 0$ is needed!

§4. Summary

- We find a universal equation which characterize boundary state

$$Q\Phi(\alpha_1+\alpha_2) + g_{open}\Phi(\alpha_1)\star\Phi(\alpha_2) = 0 \quad \cdots (*)$$

- Variations around boundary state agrees with open string spectrum on D-brane

$\Rightarrow$ (*) remains true for <u>curved</u> D-brane

- VSFT-type interpretation seems possible

$$S = \frac{1}{2}\Phi\cdot Q\Phi + \frac{g_{open}}{3}\Phi\cdot(\Phi\star\Phi)$$

However, the coupling constant is

$$g_{open}\ \left(= g_{closed}^{1/2}\right)\ !$$

$\Uparrow$

This may not be the HIKKO's closed SFT at "tachyon vacuum"

Future problems

- Derivation of Veneziano amplitude

  Interpretation of α-parameter as open string moduli?

- Is there physical closed string?

- Derivation of Cardy condition

$$\underbrace{|i\rangle\rangle}_{\text{Ishibashi state}} * |j\rangle\rangle = \sum \underbrace{N_{ij}{}^{k}}_{\text{Verlinde's fusion coefficient}} |k\rangle\rangle$$

- Appearance of non-commutative geometry

$$f^2 = f \implies f * f = f$$

  flux in transverse direction?

- Explicit computation on nontrivial background

  - Torus, Orbifold, ..
  - Deformation of closed string background

- SUSY extension