SYMPOSIUM IN HONOR OF BUNJI SAKITA

Univ. of Tokyo, Komaba

Nov 28-29

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"CONSTRUCTING COLLECTIVE STRING FIELD THEORY"

INTRODUCTION

- THE METHODS of COLLECTIVE COORDINATES AND FIELDS were pioniered BY SAKITA IN APPLICATION TO NONPERTURBATIVE PHENOMENA IN RELATIVISTIC FIELD THEORY:

COLLECTIVE COORDINATES for SOLITON QUANTIZ. EVOLVED FROM EARLIER STUDIES of THE STATIC STRONG COUPLING THEORY by SAKITA BRANCO, SENTANOVIC (1973-4)

SAKITA, GERVAIS, MYSELF (1975-76)

COORDINATES FOR SOLITONS – TODAY: MODULI of D-BRANES

THEIR EXISTENCE AT QUANTUM LEVEL WAS A NONTRIVIAL AND HIGHLY STUDIED QUESTION IN EARLY 70's

• SEPARATING A COORDINATE

FROM A FIELD

$$\Phi(x,t) = \underset{\text{SOLITON}}{\phi_0(x-\hat{X}(t))} + \underset{\text{Q. FLUCTUATIO}}{\hat{\eta}(x-\hat{X},t)}$$

$$H = M_s + \frac{(\hat{P} + \int \eta' \pi)^2}{2(M_0 + \eta'^2)} + \text{FIELD TR.}$$

$$E_s = M_s + \frac{P^2}{2 M_s} + \underbrace{}_{\frac{(P^2)^2}{8 M_s}}$$

SINCE $M_s = \frac{1}{g^2}$: $E = O(\frac{1}{g^2}) + P^2 g^2 + P^4 g^4 + \ldots$

QUANTUM LEVEL : RENORMALIZATION

Gross-Callan

COL. METHOD CAPABLE OF

$$E = \sqrt{(M_0 + \Delta M)^2 + P^2}$$

FULL RELATIVISTIC FORM (WITH QUANTUM RENORM. CORRECTION)

• GAUGE THEORIES : GERVAIS, SAKITA, WADIA

• COLLECTIVE FIELDS: FOR STUDY OF LARGE N LIMIT IN FIELD THEORY

IN GENERAL BOTH METHODS REPRESENT A CHANGE OF VARIABLES

$$\{q_i\}_{i=1\cdots N} \to \phi(x;\{q\})$$

USE OF CHAIN RULE

$$\sum_i \frac{\partial}{\partial x_i}\frac{\partial}{\partial x_i} \to \left(\frac{\partial\phi(x)}{\partial x_i}\frac{\partial\phi(y)}{\partial x_i}\right)\frac{\partial^2}{\partial\phi^2} + \left(\frac{\partial^2\phi}{\partial x}\right)\frac{\partial}{\partial\phi}$$

NONTRIVIALITY IN EVALUATION OF THE JACOBIAN

$$\int dq_i = \int \prod_x d\phi \; J(\phi)$$

• SAKITA (IN LATER STUDIES AND CONVERSATIONS) EMPHASIZED THAT BOTH METHODS (coll. coord and fields) ARE BASICALLY ONE AND THE SAME:

HE WAS INSPIRED BY A PAPER OF HOSOYA AND KIKKAWA (gauge principle)

• RECENT APPLICATIONS:

• TO $d=1$ MATRIX MODEL (S. DAS + MYSELF) 1990

GAVE THE FIRST EXAMPLE OF A

MATRIX → SPACE TRANSMUTATION (AdS_2 HOLOGRAPHY)

• MANY BODY CALOGERO SYSTEMS: AWATA, MATSUO, ODAKE, SHIRAISHI, ...

• CHERN-SIMONS TH: SAKITA, ISO, KARABALI

COND. MATTER: SAKITA, SHIZUYA

• $c<1$ STRINGS: S-D SFIELD THEORY DEVELOPED BY ISHIBASHI, KAWAI, FUKUMA, ...

CAN BE RELATED THROUGH STOCHASTIC QUANTIZATION TO COLL. SUGINO + YONEYA

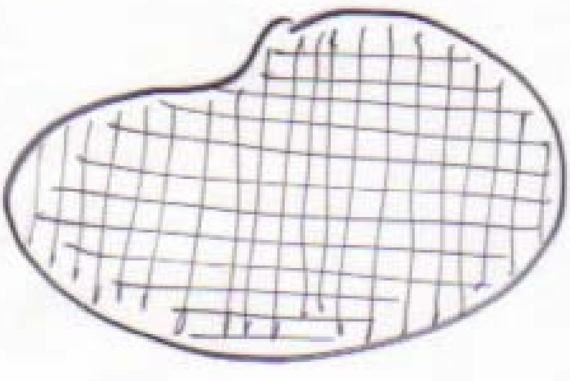
RODRIGUES + MYSELF

• 2002-2003: DERIVING CLOSED STRING FIELD THEORY FROM THE BMN LIMIT

DE MELLO-KOCH, RODRIGUES, A. DONOS + A.J.

- SO EVEN IF SUPERSYMMETRY (in HD) IS DISCOVERED SAKITA'S CONTRIBUTION AND INFLUENCE WILL REMAIN LARGE

- IT COMES BACK TO A VERY SEMINAL PAPER WRITTEN BY SAKITA + VIRASORO (1970) ON ATTEMPTING TO DERIVE DUAL MODELS FROM LOCAL FIELD THEORY



A FISHNET (PLANAR) DIAGRAM

A PRECURSOR OF OBTAINING STRINGS FROM LARGE N YANG MILLS, A SUBJECT THAT IS NOW IN FULL BLOOM

CONSTRUCTING (COLLECTIVE) STRING FIELD THEORY

JOINT WORK WITH: Robert De Mello, João Rodrigues AND Aristos Donos

THERE HAS BEEN A LONG STANDING QUEST TO DERIVE STRINGS FROM YANG-MILLS GAUGE FIELDS: LARGE N LIMIT

1970: SAKITA, VIRASORO, NIELSEN

FISHNET DIAGRAMS to GIVE DUAL MODELS

1974: 't HOOFT : PLANAR DIAGRAMS TO BE SUMMED 't Hooft : IDENTIFIED $\frac{1}{N}$ AS THE STRING COUPLING CONSTANT

$$\frac{1}{N} = g_{st}$$

1990 : FIRST WORKABLE EXAMPLE

$d=1$ MATRIX QUANTUM MECHANICS

$\longrightarrow$ $D=2$ NON-CRITICAL STRING THEORY

KAZAKOV, GROSS, KLEBANOV

POLCHINSKI DAS - A.J.

1996 : D-BRANES

$\mathcal{N}=4$ SUPER YANG MILLS THEORY FIELDS

GIVEN AN INTERPRETATION AS

D-BRANE COORDINATES

1997 : MALDACENA'S CONJECTURE

LARGE N of $\mathcal{N}=4$ SYM $\Leftrightarrow$ SUGRA II_B

$d=4$ ON

$AdS_5 \times S_5$

ADS / CFT CORRESPONDENCE (more generally)

HOLOGRAPHY : $d=4$ is A BOUNDARY

of $D=5$ ANTI DE SITTER

⋮

2002 feb : BERENSTEIN - MALDACENA - NASTASE

EXTENDED TO full 10 D STRING THEORY

2003 : $d=1$ $(c=1)$ MATRIX MODEL WAS

ALSO INTERPRETED AS A THEORY of

D-BRANES ... Verlinde, McGreevy

Klebanov, ..

Recom. • PLANE WAVE LIMIT : BERENSTEIN - MALD. - NASTASE CORRESPONDENCE

CONSIDER THE $AdS_5 \times S_5$ metric

$$ds^2 = R^2\left[-dt^2 ch\rho + d\rho^2 + sh^2\rho \, d\hat{n}_3^2\right] \quad AdS_5$$

$$+ R^2\left[d\psi^2 \cos^2\theta + d\theta^2 + \sin^2\theta \, d\hat{n}_3'^2\right] \quad S_5$$

AdS_5 : $\rho \to 0$ CENTER RATHER THEN BOUND. of AdS space

$$\rho = \frac{r}{R} \qquad R \to \infty$$

$$R^2(d\rho^2 + \rho^2 d\hat{n}_3) \to dr^2 + r^2 d\hat{n}_3$$

$$\underbrace{\vec{x} = r\hat{n} = (x_1, x_2, x_3, x_4)}_{\text{4 CARTESIAN DIMENS.}}$$

SAME IN S_5 : $\theta = \frac{y}{R} \to 0$

$$d\theta^2 + \theta^2 d\Omega_3^2 = (d\vec{y})^2$$

$$\vec{y} = (y_1, y_2, y_3, y_4)$$

- Null plane for : t, ψ

$$\tilde{x}^{\pm} = \frac{t \pm \psi}{2}$$

$$\boxed{\begin{aligned} ds^2\Big|_{11} = - dx^+ dx^- &+ \mu(\vec{x}^2 + \vec{y}^2) dx^+ \\ &+ d\vec{x}^2 + d\vec{y}^2 \end{aligned}}$$

$$* \left\{ \begin{array}{l} \vec{Z} = (\vec{x}, \vec{y}) = (x_1 x_2 x_3 x_4 , y_1 y_2 y_3 y_4) \\ \quad 8\text{ - DIMENSIONAL VECTOR} \\ SO(8) \to SO(4) \times SO(4) \end{array} \right.$$

By $F_{1234} = \text{CONST.} = F_{5678}$

- $\underline{\mathcal{N}=4}$ SUPER YANG-MILLS

$$\left.\begin{array}{l} A_\mu \\ \phi^a \\ \psi^\alpha \end{array}\right\}$$

$\phi^a \quad a = 1, 2, \cdot\ 5, 6$

Higgs

$\phi^1 + i\phi^2 = \Phi_1$

$\phi^1 - i\phi^2 = \bar{\Phi}_1$

$\phi^3 + i\phi^4 = \Phi_2$

$\phi^3 - i\phi^4 = \bar{\Phi}_2$

$\phi_5 + i\phi_6 = Z$

$\phi_5 - i\phi_6 = \bar{Z}$

IMPURITIES

Z^J

$p^+ = J$

IMPURITY #

CONSERVATION ?

NT

$J \to \infty$

CONSERVATION

- $A_\mu(x) \qquad\qquad A_\mu(t)$

$\phi^a(x) \to \phi^a(t)$

Eguchi-Kawai (HAMILTONIAN)

THE BERNSTEIN - MALDACENA - NASTASE CORRESPONDENCE ESTABLISHES A RELATIONSHIP BETWEEN

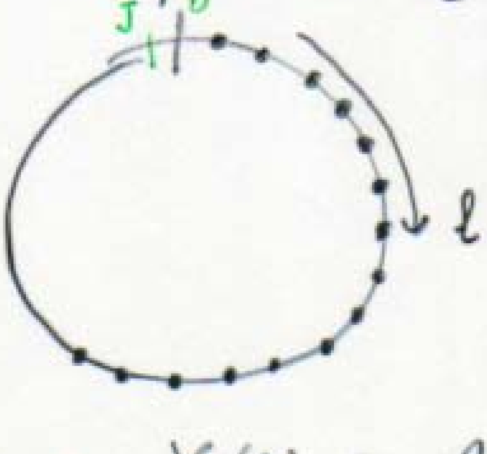
LIGHT CONE STRING THEORY ON PWAVE BACKGROUND

$$h_{BMN} = \int_0^{2\pi p^+} d\sigma \sum_a \left(P_a^2(\sigma) + (\partial_\sigma X^a)^2 + \mu (X^a)^2 \right)$$

LATTICIZE : $\sigma \to l\,\varepsilon$

$\Rightarrow \sigma \to l = 0, 1, \cdots J$

$$p^+ = \varepsilon \frac{J}{[illegible]}$$



$$X^a(\sigma) \to X_a(l) = a_a(l) + a_a^\dagger(l)$$

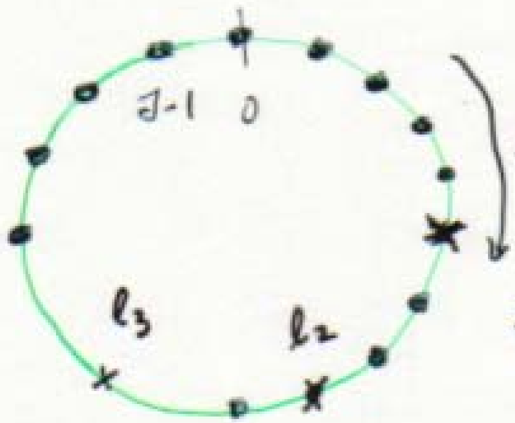
STRINGS are represented by composite degrees of freedom (OPERATO

Φ^a_{ij} - particle quanta

$$Z(x) = \Phi^5 + i\,\Phi^6$$

$$O_J(x) = \mathrm{Tr}\, Z^J(x) \qquad \text{gauge inv.}$$

$$\to \mathrm{Tr}\left(Z^{l_1} \Phi^{a_1} Z^{l_2} \Phi^{a_2} Z^{l_3} \cdots \right)$$



J - BMN lattice sites

STRING CREAT. OP.

LATTICE STRING $\qquad X^a(l) \quad l = 1 \ldots J$

BMN THEORY

- DOUBLE SCALING LIMIT of S YANG-MILLS THEORY

$$\boxed{\begin{aligned} &N \to \infty \\ &g_{YM}^2 N = \lambda \end{aligned}}$$

't Hooft's scaled coupling

$\mathrm{Tr}\, Z^J$ $\quad p = J \quad$ lattice momentum

BECOMES

P_+

$$\boxed{p^+ = \frac{J}{\sqrt{g_{YM}^2 N}} \quad \text{— FINITE}}$$

$J \to \infty$, $\sqrt{g_{YM}^2 N} \to \infty$

COLLECTIVE STRING FIELD THEORY

- Application in cond. matt.
  plasma physics
- Dynamics of large N
  systems

Change of variables (from point like) to string like:

$$\frac{\partial}{\partial \Phi}\frac{\partial}{\partial \Phi} \to \underbrace{\frac{\partial O_{J_1}}{\partial \phi}\frac{\partial O_{J_2}}{\partial \phi}}_{\mathrm{I}} \frac{\partial}{\partial O_{J_1}}\frac{\partial}{\partial O_{J_2}}$$

$$+ \underbrace{\frac{\partial^2 O_J}{\partial \phi \partial \phi}}_{\mathrm{II}} \frac{\partial}{\partial O_J}$$

B. SAKITA & A. J.

• Eguchi-Kawai type : MATRIX MODEL

LOOP SPACE IS SAME for E-K

$$H = -\frac{\partial}{\partial \bar{Z}}\frac{\partial}{\partial Z} + \bar{Z}Z + \frac{\partial}{\partial \phi_a}\frac{\partial}{\partial \phi_a} + \phi_a^2 = H_0$$

MATRIX MODEL

$$+ g_{YM}^2 \sum_{OCN} [\phi^a, \phi^b][\phi^a, \phi^b] = H_1$$

• OBSERVABLES (LOOPS)

$$O^J = \mathrm{Tr}(Z^J) \qquad \bar{O}^J = \mathrm{Tr}(\bar{Z}^J)$$

$$O^J_n = \sum \mathrm{Tr}(\Phi^n Z^J)$$

SUGRA

symmetrization

$$\bar{O}^J_n = \sum \mathrm{Tr}(\Phi^n \bar{Z}^J)$$

• Consider the simplest set

$$\boxed{O^J = \mathrm{Tr}\, Z^J}$$

Coll. hamiltonian: $H_0 = -\frac{\partial}{\partial \bar{Z}_{ij}} \frac{\partial}{\partial Z_{ji}}$

$$H_0 \to - \underbrace{\frac{\partial \bar{O}^{J_2}}{\partial \bar{Z}_{ij}} \frac{\partial O^{J_1}}{\partial Z_{ji}}} \frac{\partial}{\partial \bar{O}_{J_2}} \frac{\partial}{\partial O_{J_1}}$$

$$\underbrace{J_1 J_2 \,\mathrm{Tr}\left(\bar{Z}^{J_2-1} Z^{J_1-1}\right)}$$

Joining of loops O^{J_1} $\bar{O}^{J_2}$

New Mixed Loop

$$H_0 = \sum_{J_1, J_2} \bar{\Phi}(J_1-1, \bar{J}_2-1)\, {}_{J_2}\bar{\Pi}^{J_2}\, {}_{J_1}\Pi^{J_2}$$

• Joining and splitting generates an enormous # of new non stringy loops / challenge. meaning

FOR THE **SIMPLEST** TRACES

$$O_J = \mathrm{Tr}\left(Z^J\right) \qquad \bar{O}_J = \mathrm{Tr}\left(\bar{Z}^J\right)$$

FOR EXAMPLE ONE SEES ALMOST THE SAME JOINING / SPLITTING PROCESS AS IN OUR ONE MATRIX MODEL: THE KINETIC OP.

$$\mathrm{Tr}\left(\frac{\partial}{\partial \bar{Z}}\frac{\partial}{\partial Z}\right) \to J_1 J_2 \, \mathrm{Tr}\left(\bar{Z}^{J_1-1} Z^{J_2-1}\right) \frac{\partial}{\partial \bar{\Phi}_{J_1}} \frac{\partial}{\partial \Phi_{J_2}}$$

NEW FEATURE: A HOLOMORPHIC AND ANTIHOLOMORPHIC TRACE JOIN INTO A NEW MIXED TRACE

$$\mathrm{Tr}\left(\bar{Z}^{J_1-1} Z^{J_2-1}\right)$$

NOT IN THE BMN LIST OF PHYSICAL STATES

• RESOLUTION : SPLITTING OPERATION (FACTORIZATION)

$$\underset{J_1 > J_2}{\Phi(J_1-1, \overline{J_2-1})} = \sum_{J_3}{}' \underbrace{\Phi(J_1 - J_3, J_2)}\ \underbrace{O^{J_3}}$$

REPLACE $\Phi(J, \bar{J}) \Rightarrow \langle \quad \rangle = \phi_0(J, \bar{J})$

$$\phi_0(J, J) = \int dz\, d\bar{z}\ \mathrm{tr}(\bar{Z}^J Z^J)\, e^{-\bar{Z}Z}$$

Mixed Loops are replaced by expectation values :

$$H_0 \Rightarrow \frac{1}{N} \underbrace{\sqrt{J_1 J_2 J_3}}\ O^{J_3}\ \bar{\Pi}_{J_2}\ \Pi_{J_1}\ \delta_{J_2+J_3, J_1}$$

Effective form factor comes from ϕ_0 - expectation BACKGROUND FIELD

ANOTHER (SMALLER) QUESTION:

STILL TWICE AS MANY DEGREES OF FREEDOM IN COMPARISON WITH Light CONE SFT

$$\mathcal{O}^J = \mathrm{Tr}(Z^J) \qquad \Pi_J = \frac{\partial}{\partial \mathcal{O}^J}$$

$$\bar{\mathcal{O}}^J = \mathrm{Tr}(\bar{Z}^J) \qquad \bar{\Pi}^J = \frac{\partial}{\partial \bar{\mathcal{O}}^J}$$

$$\mathcal{O}_J = A_J + \tilde{A}_J^+ \qquad \Pi_J = -\frac{i}{2}\left(A_J^+ - \tilde{A}_J \right)$$

$$\bar{\mathcal{O}}^J = A_J^+ + \tilde{A}_J^* \qquad \bar{\Pi}_J = \frac{i}{2}\left(A_J - \tilde{A}_J^+ \right)$$

left - right moving

Infinite momentum: ONLY RIGHT MOVING

$$\mathcal{O}_J \to A_J \qquad \Pi_J \to -\frac{i}{2} A_J^+$$

$$\bar{\mathcal{O}}_J \to A_J^+ \qquad \bar{\Pi}_J \to \frac{i}{2} A_J$$

NOTICE: CONJUGATES

• YANG-MILLS INTERACTION AND STRINGY V_3 VERTEX

We need to take into account the effect of Y-M interaction term

$$H = H_0 + H_1 \qquad H_1 = g^2_{YM} \sum_{a c b} \mathrm{Tr}\left([\phi_a, \phi_b]^2\right)$$

The simplest way to take H_1 into acc. together with H_0 is to swich to coherent-st. picture right away:

$$\left.\begin{aligned} Z &= A + \tilde{A}^+ \to A \\ \bar{Z} &= A^+ + \tilde{A} \to A^+ \\ \phi &\to b_\phi \\ \psi &\to b_\psi \end{aligned}\right\}$$

matrix creation-anih. operators

$$H_0 = A\frac{\partial}{\partial A} + b\frac{\partial}{\partial b} + b\frac{\partial}{\partial b}$$

$$H_1 = g^2_{YM}\left([b,A]\left[\frac{\partial}{\partial b}, \frac{\partial}{\partial A}\right] + [c,A]\left[\frac{\partial}{\partial c}, \frac{\partial}{\partial A}\right] + [b,c]\left[\frac{\partial}{\partial b}, \frac{\partial}{\partial c}\right]\right)$$

• Mixing of 2-impurity stringy states

$$A^J = \mathrm{Tr}(\hat{A}^{+J})$$

$$A^J_{2,n} = \sum q^l \, \mathrm{Tr}\left(b^+_\phi A^{+l} b^+_\psi (A^+)^{J-l} \right)$$

$$A^J_\phi = \mathrm{Tr}(b^+_\phi A^{+J}) \quad , \quad A^J_\psi = \mathrm{Tr}\left(b^+_\psi A^{+J} \right)$$

$H_1 \to$ acting on $A^{+J_1}_m A^{+J_2}$ — or general "Fock" state of A's.

$$H_1 = g^2_{YM} N \cdot \frac{8\pi^2 n^2}{J^2} A_n \frac{\partial}{\partial A^J_n}$$ Energy correction

$$+ g^2_{YM} N \cdot \frac{1}{N} \sum_{J_1+J_2=J} \sum_m \sqrt{\frac{1-y}{y}} \left(\frac{8m}{ny-m} \right) \sin^2 \pi n y \frac{\partial}{\partial A^{J_1}_m} \frac{\partial}{\partial A^{J_2}}$$

correction to V_3

$$\text{Energy } \mathcal{E}_m = J + 8\pi^2 \lambda' n^2 + O(\lambda'^2)$$

Althogether the coherent representation of H_1 reads

$$H_1 \to \sum_a E_a \frac{\partial}{\partial A_a} A_a + g_{ym}^2 D_{m,my} A_n^{+J} A_m^{J_1} A^{J_2}$$

$$+ g_{ym}^{\sim} D_{my,m} A_n^{J} \frac{\partial}{\partial A_m^{J_1}} \frac{\partial}{\partial A^{J_2}}$$

Has to be combined with H_0

$$H_0 \to \sum_i E_i^0 \alpha_i^+ \alpha_i + \frac{1}{N} (E_j^0 + E_l^0 - E_i^0) \bar{C}_{ijl}) \alpha_j^+ \alpha_l^+ \alpha_i$$

$$+ \frac{1}{N} (E^0 - E - E) C_{ijl} \alpha_i^+ \alpha_j \alpha_l$$

with a clear (nonlinear) relation between A's and α's since

$$H_0 \to \sum_i E_i A_i \frac{\partial}{\partial A_i}$$

in the coherent rep. picture.

The TRANSFORMATION : PHYSICAL $\leftrightarrow$ COHERENT

$$A_i^+ = \alpha_i^+ + \frac{1}{4N} \bar{C}_{i;pq} \alpha_p^+ \alpha_q^+ + \frac{1}{2N} C_{p;iq} \alpha_q \alpha_p^+$$

$$\frac{\partial}{\partial A_i^+} = \alpha_i - \frac{1}{4N} C_{i;pq} \alpha_p \alpha_q - \frac{1}{2N} \bar{C}_{p;iq} \alpha_p \alpha_q^+$$

LEADING TO

$$H_0 + H_1 \to (E_i^0 + E_i^1)\, \alpha_i^+ \alpha_i$$

$$+ \left((E_j + E_l - E_i) \bar{C}_{i;jl} + D_{i;jl} \right) \alpha_j^+ \alpha_l^+ \alpha_i$$

$$+ \left((E - E - E) C_{i;jl} + D_{i;jl} \right) \alpha_i^+ \alpha_j \alpha_l$$

Total 3-vertex

- Result :

$$\left[-\tfrac{1}{2}C_{m,q}\left(n^2-\tfrac{q^2}{J^2}\right)+D_{m,q}\right]\alpha_n^{+J}\alpha_q^{J_2}\alpha^{J_1}$$

$$\left[-\tfrac{1}{2}C\,(m^2-0-0)+D_{m,}\right]\alpha_m^{+J}\,\alpha_\phi^{J_2}\,\alpha_\psi^{J_1}$$

$$\left[\tfrac{1}{2}C\left(m^2-\tfrac{p^2}{y^2}\right)+D\right]\alpha^{+}_{J_1}\alpha_p^{+J_2}\alpha_m^{J}$$

$$\left[\tfrac{1}{2}C(m^2-0-0)+0\right]\alpha_\phi^{+}\alpha_\psi^{+}\alpha_m^{J}$$

$$\frac{1}{\sqrt{J}}\sqrt{\frac{1-z}{z}}\;\sin^2\pi n z$$

$$\frac{1}{\sqrt{J}}\sqrt{\frac{1-y}{y}}\;\sin^2\pi m y$$

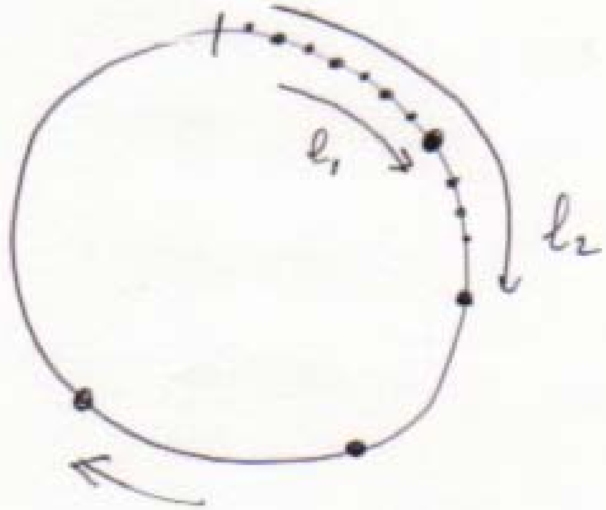
$$-\frac{1}{\sqrt{J}}\;\sin^2(\pi m y)$$

EXACT AGREEMENT WITH FORM OF STRING FIELD TH.

GENERAL STATES

OPERATORS IN SYM — STATES IN STRING THEORY

$$\mathrm{Tr}\left(T_{\ell}\,\phi_{a_1}(\ell_1)\,\phi_{a_2}(\ell_2)\cdots\phi_{a_k}(\ell_k)\,Z^J\right)$$



$$\alpha^{+}_{a_1}(\ell_1)\,\alpha^{+}_{a_2}(\ell_2)\cdots\alpha^{+}_{a_k}(\ell_k)\,|0,p^+\rangle$$

$$p^+ \leftrightarrow J$$

$$\boxed{\phi(\ell) = Z^{\ell}\,\phi\,Z^{-\ell}}$$

FULL LATTICE STRING F.T. : INTERACTION

$$\bar{\Phi}_J(\{l\}) = \mathrm{Tr}\left(\bar{Z}^J \tilde{T}_l \prod_{i=1}^{n} \bar{\phi}(l_i)\right), \qquad \bar{\phi}(l_i) = \bar{Z}^{-l_i} \bar{\phi} \bar{Z}^{l_i}. \tag{4.29}$$

$\tilde{T}_l$ is a second l ordering operator - it orders the $\bar{\phi}(l)$ factors so that l_i *decreases* from left to right. The loops are orthogonal at large N, i.e.

$$\langle \Phi_J(\{k\}) \bar{\Phi}_J(\{l\}) \rangle = \delta_{JJ} \prod_{i=1}^{n} \delta_{k_i l_i} \frac{1}{(2\mu)^{J+n}}.$$

Introduce

$$P_J(\{l\}))_{ij} = \sum_{a=1}^{J} T_l\Big(\Big[\prod_{i=1}^{n} \phi(l_i - a) Z^{J-1}\Big]_{ij}\Big) = \frac{\partial \Phi_J(\{l\})}{\partial Z_{ji}}$$

In the last formula we take $l_i - a$ mod J so that $0 \le l_i - a \le J-1$. Introduce

$$Q_J(\{l\}))_{ij} = \sum_{j=1}^{n} T_l\Big(\prod_{i=1, i\neq j}^{n} \Big[\phi(l_i - l_j) Z^J\Big]_{ij}\Big) = \frac{\partial \Phi_J(\{l\})}{\partial \phi_{ji}}$$

In the last formula we take $l_i - l_j$ mod J. At leading order in N we have

$$\langle \mathrm{Tr}\left[T_l\Big(\prod_{i=1}^{n} \phi(l_i)\Big) Z^J \bar{Z}^J \tilde{T}_l\Big(\prod_{j=1}^{\bar{n}} \bar{\phi}(\bar{l}_j)\Big)\right] \rangle = \delta_{n\bar{n}} \delta_{J\bar{J}} N^{n+J+1} \prod_{i=1}^{n} \delta_{l_i \bar{l}_i} \frac{1}{(2\mu)^{J+n}}.$$

Next, we need to study the quantity

$$\begin{aligned}
\frac{\partial \Phi_{J_1}(\{l\})}{\partial Z_{ij}} \; & \frac{\partial \bar{\Phi}_{J_2}(\{\bar{l}\})}{\partial \bar{Z}_{ji}} + \frac{\partial \Phi_{J_1}(\{l\})}{\partial \phi_{ij}} \frac{\partial \bar{\Phi}_{J_2}(\{\bar{l}\})}{\partial \bar{\phi}_{ji}} \\
&= \mathrm{Tr}\left(P_{J_1}(\{l\}) \bar{P}_{J_2}(\{\bar{l}\})\right) + \mathrm{Tr}\left(Q_{J_1}(\{l\}) \bar{Q}_{J_2}(\{\bar{l}\})\right) \\
&= \sum_{a=1}^{J_1} \sum_{b=1}^{J_2} \mathrm{Tr}\left[T_l\Big(\prod_{i=1}^{n_1} \phi(l_i - a)\Big) Z^{J_1 - 1} \bar{Z}^{J_2 - 1} \tilde{T}_l\Big(\prod_{j=1}^{n_2} \bar{\phi}(\bar{l}_j - b)\Big)\right] \\
&+ \sum_{k=1}^{n_1} \sum_{m=1}^{n_2} \mathrm{Tr}\left[T_l\Big(\prod_{i=1, i\neq k}^{n_1} \phi(l_i - l_k)\Big) Z^{J_1} \bar{Z}^{J_2} \tilde{T}_l\Big(\prod_{j=1, j\neq m}^{n_2} \bar{\phi}(\bar{l}_j - \bar{l}_m)\Big)\right]
\end{aligned}$$

We would like to derive a $d = 0$ Schwinger-Dyson equation for this quantity. It is helpful to introduce

- The kinetic term of Y-M

$$\mathrm{Tr}\left(Z\frac{\partial}{\partial Z}+\phi\frac{\partial}{\partial\phi}\right)\to\mathrm{Tr}\left(Z\frac{\partial O}{\partial Z}\right)\frac{\partial}{\partial O}$$

$$\to\quad \Psi^+\ b^+(\ell)\,b(\ell)\ \Psi$$

- Quartic (potential) term of YM

$$\underline{g_{YM}^2 N}\quad \Psi^+\left(b^+_{\ell m}b_{\ell m}-b_\ell-b_\ell\right)^2\Psi$$

$$O\leftrightarrow\Psi^+\qquad\qquad \frac{\partial}{\partial O}\leftrightarrow\Psi$$

$$\underline{g_{YM}^2 N\sim\frac{1}{\varepsilon^2}}$$

string lattice
spacing related
to 't Hooft coupling

- GOING TO GENERA "STRINGY" STATES

$$\mathrm{Tr}\left(\phi_{a_1}(l_1)\, \phi_{a_2}(l_2) \cdots \phi_{a_k}(l_k)\, Z^J \right)$$

IT IS INSTRUCTIVE TO FIRST EXIBIT THE QUADRATIC PART of SFT:

$$H_2 = \Psi^+\{(l_i)\}\, \hat{h}\, \Psi(\{l\})$$

where the single string hamiltonian is

$$\hat{h}_{BMN} \sim \sum_l b^+(l)\, b(l) + \frac{1}{\varepsilon^2} \sum_l \left(b^+_{l+1} + b_{l+1} - b^+_l - b_l \right)^2$$

REPRODUCED FROM Y-M (CHEAT - AN. BASI)

$$H_{YM} = \mathrm{Tr}\Big\{ \bar{Z} \frac{\partial}{\partial \bar{Z}} + \sum_{a=1}^{4} \bar{\phi}_a \frac{\partial}{\partial \bar{\phi}_a}$$

$$+ g^2_{YM} \mathrm{Tr}\left([\bar{\phi}, \bar{Z}]\left[\frac{\partial}{\partial \bar{\phi}}, \frac{\partial}{\partial \bar{Z}}\right] + \left[\frac{\partial}{\partial \bar{Z}}, \bar{\phi}\right]\left[\frac{\partial}{\partial \bar{\phi}}, \bar{Z}\right] \right.$$

+ 2 more terms

• SPLITING INTO CREATION - ANIHILATION OPS WE HAVE CONTRIBUTIONS FROM

$$H_4 \to -g_{YM}^2 \Big([b^+, A^+] [\tfrac{\partial}{\partial b^+}, \tfrac{\partial}{\partial A^+}] + [\tfrac{\partial}{\partial b^+}, A^+][b^+, \tfrac{\partial}{\partial A^+}]$$

$$+ [b^+, A^+][b^+, \tfrac{\partial}{\partial A^+}] + [\tfrac{\partial}{\partial b^+}, A^+][\tfrac{\partial}{\partial b^+}, \tfrac{\partial}{\partial A^+}] \Big)$$

• the FIRST TWO terms WILL CONSERVE THE # of IMP. The LAST two will NOT

CONSIDER THE FIRST (two) TERMS: SIMPLE

$$\mathrm{Tr}(b A A^+ b^+) : \mathrm{Tr}(\cdots b^+ A^+ \cdots) \Rightarrow N\,\mathrm{Tr}(\cdots b^+ A^+ \cdots)$$

$$\mathrm{Tr}(A b b^+ A^+) : \mathrm{Tr}(\cdots A^+ b^+) \Rightarrow N\,\mathrm{Tr}(\cdots A^+ b^+)$$

$$\mathrm{Tr}(b A b^+ A^+)\ \mathrm{Tr}(\cdots A^+ b^+ \cdots) \Rightarrow N\,\mathrm{Tr}(\cdots b^+ A^+ \cdots)$$

PERMUTATION

$+\,O(1)$ splitting effects

- SPIN CHAIN NOTATION

$$\mathrm{Tr}\left[b^{+}(l_1)\, b^{+}(l_2) \cdots b^{+}(l_n) (A^{+})^{J} \right] \Leftrightarrow a^{+}_{l_1} a^{+}_{l_2} \cdots a^{+}_{l_n} |0,J\rangle$$

STRING STATE

$$b(l) \equiv Z^{l}\, b\, Z^{-l}$$

$$\mathrm{Tr}\left(Z^{l_1} b^{+} Z^{l_2 - l_1} b^{+} \cdots b^{+} Z^{J - l_n} \right)$$

$$\Updownarrow$$

$$| \underbrace{Z Z \cdots Z}_{l_1}\; b^{+} \underbrace{Z \cdots Z}_{l_2 - l_1}\; b^{+} \cdots b^{+} Z \cdots Z \rangle$$

$$\Updownarrow$$

SPINS $\pm \frac{1}{2}$

$$| + + \cdots + - + + \cdots + - \quad - + + + + \rangle$$

$$\hat{h}\, | + + + - + + + - \cdots \rangle = \lambda \sum_{i \neq j} (1 - P_{ij}) | \;\rangle$$

$$\hat{h} = \frac{\lambda}{4} \sum_{\langle ij \rangle} (1 - 4 \vec{S}_i \cdot \vec{S}_j)$$

MINAHAN
-ZAREMBO

NOTE : $i = 1 \cdots J+n$

lattice

RETURNING TO OUR STRING-LIKE NOTATION

WE GET

$$\hat{h}_{col} = \lambda \sum_{l=0}^{J-1} \left(b_{l+1}^{+} b_{l+1} + b_l^{+} b_l - 2 b_{l+1}^{+} b_l - 2 b_l^{+} b_{l+1} \right.$$

$$\left. + \underbrace{b_{l+1}^{+2} + b_l^{+2} - 2 b_l^{+} b_{l+1}^{+} + b_{l+1}^{2} + b_l^{2} - 2 b_l b_{l+1}}_{\text{from impurity nonconservation}} \right)$$

$\hat{h}_{col}\big|_{CONS}$ = SPIN CHAIN

$\hat{h}_{col}\big|_{NONCONSERVING}$ MORE GENERAL CONTRIB

$$\hat{h}_{col} = \hat{h}_{BMN\,STRING} = \frac{1}{\varepsilon^2} \sum_{l=0}^{J-1} \left(b_{l+1}^{+} + b_{l+1} - b_l^{+} - b_l \right)^2$$

- NOTE: KUNZ type OSCILLATORS

$$b_l b_l^{+} - \cancel{b_l^{+} b_l} = 1$$

AS BMN have ALREADY NOTED WHEN FOURIER TRANSFORMED + $1/J$: equivalent to reg.

$\phi^n Z^J \qquad \phi Z \phi Z \phi Z$

$$\Omega \Rightarrow \quad \Phi_{J_3}(\{p\})\mathrm{Tr}\left(T[\prod_{i=1}^{n_1-n_3}\phi(q_i)]Z^{J_1-J_3-2}\bar{Z}^{J_3-2}\bar{T}[\prod_{i=1}^{n_3}\bar{\phi}(\bar{l}_i)]\right)$$

After rescaling to obtain normalized two point functions, our cubic interaction takes the form

$$\frac{2\mu}{N}\delta_{J_1-J_3,J_2}\delta_{n_1-n_3,n_2}\prod_{i=1}^{n_2}\delta_{q_i\bar{l}_i}\prod_{j=1}^{n_3}\delta_{p_j\bar{l}_{j+n_2}}\Phi_{J_3}(\{p\})\Pi_{J_1}(\{q\})\bar{\Pi}_{J_2}(\{\bar{l}\})$$

This interaction term of collective field theory matches up with the string field theory vertex. It is represented by by

$$|V\rangle = e^{\sum_{l=0}^{\alpha_1-1} a_1^\dagger(l)a_3^\dagger(l)+\sum_{l=\alpha_1}^{\alpha_3-1} a_2^\dagger(l-\alpha_1)a_3^\dagger(l)}|0\rangle.$$

Using the notation (we order the l_i's so that $l_i \geq l_j$ if $i > j$)

$$|\alpha_1,\{l_1,n_1\},\{l_2,n_2\},...\{l_N,n_N\}\rangle \equiv \frac{(a(l_1)^\dagger)^{n_1}}{n_1!}\frac{(a(l_2)^\dagger)^{n_2}}{n_2!}...\frac{(a(l_N)^\dagger)^{n_N}}{n_N!}|0,\alpha_1\rangle,$$

for a state with occupation numbers n_i at sites l_i, we find

$$\begin{aligned}
&\langle V|\alpha_1,\{l_1,n_1\},...,\{l_N,n_N\}\rangle_1|\alpha_2,\{m_1,p_1\},...,\{m_M,p_M\}\rangle_2\\
&\qquad|\alpha_3,\{q_1,r_1\},...,\{q_{N+M},r_{N+M}\}\rangle_3\\
&=\delta_{\alpha_1+\alpha_2,\alpha_3}\delta_{l_1q_1}\delta_{n_1r_1}...\delta_{l_Nq_N}\delta_{n_Nr_N}\delta_{m_1+\alpha_1,q_{N+1}}\delta_{p_1r_{N+1}}\\
&\qquad\times...\delta_{m_M+\alpha_1,q_{N+M}}\delta_{p_Mr_{N+M}}.
\end{aligned} \tag{4.30}$$

demonstrating the agreement.

RECENT RESULT: $\Rightarrow$ A. DONOS, R. DE MELLO, J. RODRIGUES, A.J. hep-th/0305042

The (loop) interaction generated from SY-Mills theory coincides with the pp wave string field theory interaction

pp wave: Exactly solvable string

$$S = \int_0^{2\pi p^+} d\tau \int d\sigma \left(\sum_{i=1}^{8} P_i(\sigma)^2 + \underbrace{\mu \vec{X}^2}_{\text{mass term}} + (\partial_\sigma \vec{X})^2 \right)$$

Gives the 3-string vertex

$$|V_3\rangle = \hat{P}\; e^{\frac{1}{2}\hat{\alpha}^r_m N^{rs}_{nm} \hat{\alpha}^s_m} |0\rangle_{123}$$

Defined in the 3-string Hilbert space

$$\alpha^r_n \qquad r = 1, 2, 3$$

n ← modos

- Prefactor: insertion at interaction point

Shown in 0305042 (Susy)

$$\hat{P} = E_3 - E_1 - E_2 + \alpha^3(\sigma_0)' \,\alpha^{(2)}(\sigma_0)' + \alpha^3(\sigma_0)\,\alpha^{(2)}(\sigma_0)$$

- CONCLUSION

- WE NOW HAVE SEVERAL FAIRLY WELL UNDERSTOOD CASES OF FULL GAUGE TH. / STRING THEORY CORRESPONDENC[E]

The SIMPLEST 1 MATRIX $d=1$: VARIOUS 2D STRING THEORIES

$\mathcal{N}=4$ $d=4$ SUPER YM : STRING TH. IN pp WAVE BACKGROUN[D]

- CONSTRUCTION OF CLOSED SFT

- EXPECTED TO EXTEND TO FURTHER EXAM[PLES]

- HALF $SO(4) \times SO(4)$

$$\begin{array}{ccc} \uparrow & & \uparrow \\ D_\mu \phi & & \phi^a \text{ Higgs } \checkmark \end{array}$$

- NOTE: The Light-Cone pp string theory has 8 transverse coordinates

$$\hat{i} = (\underbrace{1, 2, 3, 4}, \underbrace{5, 6, 7, 8})$$

It is the later 4 that were succesfully recovered presently. Work in progress to reconstruct the other 4 $(1, 2, 3, 4)$.

- One is not that far from fulfilling the longheld goal of deriving string theory from Y.R.

OUTSTANDING QUESTIONS

EXTENSION TO COMPLETE TREATMENT OF VECTOR IMPURITIES $D_\mu Z$ $SO(4)'$

SYMMETRY $SO(4) \times SO(4)'$ NOT OBVIOUS IN YANG-MILLS THEORY

QUESTION OF HOLOGRAPHY: AdS BOUNDARY

SEE DOBASHI, SHIMADA, YONEYA

ALSO YONEYA & STRINGS 2003

OTHER BACKGROUNDS

NO SUPERSYMMETRY