Effects of spacetime noncommutativity on the angular power spectrum of the cosmic microwave background (CMB)

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§1. Introduction

— Angular power spectrum C_ℓ of the CMB

§2. Mechanism of the large-scale damping due to the noncommutativity in short distance scale

§3 Example of the large-scale damping — fuzzy sphere

§4 Holographic interpretation of the large-scale damping

§5 Conclusion and outlook

§1. Introduction

- Inflationary model of universe

  gives a desired initial condition in the big bang cosmology.

  ⇒ · Vacuum fluctuations of an inflaton field give all of the global structure of the universe.
  (small anistropy)

- String theory

  is the most promising candidate for a unified theory including quantum gravity.

  ⇒ Existence of the minimal length scale L_s $(\sim O(l_s))$

  ⇒ Space-time is expected to loose smooth Riemannian structure in short distance scale, and will be noncommutative there.

- Main aim of this talk is to show that

  a discrepancy between the recent observational data on the CMB (COBE, WMAP, ⋯) and the prediction from standard inflationary models can be explained as an effect coming from the space-time noncommutativity in the very early universe.

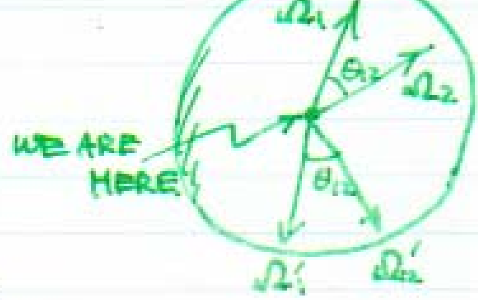
Review of CMB

- CMB : Planck distribution with $T \simeq 2.725\,\mathrm{K}$ $(\pm 0.002\,\mathrm{K})$
- anisotropy $\left(\frac{\delta T}{T} \sim 10^{-5}\right)$:

angular power spectrum

$$\left\langle \frac{\delta T}{T}(\Omega_1)\frac{\delta T}{T}(\Omega_2)\right\rangle = \sum_{\ell=1}^{\infty} \frac{2\ell+1}{4\pi} C_\ell\, P_\ell(\cos\theta_{12}) \qquad (\Omega = (\theta, \varphi))$$

$(|C_\ell| \sim 10^{-5})$

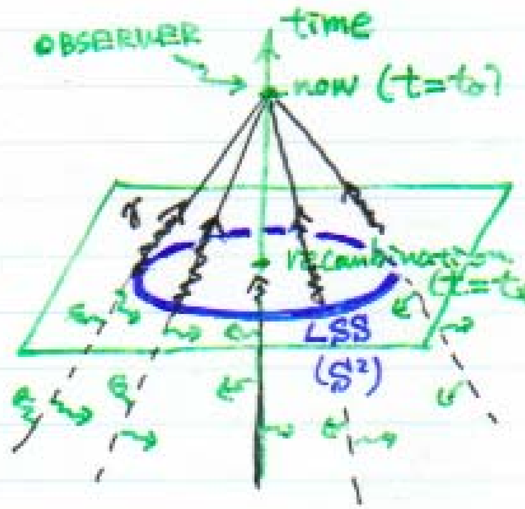


where

$\langle\ \rangle$: sample average over the data taken from various parts in the celestial sphere.

- photons we observe are coming from the last-scattering surface (LSS) where the recombination occurs.

  (Before that $(t < t_*)$, photons are scattered by free electrons and don't reach us.)

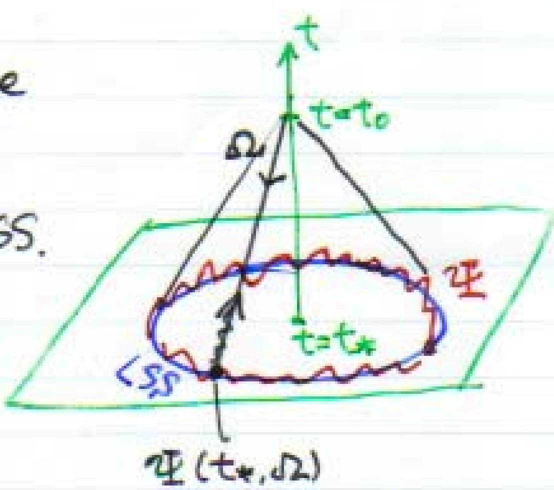


- origin of anisotropy

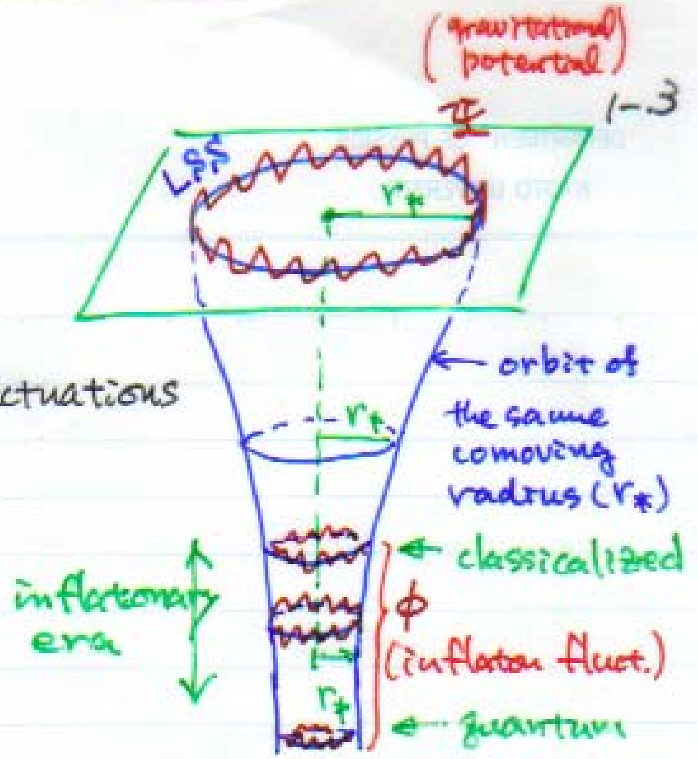
  δT is the redshift of temperature due to the fluctuations of gravitational potential Ψ on the LSS.

  $$\frac{\delta T}{T}(t_0, \Omega) = \frac{1}{3}\Psi(t_*, \Omega)$$

  : Sachs-Wolfe effect



Furthermore,
the conservation law in the cosmological perturbation theory relates Ψ to the classicalized fluctuations of inflaton field within the inflationary era.



$\Downarrow$

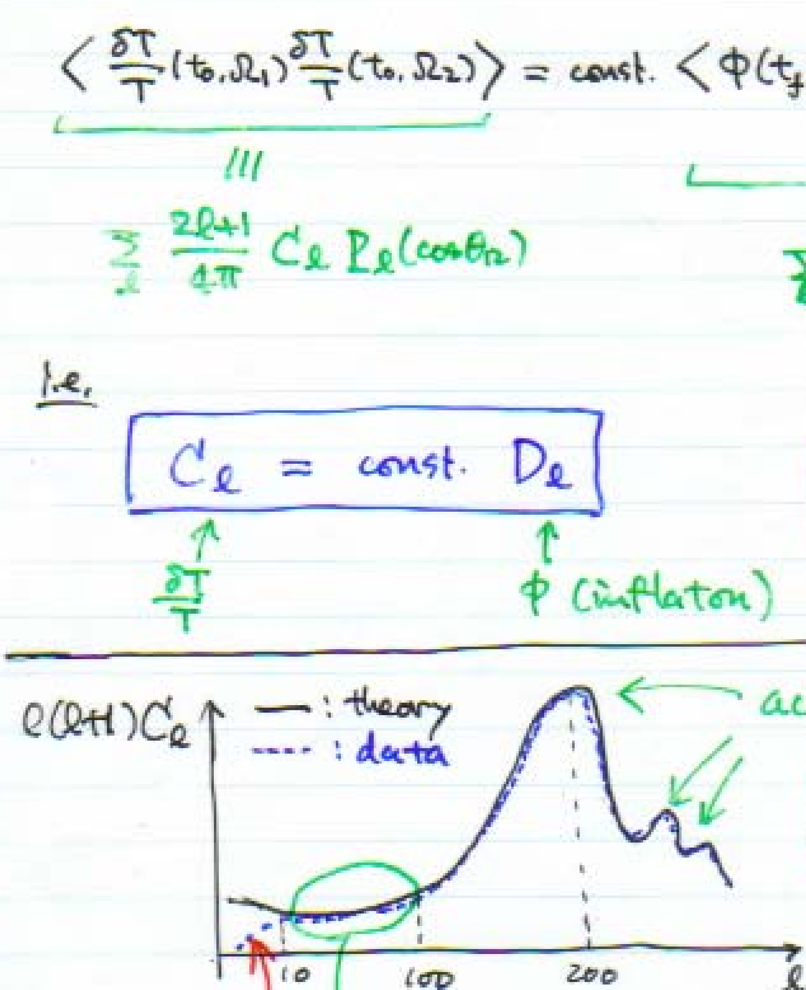
$$\underbrace{\left\langle \frac{\delta T}{T}(t_0,\Omega_1)\frac{\delta T}{T}(t_0,\Omega_2)\right\rangle}_{\displaystyle \sum_l \frac{2l+1}{4\pi} C_l P_l(\cos\theta_{12})} = \text{const.}\ \underbrace{\left\langle \phi(t_f,r_*,\Omega_1)\,\phi(t_f,r_*,\Omega_2)\right\rangle}_{\displaystyle \sum_l \frac{2l+1}{4\pi} D_l P_l(\cos\theta_{12})}$$

(t_f : the exit time of inflation)

i.e.

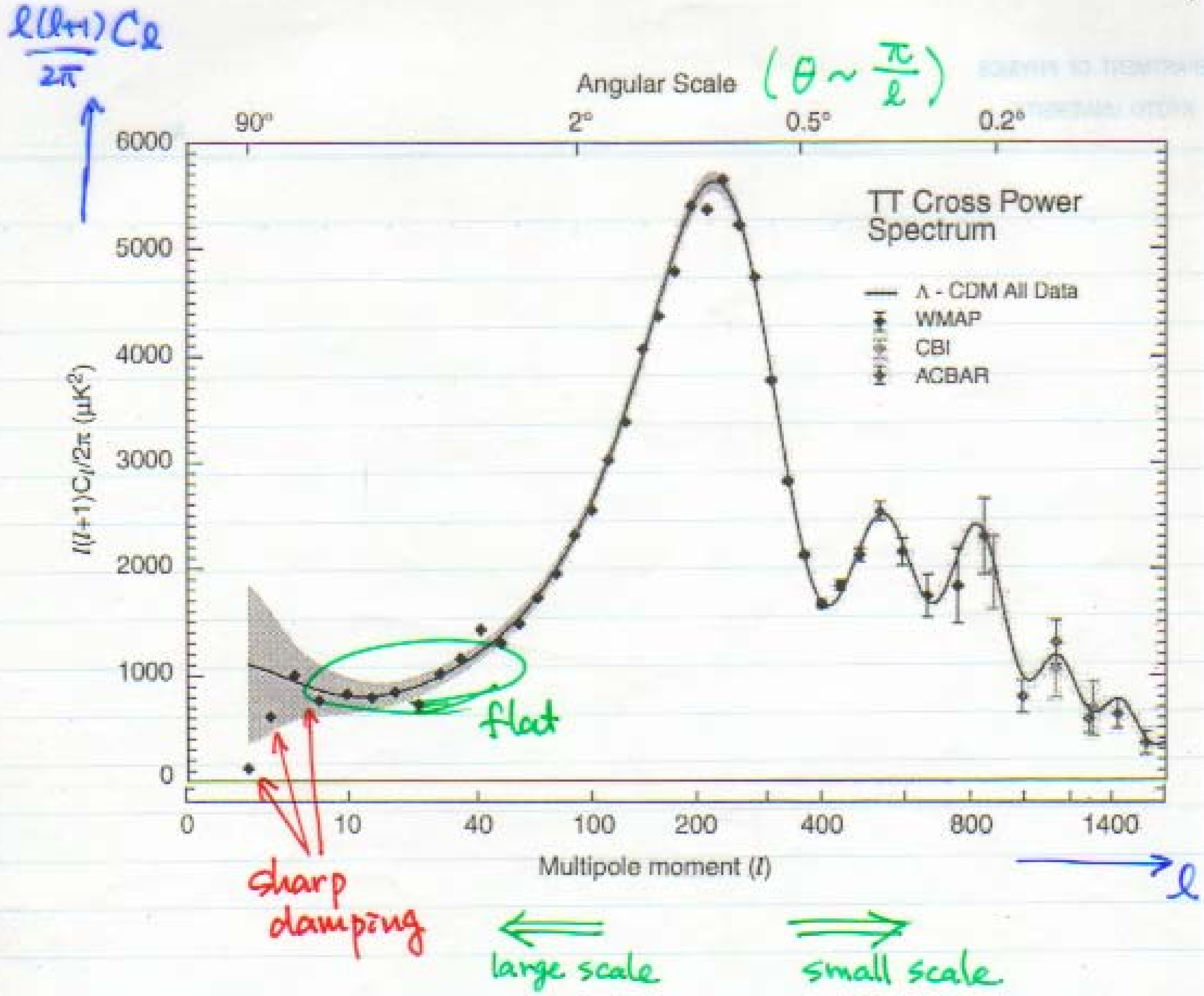
$$\boxed{C_l = \text{const.}\ D_l} \qquad (\Rightarrow \text{figure})$$

$\uparrow \frac{\delta T}{T}$ $\qquad$ $\uparrow \phi$ (inflaton)



acoustic peaks
(resonance due to the strong binding between photons and baryons.)
$\Downarrow$
determine cosmological parameters

- $\boxed{\text{almost flat}}$
  $\rightarrow$ strong evidence for inflationary models
  ($\because$ $D_l \propto 1/l(l+1)$ as we see later)
- $\boxed{\text{sharp damping for } l<10}$
  $\rightarrow$ difficult to explain by standard inflationary models without fine-tunings



$$\left\langle \frac{\delta T}{T}(\Omega_1)\,\frac{\delta T}{T}(\Omega_2)\right\rangle = \sum_{\ell=1}^{\infty} \frac{2\ell+1}{4\pi}\, C_\ell \cdot P_\ell(\cos\theta_{12})$$

- <u>Conventional explanation for the sharp damping</u>

"<u>Cosmic variance</u>" : lower ℓ $\quad$ $(\#(\text{indep}) = 2\ell+1)$

$\Rightarrow$ fewer independent samples

$\Rightarrow$ statistical mean value based on fewer samples can deviate largely from the real (theoretical) value.

$$\left(\frac{\Delta C_\ell}{C_\ell} \sim \sqrt{\frac{2}{2\ell+1}} \right)$$

But, (data possible) $\lesssim 0.3\%$

- <u>our claim</u> :

(large-scale damping)

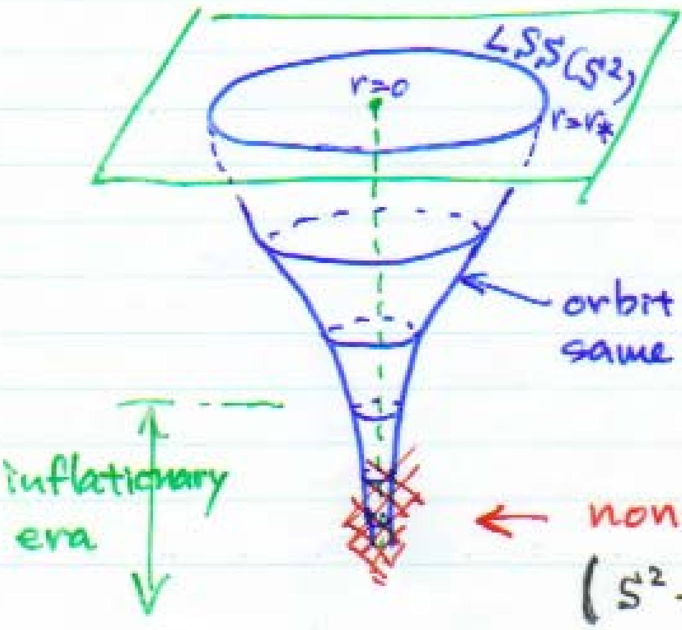
$$\boxed{\begin{array}{l}\text{This sharp damping for } \ell<10 \text{ could be understood}\\ \text{as a generic feature to hold for the CMB}\\ \underline{\text{If}} \text{ we assume the noncommutativity}\\ \text{in the very early universe.}\end{array}}$$

In this talk,

noncommutativity is introduced only to angular coordinates Ω $\;(= (\theta,\varphi))$

because their noncommutativity would be most relevant to the deviation of C_ℓ from the standard value.

$(\ell(\ell+1)C_\ell : \text{const})$



(r_*: comoving radius of LSS)

orbit of the same comov. coord ($r=r_*$)

$\leftarrow$ noncommutative

($S^2 \rightarrow$ fuzzy sphere)

NB noncommutativity in string theory

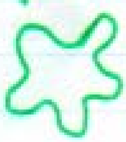
- string theory

  $\Rightarrow \exists$ minimal length l_s

  $\Rightarrow |\Delta X| \gtrsim L_s \quad (L_s = O(l_s))$

- string probe of energy E : [Gross-Mende]

$$|\Delta X| = \underbrace{\frac{1}{E}}_{\substack{\text{particle-like} \\ \text{resolution}}} + \underbrace{l_s^2 E}_{\text{stretching}} \gtrsim l_s$$



- spacetime uncertainty principle [Yoneya, Jevicki-Yoneya]

$$\Delta T \cdot \Delta X \gtrsim l_s$$

- There is a classical soln which exibits the noncommutativity. [Seiberg-Witten, ...]

However, in generic cases the NC would appear as a result of (possibly very complicated) dynamics of strings.

§2 Mechanism of the large-scale damping due to the noncommutativity in short distance scale

Review of inflation

· flat FRW metric

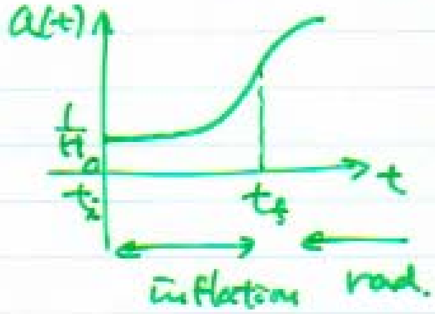
$$\boxed{ds^2 = -dt^2 + a^2(t)\, d\vec{x}^2}$$

$\left(\begin{array}{l} \vec{x} = (x^1, x^2, x^3) : \text{comoving coord.} \\ a(t) : \text{scale factor} \end{array} \right)$

$$= g_{\mu\nu}(x)\, dx^\mu dx^\nu$$

· During inflation

$$\boxed{a(t) = \frac{1}{H} e^{H(t-t_i)}}$$



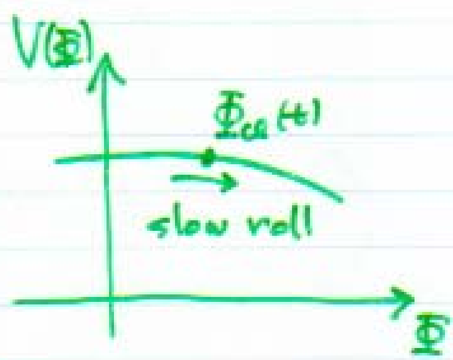
· inflaton field $\Phi(x) = \Phi(t, \vec{x})$

$$S[\Phi(x)] = \int d^4x \sqrt{-g} \left[-\frac{1}{2} g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi - V(\Phi) \right]$$

· Gaussian fluctuation around a slow-rolling soln :

$$\Phi(t, \vec{x}) = \Phi_{cl}(t) + \phi(t, \vec{x})$$

$\uparrow$ slow-rolling soln $\qquad \uparrow$ fluctuation



$$\Downarrow$$

$$S[\Phi = \Phi_{cl} + \phi] \cong S[\Phi_{cl}] + S_2[\phi]$$

$$\boxed{S_2[\phi] = \int d^4x \sqrt{-g} \left[-\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi} \cancel{- \frac{1}{2} a^2 V''(\Phi_{cl}) \cdot \phi^2} \Big]$$

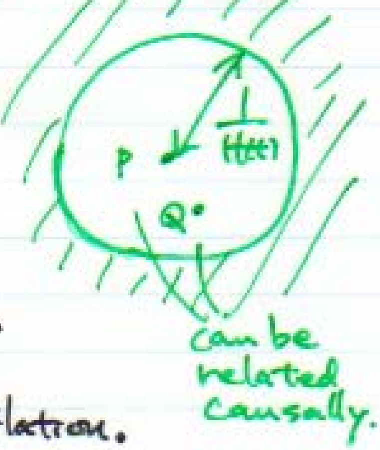
neglected in the slow-roll approx.

: describes a free scalar field in the de-Sitter spacetime.

- Hubble parameter

$$H(t) \equiv \frac{\dot{a}(t)}{a(t)} \quad (= \text{const during inflation})$$

$$\Downarrow$$



- Hubble length

$$\boxed{\frac{1}{H(t)}}$$

- distance scale within which points can be causally related.
- constant in time during inflation.

(proper)
- physical wave length $\quad \left(\phi(t,\vec{x}) = \sum_{\vec{k}} e^{i\vec{k}\vec{x}} \phi_{\vec{k}}(t) \right)$

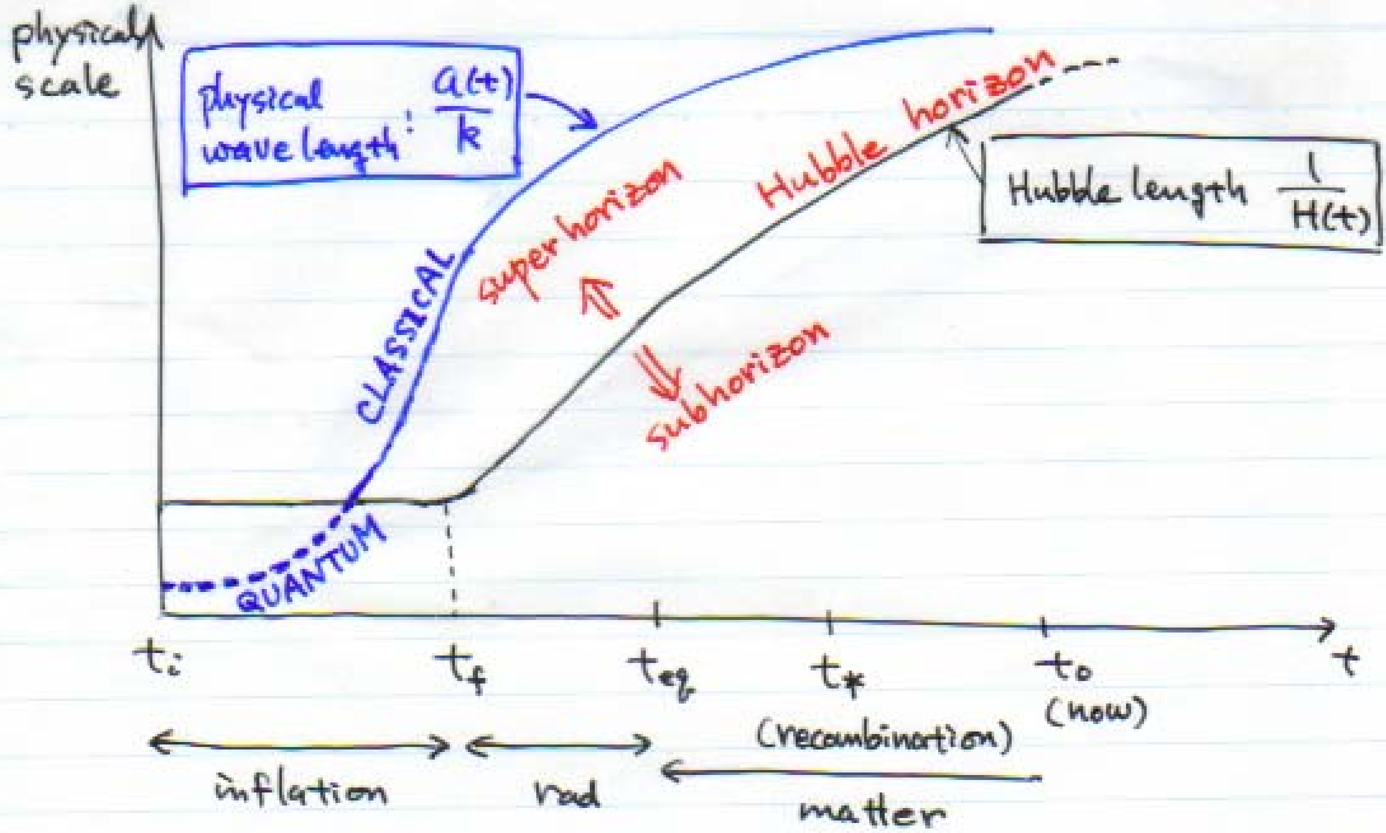
For a quantum mode of comoving wave number $\vec{k}$, its physical wave length is given by

$$\boxed{\frac{\lambda_{phys}}{2\pi} = \frac{a(t)}{k}} \quad (k = |\vec{k}|) \Leftarrow \text{time-dependent}$$

$$\Downarrow$$

$\frac{a(t)}{k} \lesssim \frac{1}{H(t)}$	$\Rightarrow$	The mode $\phi_{\vec{k}}(t)$ can fluctuate quantum mechanically.
$\frac{a(t)}{k} \gtrsim \frac{1}{H(t)}$	$\Rightarrow$	The mode $\phi_{\vec{k}}(t)$ is frozen to a classical value. ("classicalization")

Pictorially.

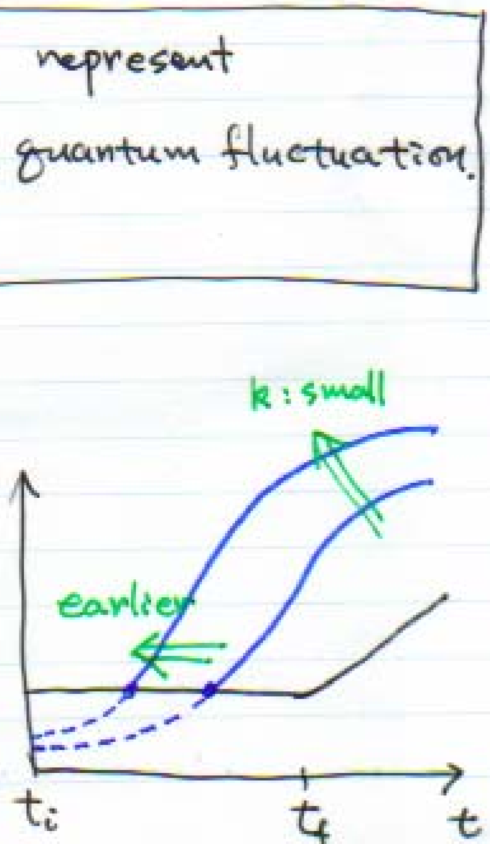


NB

- Inflation models assume that all anisotropies observed are coming from vacuum fluctuations of a scalar field(s). (inflaton)
  In particular,

$$\boxed{\begin{array}{l}\text{the anisotropies in the \underline{superhorizon} represent} \\ \text{the classicalized fluctuations of a quantum fluctuation.} \\ \text{("fossil of a quantum life")} \\ \text{(化石)}\end{array}}$$

- Larger-scale modes ($\Leftrightarrow$ smaller k) cross the Hubble horizon earlier.



$\boxed{\text{mechanism of the large-scale damping}}$ (M.F.-Kono-Miwa)

· STEP 1

· Mode expansion of the inflaton field

$$\phi(t,\vec{x}) = \sum_A \left(a_A \psi_A(t,\vec{x}) + a_A^\dagger \psi_A^*(t,\vec{x}) \right) \qquad \left(\begin{matrix}\text{e.g.}\\ A = \vec{k}\end{matrix}\right)$$

$$\left(\begin{matrix} \psi_A(t,\vec{x}) : \text{normalized positive energy soln} \\ [a_A, a_B^\dagger] = \delta_{AB} \end{matrix} \right)$$

We introduce the ordering in $\{A\}$ s.t.

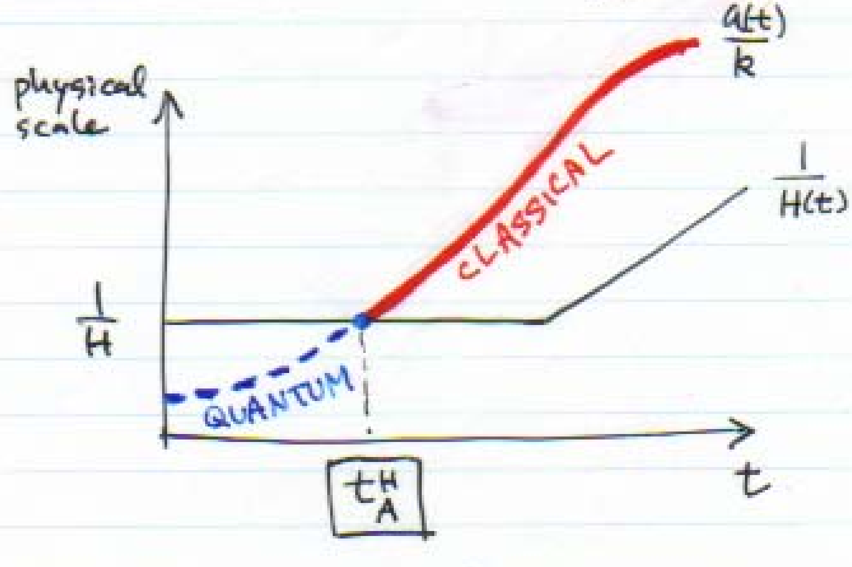
$$A < B \Rightarrow A : \text{larger scale than } B \qquad \left(\begin{matrix}\text{e.g.}\\ \vec{k} < \vec{k}' \\ \text{if } |\vec{k}| < |\vec{k}'|\end{matrix}\right)$$

Setting

$$\boxed{t_A^H : \text{the moment at which the mode } A \text{ crosses the horizon,}}$$

we have

$$\boxed{\begin{matrix} t \lesssim t_A^H : \text{quantum} \\ t \gtrsim t_A^H : \text{classical} \end{matrix}}$$



STEP 2

In a noncommutative spacetime,

there exists a cutoff on the modes of higher A.

(short distance)

In the expanding universe,

the cutoff can be a monotonically increasing function of time:

(to be discussed later)

$$\boxed{A \leq N(t)}$$

monotonically increasing fcn of t

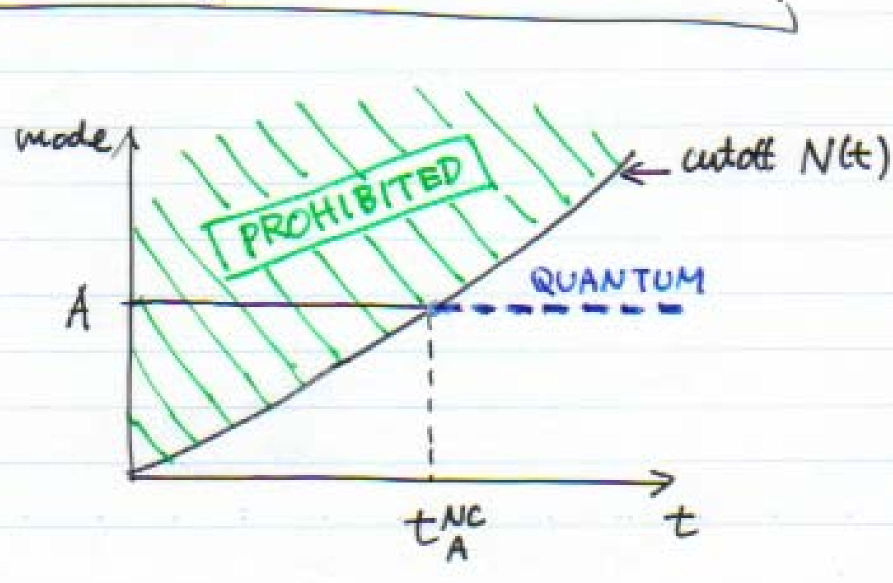
Setting

t_A^{NC} : the moment at which $A = N(t_A^{NC})$,

we have

$t < t_A^{NC}$: NO quantum fluctuation

$t \geq t_A^{NC}$: $\exists$ quantum fluctuation



• STEP 3

Suppose that there exists $A = A_c$ s.t.

$$t^H_{A_c} = t^{NC}_{A_c}$$

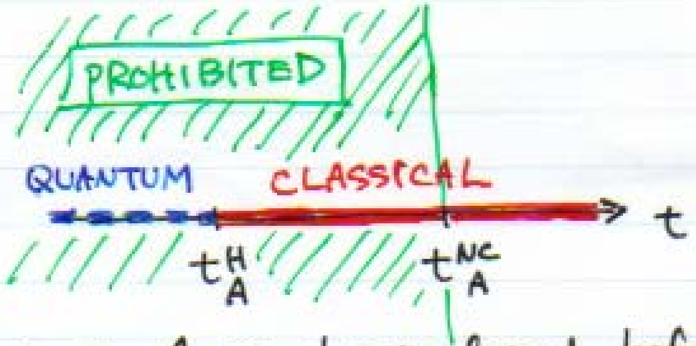
and that

$$\boxed{A \lesseqgtr A_c \iff t^H_A \lesseqgtr t^{NC}_A}$$

holds.

$$\Downarrow$$

(1) $\boxed{A < A_c}$ (larger scale)



The mode A is classicalized before its quantum fluct. begins.

$\Rightarrow$ $\boxed{\text{classical fluct.} = 0}$ ("No quantum life, no classical fossil.")

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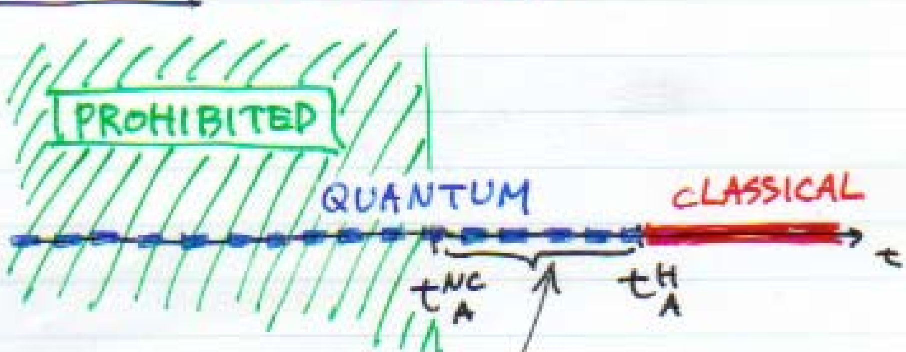
(2) $\boxed{A > A_c}$ (short scale)

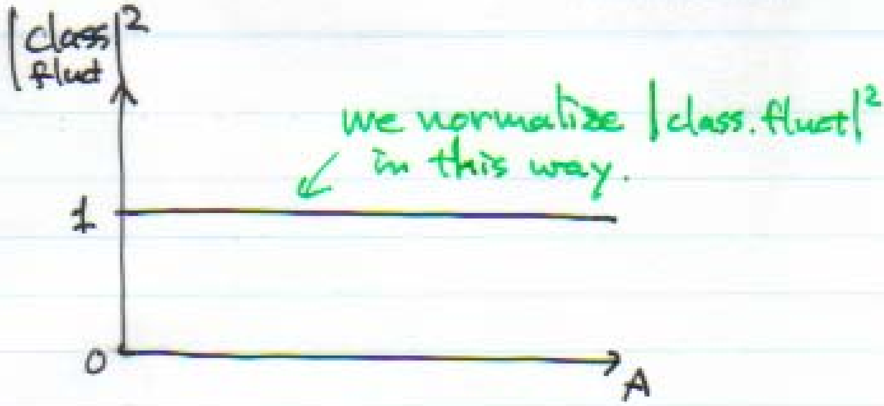


There can exist a period during which the mode A can fluctuate quantum mechanically.

$\Rightarrow$ $\boxed{\text{classical fluct.} \neq 0}$ ("Once having a quantum life, a fossil can be created.")

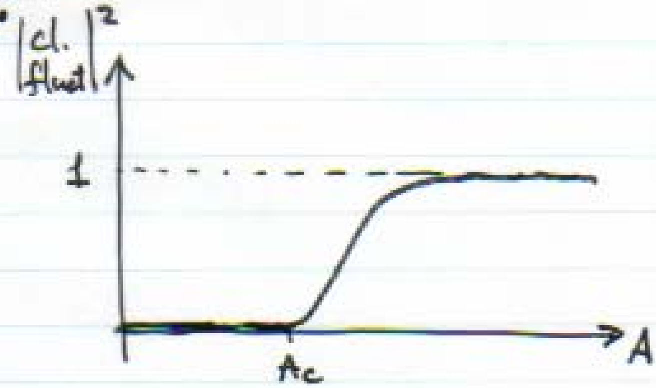
Schematically, (the classicalized fluct.)^2 should behave as follows:

• commutative case:



$\Downarrow$

• noncommutative case:

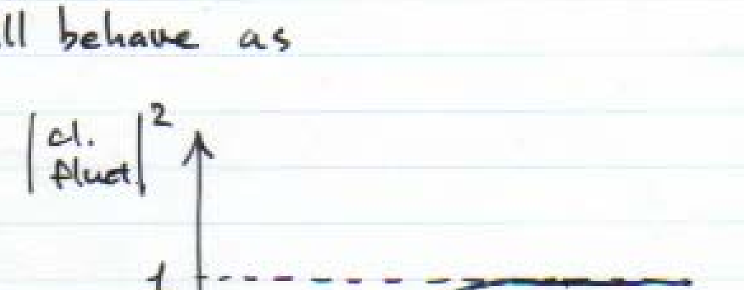


$\Downarrow$

In general, the classicalization proceeds only gradually around $t = t^H_A$.

$\Downarrow$

|The class. fluct.|^2 will behave as



$\Uparrow$

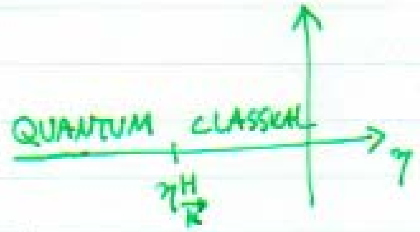
large-scale damping!

NB "classicalization"

$$ds^2 = a^2(\eta)\left(-d\eta^2 + d\vec{x}^2\right)$$
$$= \left(-\frac{1}{H\eta}\right)^2\left(-d\eta^2 + d\vec{x}^2\right) \quad (\eta<0$$

$$\phi(\eta,\vec{x}) = \int d^3\vec{k}\,\left(a_{\vec{k}}\,\psi_{\vec{k}}(\eta,\vec{x}) + a^{\dagger}_{\vec{k}}\,\psi^{*}_{\vec{k}}(\eta,\vec{x})\right) \quad (A=\vec{k})$$

with
$$\psi_{\vec{k}}(\eta,\vec{x}) = \frac{H}{\sqrt{2\cdot k^3}}\,(1+ik\eta)\,e^{-ik\eta}\,e^{i\vec{k}\cdot\vec{x}}$$



After crossing the Hubble horizon,

$$\eta \gg \eta^H_{\vec{k}} = -\frac{1}{k} \quad \left(\text{i.e. } |k\eta| \ll 1\right)$$

we have
$$\begin{cases} \psi_{\vec{k}}(\eta,\vec{x}) \to \dfrac{H}{\sqrt{2k^3}}\,e^{i\vec{k}\cdot\vec{x}} \\ \psi^{*}_{\vec{k}}(\eta,\vec{x}) \to \dfrac{H}{\sqrt{2k^3}}\,e^{-i\vec{k}\cdot\vec{x}} \end{cases}$$

$\therefore$
$$\phi(\eta,\vec{x}) \longrightarrow \int d^3\vec{k}\,\frac{H}{\sqrt{2k^3}}\,\underbrace{\left(a_{\vec{k}} + a^{\dagger}_{-\vec{k}}\right)}_{\equiv\, c_{\vec{k}}}\,e^{i\vec{k}\cdot\vec{x}}$$

The coefficients
$$c_{\vec{k}} \equiv a_{\vec{k}} + a^{\dagger}_{-\vec{k}}$$

satisfy <u>commuting</u> algebra:
$$\begin{cases} [c_{\vec{k}}, c^{\dagger}_{\vec{k}'}] = \underline{0} \\ [c_{\vec{k}}, c_{\vec{k}'}] = [c^{\dagger}_{\vec{k}}, c^{\dagger}_{\vec{k}'}] = 0 \end{cases}$$

§3 Example of the large-scale damping — fuzzy sphere

- flat FRW metric with conformal time:

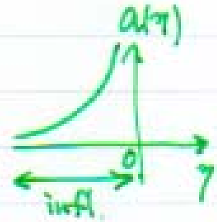
$$\boxed{\begin{aligned} ds^2 &= -dt^2 + a^2(t)\, d\vec{x}^2 \\ &= a^2(\eta)\left(-d\eta^2 + d\vec{x}^2\right) \\ &= a^2(\eta)\left(-d\eta^2 + dr^2 + r^2 d\Omega^2\right) \end{aligned}}$$

η : conformal time $\left(\frac{dt}{a} = d\eta\right)$

$\Omega = (\theta, \varphi)$

During inflation,

$$a = \frac{1}{H} e^{H(t-t_i)} \quad \Rightarrow \quad \boxed{a(\eta) = -\frac{1}{H\eta} \quad (\eta < 0)}$$



($\eta = -0$: exit time of inflation)

- Noncommutative inflationary universe

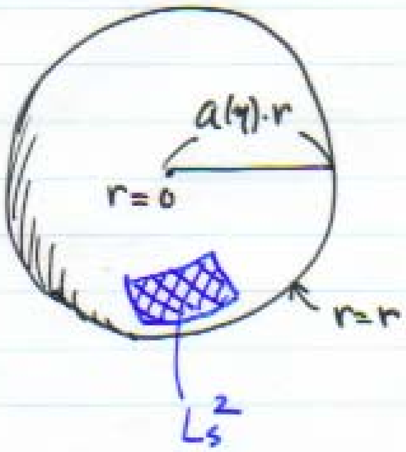
— In order to investigate the effects of NC on the angular power spectrum,

> we introduce the noncommutativity
> only to the angular coordinates $\Omega = (\theta, \varphi)$.

[M.F. - Kono - Miwa]

$$\left(ds^2 = \left(-\frac{1}{H\eta}\right)^2 \left(-d\eta^2 + dr^2 + r^2 \underbrace{d\Omega^2}_{\text{NC}}\right) \right)$$

> For each (η, r), we require that the sphere is fuzzy such that there are at most 1-bit degrees of freedom per <u>physical</u> (proper) Planck area $(L_s)^2$

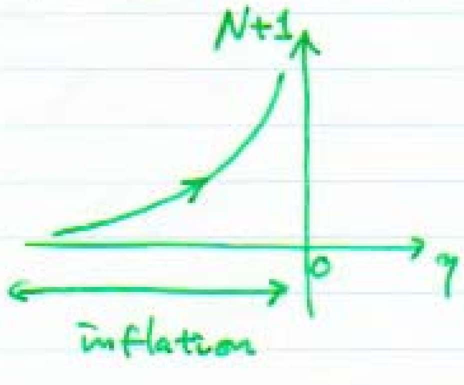


$\therefore$ The maximum degrees of freedom on the sphere of comoving radius r is given by

$$\boxed{\frac{4\pi\,(a(\eta)\,r)^2}{L_s^2} = \frac{4\pi r^2}{(L_s H)^2\cdot\eta^2} \equiv N(\eta,r)+1}$$

$(a(\eta) = -1/H\eta)$

This is actually a monotonically increasing function of η.



On this fuzzy sphere, the Hilbert space is $(N+1)$-dimensional.

$\Downarrow$

Operators are expressed by $(N+1)\times(N+1)$ (hermitian) matrices.

- Natural basis

$\hat{Y}_{lm}$: truncated spherical harmonics

$(l=0,1,\cdots,\underline{N}\ ;\ m=-l,\cdots,l)$

[construction]

1) We assume that the coordinates x_i $(i=1,2,3)$ on the fuzzy sphere satisfy the following NC algebra:

$$\begin{cases} [\hat{x}_i, \hat{x}_j] = i\theta\epsilon_{ijk}\hat{x}_k \\ \sum_{i=1}^{3}(\hat{x}_i)^2 = r^2\hat{1} \end{cases}$$

2) Setting

$$\hat{J}_i = \theta^{-1}\hat{x}_i, \quad \text{and} \quad \theta^2 = \frac{4r^2}{N(N+2)},$$

we have

$$\begin{cases} [\hat{J}_i, \hat{J}_j] = i\epsilon_{ijk}\hat{J}_k \\ \sum_{i=1}^{3}(\hat{J}_i)^2 = \frac{N}{2}\left(\frac{N}{2}+1\right)\hat{1} \quad \left(= \frac{r^2}{\theta^2}\hat{1}\right) \end{cases}$$

$\Downarrow$

$\{\hat{J}_i\}$ gives spin-$\frac{N}{2}$ representation of $SU(2)$

$\left(\dim = 2\cdot\frac{N}{2}+1 = N+1\right)$

$\Downarrow$

$\hat{x}_i = \theta\hat{J}_i$ is an $(N+1)\times(N+1)$ matrix.

3) Write commutative spherical harmonic $Y_{lm}(\Omega)$ in the form

$$Y_{lm}(\Omega) = \frac{1}{r^l} \sum_{\{i_k\}} \underbrace{f^{lm}_{i_1 i_2 \cdots i_l}}_{\text{traceless symmetric}} x_{i_1} x_{i_2} \cdots x_{i_l}$$

Then $\hat{Y}_{lm}$ are defined by

$$\hat{Y}_{lm} = \frac{1}{r^l} \sum_{\{i_k\}} f^{lm}_{i_1 i_2 \cdots i_l} \hat{x}_{i_1} \hat{x}_{i_2} \cdots \hat{x}_{i_l}$$

$$(l = 0, 1, \cdots, N)$$

· Properties

$$\begin{cases} [\hat{J}_3, \hat{Y}_{lm}] = m \hat{Y}_{lm} \\ [\hat{J}_1 \pm i \hat{J}_2, \hat{Y}_{lm}] = \sqrt{(l \mp m)(l \pm m + 1)}\, \hat{Y}_{l, m\pm 1} \\ \sum_{i=1}^{3} [\hat{J}_i, [\hat{J}_i, \hat{Y}_{lm}]] = l(l+1) \hat{Y}_{lm} \\ \frac{1}{N+1} \mathrm{Tr}\left(\hat{Y}^{\dagger}_{lm} \hat{Y}_{l'm'} \right) = \delta_{ll'} \delta_{mm'} \\ \quad \vdots \end{cases}$$

NB

$\{l\}$ should be truncated at N.

In fact,

$$\underbrace{\sum_{l=0}^{N} (2l+1)}_{\text{\# of } \{\hat{Y}_{lm}\}} = (N+1)^2 \quad (= \text{dim of } \{(N+1)\times(N+1) \text{ matrices}\})$$

· **NB**

$$[\hat{x}_i, \hat{x}_j] = i\theta \epsilon_{ijk} \hat{x}_k$$

$\Rightarrow$
- $\theta = \frac{2r}{\sqrt{N(N+2)}} \sim \frac{(a(\eta) r)^3}{a(\eta) \cdot L_s^2}$: noncommutative param. for the (dimensionless) comoving coordinates
- L_s : dimensionful noncommutative scale measured in the physical length.

Then,

the inflaton field $\phi(\eta, r, \Omega)$ in a <u>COMMUTATIVE</u> spacetime becomes a matrix-valued 2D field in the <u>NC</u> spacetime:

<u>Commutative</u>: $\displaystyle \phi(\eta, r, \Omega) = \sum_{l=0}^{\infty} \sum_{m} \phi_{lm}(\eta, r)\, Y_{lm}(\Omega)$

$$\Downarrow$$

<u>noncommutative</u>: $\displaystyle \hat{\phi}(\eta, r) = \sum_{l=0}^{N(\eta, r)} \sum_{m} \phi_{lm}(\eta, r)\, \hat{Y}_{lm}$

$\left(N(\eta, r) = \dfrac{4\pi r^2}{(l_s H)^2 \eta^2} - 1 \right)$

Accordingly, the action becomes

$$\begin{aligned} S_2[\hat{\phi}] &= \int d\eta\, dr\, a^2(\eta)\, r^2 \frac{1}{N+1} \mathrm{Tr}\left(\frac{1}{2}(\partial_\eta \hat{\phi})^2 - \frac{1}{2}(\partial_r \hat{\phi})^2 - \frac{1}{2r^2}[\hat{J}_i, \hat{\phi}]^2 \right) \\ &= \int d\eta\, dr \sum_{l=0}^{N(\eta, r)} \sum_{m} a^2(\eta) \cdot r^2 \cdot \frac{1}{2} \left(|\partial_\eta \phi_{lm}|^2 - |\partial_r \phi_{lm}|^2 - \frac{l(l+1)}{r^2} |\phi_{lm}|^2 \right) \end{aligned}$$

· normalized positive energy sol^n (rough argument)

$\boxed{\text{commutative case}}$

$$\phi(\eta,r,\Omega) = \int_0^\infty dk \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \left(a_{klm} \psi_{klm}(\eta,r,\Omega) + h.c. \right)$$

$$\psi_{klm}(\eta,r,\Omega) = \frac{H}{\sqrt{\pi k}} (1+ik\eta)\, j_l(kr)\, Y_{lm}(\Omega)$$

$$\Downarrow$$

$\boxed{\text{noncommutative case}}$

$$\phi(\eta,r,\Omega) \simeq \int_0^\infty dk \sum_{l=0}^{N(\eta,r)} \sum_{m=-l}^{l} \left(a_{klm} \psi_{klm}(\eta,r,\Omega) + h.c. \right)$$

From this, we see that each mode $\boxed{A=(k,l,m)}$ has

(1) $\boxed{\eta^H_A = -\frac{1}{k}}$

$\left(\because \ \frac{1}{H} = \frac{a(\eta)}{k}\Big|_{\eta=\eta^H_A} = -\frac{1}{Hk\eta}\Big|_{\eta=\eta^H_A} \right)$

(2) $\boxed{\eta^{NC}_A = -\alpha_l r \quad \left(\alpha_l \equiv \frac{1}{L_s H}\sqrt{\frac{4\pi}{l+1}} \right)}$

$\left(\because \ l = N(\eta^{NC}_A, r) = \frac{4\pi r^2}{(L_s H)^2 (\eta^{NC}_A)^2} - 1 \right)$

A_c can be evaluated by noting that

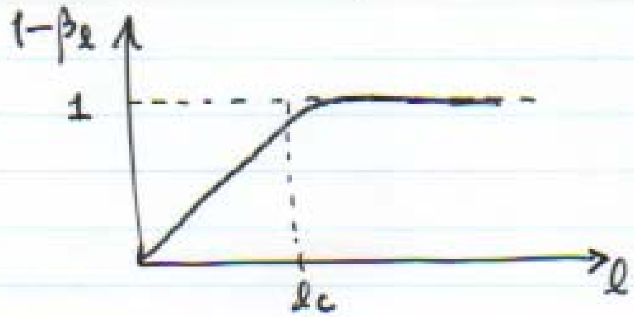
$$j_l(kr) \sim \frac{1}{kr}\cos\left(kr - \frac{l+1}{2}\pi\right) \Rightarrow kr \sim \frac{l+1}{2}\pi \Rightarrow \eta^H_A \simeq -\frac{2r}{\pi(l+1)}$$

and we have

$$\eta^{NC}_{A_c} = \eta^H_{A_c} \Rightarrow \boxed{l_c \simeq \frac{(L_s H)^2}{\pi^3} - 1}$$

$\left(\because \ l_c = O(1) \Rightarrow L_s H = O(1) \right)$

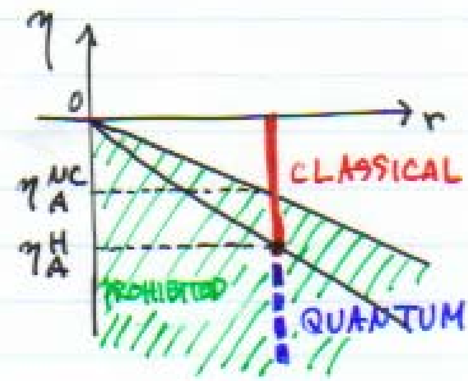
Thus, |class. fluct.$|^2$ can be roughly estimated by multiplying that of the commutative case with the following damping factor ($\equiv 1-\beta_\ell$):



$\left(\ell_c \simeq \frac{(L_s H)^2}{\pi^2} - 1\right)$

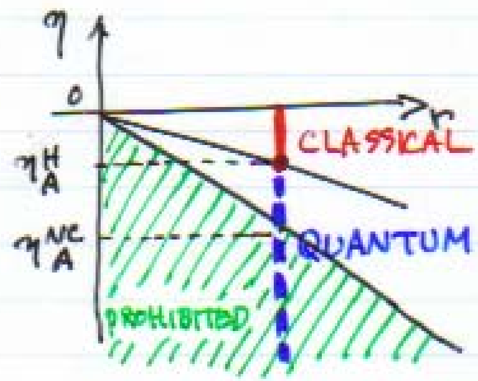
NB

$\ell < \ell_c$



(no quantum life, no classical fossil)

$\ell > \ell_c$



(once having a quantum life, there can be a class. fossil.)

- <u>angular power spectrum</u>

$\boxed{\text{Commutative case}}$ (review)

$$\phi(\eta, r, \Omega) = \sum_{\ell=0}^{\infty} \sum_{m} \phi_{\ell m}(\eta, r)\, Y_{\ell m}(\Omega)$$

$$S_2[\phi] = \sum_{\ell=0}^{\infty} \sum_{m} \int d\eta\, dr\, \frac{a^2(\eta) r^2}{2} \left(|\partial_\eta \phi_{\ell m}|^2 - |\partial_r \phi_{\ell m}|^2 - \frac{\ell(\ell+1)}{r^2} |\phi_{\ell m}|^2 \right)$$

$$\left(a(\eta) = -\frac{1}{H\eta} \ (\eta < 0) \ , \quad \eta \to -0 : \text{exit time of inflation} \right)$$

$$\underset{(m \geq 0)}{\phi_{\ell m}(\eta, r)} = \int_0^\infty dk \left(b_{k\ell m} \psi^{(0)}_{k\ell m}(\eta, r) + d^\dagger_{k\ell m} \psi^{*(0)}_{k\ell m} \right)$$

$$\left(\psi^{(0)}_{k\ell m}(\eta, r) = \frac{H}{\sqrt{\pi k}} (1 + ik\eta)\, j_\ell(kr) \ , \quad \left\{ \begin{aligned} &[b_{k\ell m}, b^\dagger_{k'\ell' m'}] \\ &= \delta(k-k')\, \delta_{\ell\ell'} \delta_{mm'} \end{aligned} \right. \right)$$

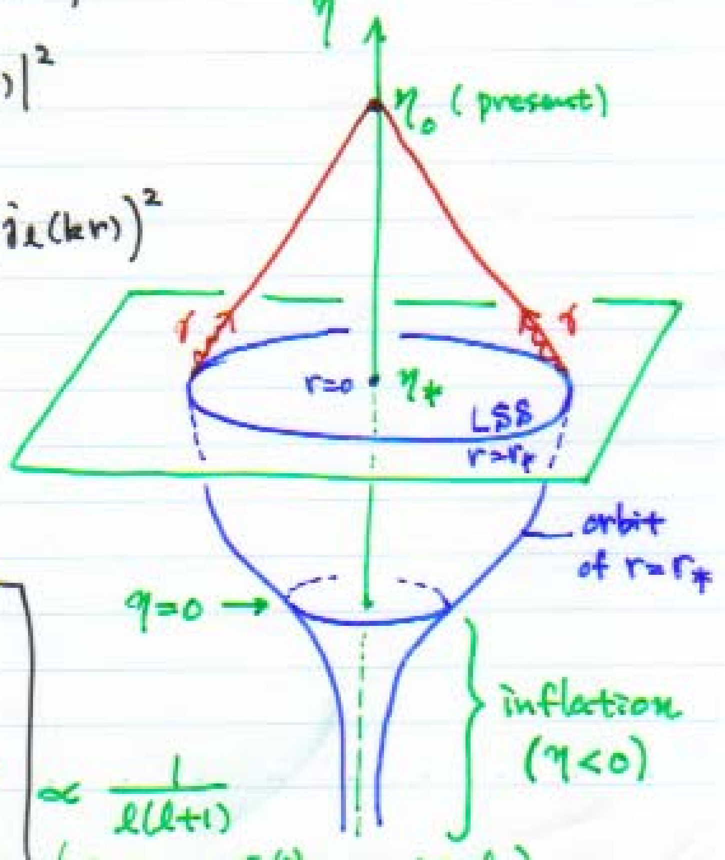
We define

$$\begin{aligned} D^{(0)}_\ell(\eta, r) &\equiv \langle \phi^\dagger_{\ell m}(\eta, r)\, \phi_{\ell m}(\eta, r) \rangle \\ &= \int_0^\infty dk\, |\psi_{k\ell m}(\eta, r)|^2 \\ &= \frac{H^2}{\pi} \int_0^\infty dk\, \frac{k^2 \eta^2 + 1}{k} \left(j_\ell(kr) \right)^2 \end{aligned}$$

$\Downarrow$

The angular power spectrum for the orbit of the LSS ($r = r_*$) is given by

$$\boxed{\begin{aligned} D^{(0)}_\ell &\equiv \lim_{\eta \to -0} D^{(0)}_\ell(\eta, r_*) \\ &= \frac{H^2}{\pi} \int_0^\infty \frac{dk}{k} \left(j_\ell(kr_*) \right)^2 \end{aligned}} \propto \frac{1}{\ell(\ell+1)}$$



Noncommutative case

$$\hat{\phi}(\eta, r) = \sum_{l=0}^{N(\eta,r)} \sum_m \phi_{lm}(\eta, r)\, \hat{Y}_{lm}$$

$$S_2[\hat{\phi}] = \int d\eta\, dr \sum_{l=0}^{N(\eta,r)} \sum_m \frac{a^2(\eta)\cdot r^2}{2}\left(|\partial_\eta \phi_{lm}|^2 - |\partial_r \phi_{lm}|^2 - \frac{l(l+1)}{r^2}|\phi_{lm}|^2\right)$$

$$\boxed{\underset{(m \geq 0)}{\phi_{lm}(\eta, r)} = \int_0^\infty dk\left(b_{klm}\, \psi_{klm}(\eta, r) + d^\dagger_{klm}\, \psi^*_{klm}(\eta, r)\right)}$$

Two fundamental moments:

$(A \equiv (k, l, m))$

$\eta^H_A = -\frac{1}{k}$: the moment at which quantum fluctuations become classical

$\eta^{NC}_A = -\alpha_l r_*$: the moment at which quantum fluctuations start to exist.

$\left(\alpha_l = \frac{1}{L_s H}\sqrt{\frac{4\pi}{l+1}}\right)$

We adopt the following ansatz:

Ansatz :

1) Those modes that are absent in the subhorizon take the vanishing value also in the superhorizon:

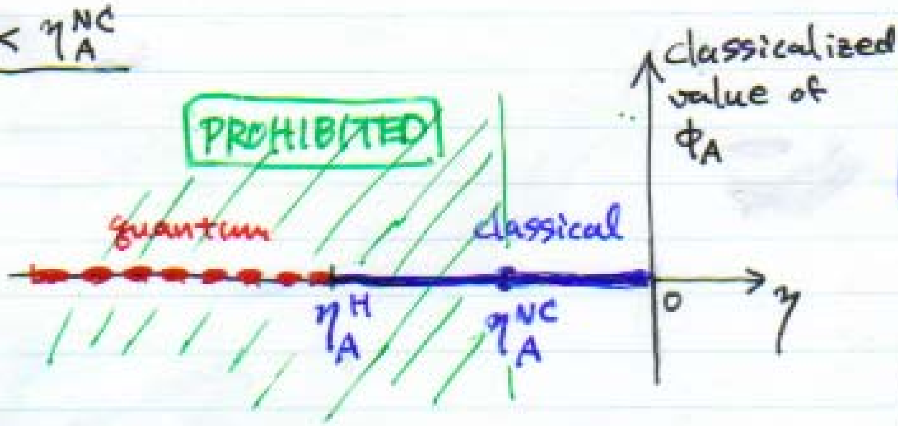
$$\eta_A^{H} < \eta_A^{NC} \;\Rightarrow\; \varphi_A(\eta, r_*) = 0$$

2) Those modes that can exist in the subhorizon behave in the same way as in the commutative case:

$$\eta_A^{NC} < \eta_A^{H} \;\Rightarrow\; \varphi_A(\eta, r_*) = \frac{H}{\sqrt{\pi k}}(1+ik\eta)\, j_\ell(kr_*)\, Y_{\ell m}(\Omega)$$

Schematically, this ansatz is expressed as

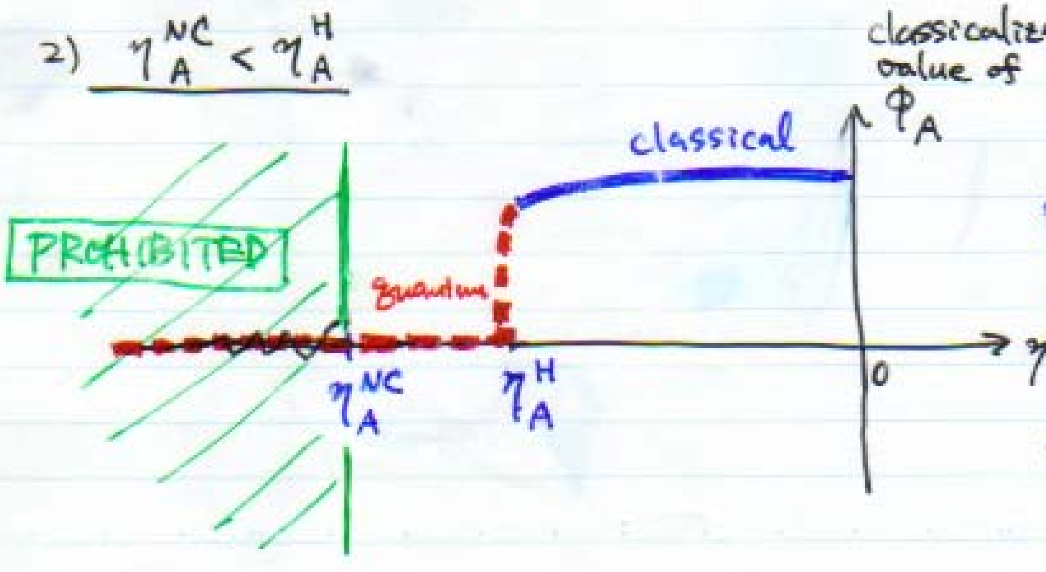
1) $\eta_A^{H} < \eta_A^{NC}$



NO QUANTUM LIFE, THEN NO FOSSIL

- - - : quantum fluctuation
—— : classicalized value

2) $\eta_A^{NC} < \eta_A^{H}$



ONCE HAVING A QUANTUM LIFE, THE SAME SIZE OF FOSSIL TO BE OBSERVED.

This ansatz is equivalent to replacing the positive mode function by

commutative

$$\psi_A(\eta, r_*) = \underbrace{\theta(\eta_A^H - \eta_A^{NC})}_{\substack{\| \\ \theta(-\frac{1}{k} + \alpha_\ell r_*) \\ \| \\ \theta(k - \frac{1}{\alpha_\ell \cdot r_*})}} \cdot \psi_A^{(0)}(\eta, r_*)$$

$(A = (k, \ell, m))$

$\left(\alpha_\ell = \frac{1}{L_s H}\sqrt{\frac{4\pi}{\ell+1}}\right)$

Accordingly, the angular power spectrum is given by

$$D_\ell^{(0)} \to \boxed{D_\ell = \langle \phi^\dagger_{\ell m}(\eta, r_*) \cdot \phi_{\ell m}(\eta, r_*)\rangle\Big|_{\eta \to -0}}$$

$$= \int_0^\infty dk \cdot |\psi_{k\ell m}(\eta, r_*)|^2 \Big|_{\eta \to -0}$$

$$= \frac{H}{\pi}\int_{1/\alpha_\ell r_*}^\infty \frac{dk}{k}\left(j_\ell(kr_*)\right)^2$$

$$= \boxed{\frac{H}{\pi}\int_{1/\alpha_\ell}^\infty \frac{dx}{x}\left(j_\ell(x)\right)^2} \qquad (x = kr_*)$$

We see that

<u>UV</u> cutoff $\frac{1}{L_s}$ in the <u>physical</u> scale gives <u>IR</u> cutoff $\frac{1}{\alpha_\ell \cdot r_*}$ in the <u>comoving</u> wave number.

Beyond the slow-roll approximation,

D_ℓ is expressed with a function $n(k)$ (~ 1) as

$$D_\ell^{(0)} \longrightarrow D_\ell = \frac{H}{\pi} \int_{1/\alpha_\ell \cdot r_*}^{\infty} \frac{dk}{k} k^{n(k)-1} \left(j_\ell(kr_*)\right)^2 \quad (n \sim 1)$$

If n is constant,

$$D_\ell = \frac{H}{\pi} (r_*)^{1-n} \int_{1/\alpha_\ell}^{\infty} dx \; x^{n-2} \left(j_\ell(x)\right)^2$$

$$= D_\ell^{(0)} (1 - \beta_\ell)$$

Here, $\left(\alpha_\ell = \frac{1}{L_s H} \sqrt{\frac{4\pi}{\ell+1}}\right)$

$$\boxed{\begin{aligned} \beta_\ell &= \frac{1}{D_\ell^{(0)}} \frac{H}{\pi} \cdot r_*^{1-n} \cdot \int_0^{1/\alpha_\ell} dx \cdot x^{n-2} \left(j_\ell(x)\right)^2 \\ &= \frac{4}{\sqrt{\pi}} \frac{\Gamma\left(\frac{4-n}{2}\right) \Gamma\left(\ell + \frac{5-n}{2}\right)}{\Gamma\left(\frac{3-n}{2}\right) \Gamma\left(\ell + \frac{n-1}{2}\right)} \int_0^{1/\alpha_\ell} dx \; x^{n-2} \left(j_\ell(x)\right)^2 \end{aligned}}$$

Because of the relation

$$C_\ell = (\ell\text{-indep. const}) \times D_\ell$$

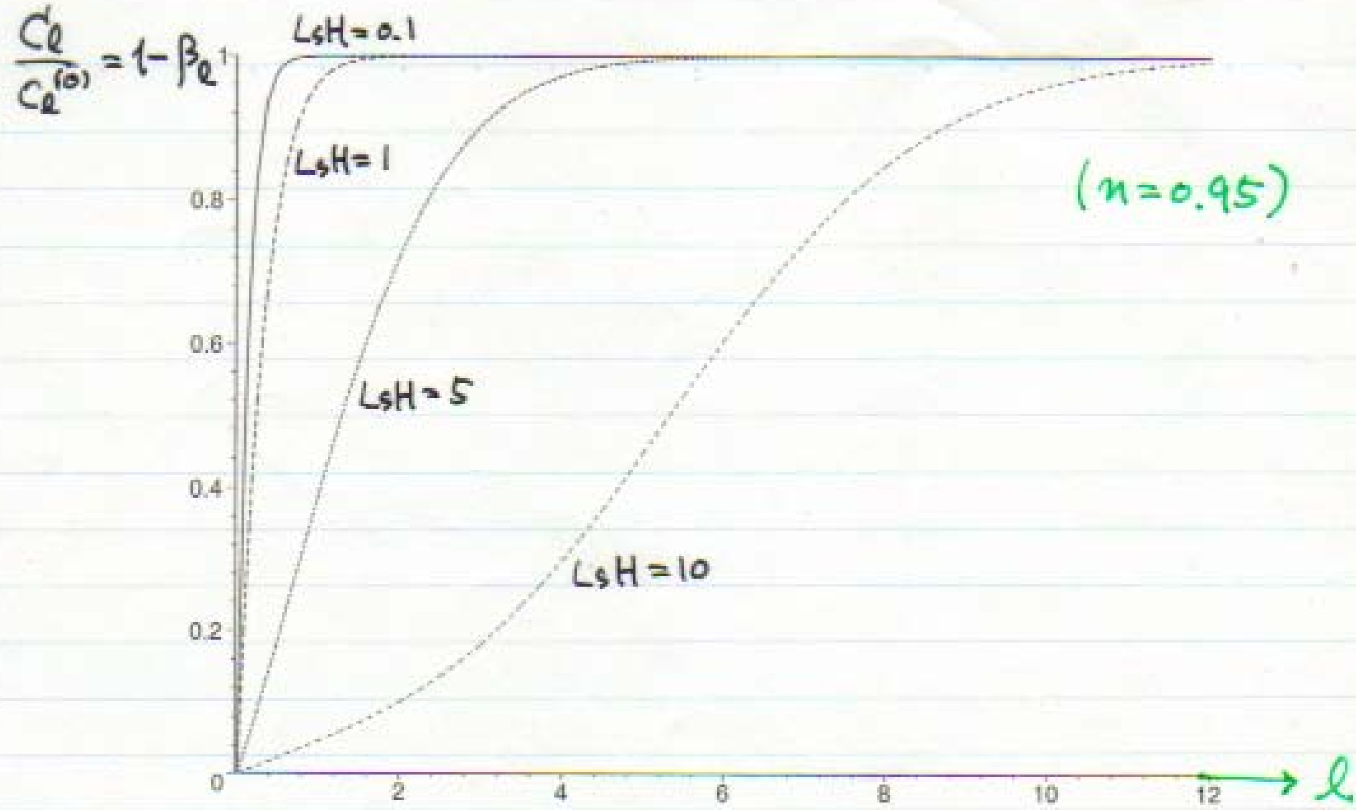
$$\left(\left\langle \frac{\delta T}{T}(\Omega_1) \frac{\delta T}{T}(\Omega_2) \right\rangle = \sum_\ell \frac{2\ell+1}{4\pi} C_\ell \cdot P_\ell(\cos\theta_{12}) \right)$$

for the angular power spectrum of CMB,

we obtain

$$\boxed{\underset{\uparrow\ \text{NC}}{C_\ell} = \underset{\uparrow\ \text{commutative case}}{C_\ell^{(0)}} \underbrace{(1 - \beta_\ell)}_{\text{damping factor}}}$$

The result is



$$\frac{\delta T}{T}(\eta_0, \Omega) = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\Omega)$$

$$\left\langle \frac{\delta T}{T}(\eta_0, \Omega_1) \frac{\delta T}{T}(\eta_0, \Omega_2) \right\rangle = \sum_{\ell} \frac{2\ell+1}{4\pi} C_\ell P_\ell(\cos\theta_{12})$$

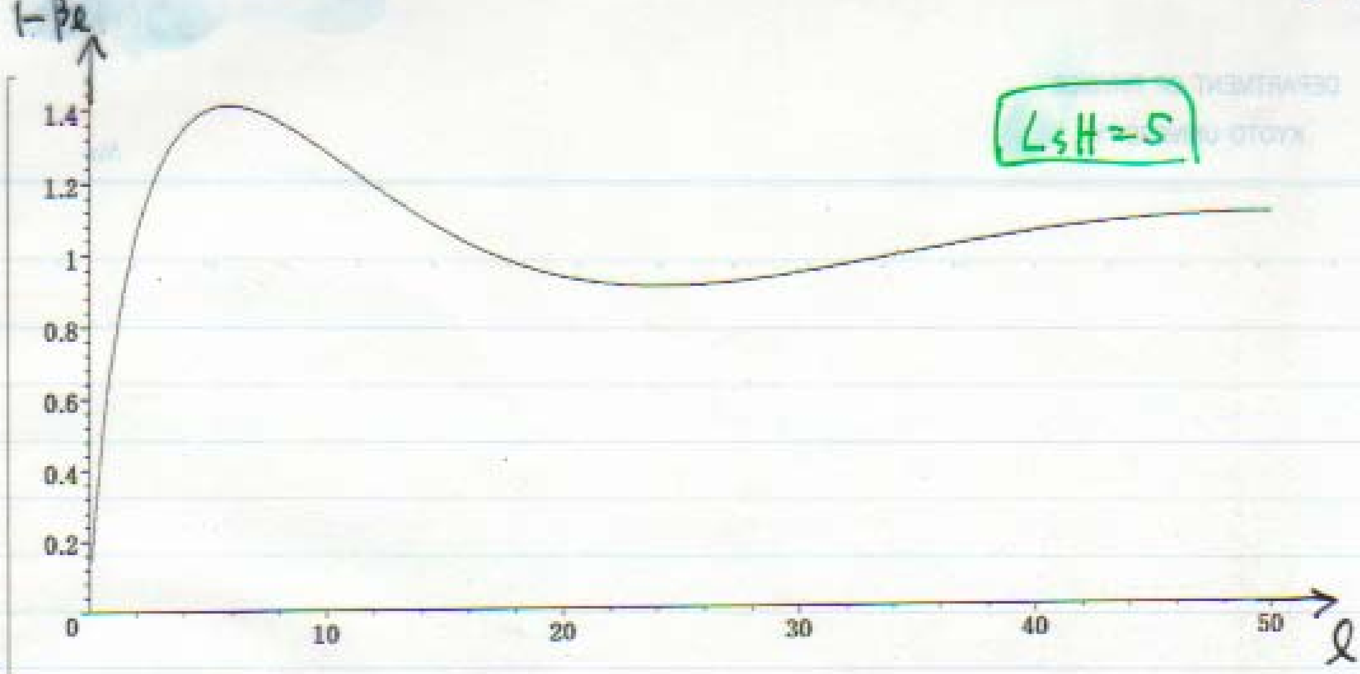
$$C_\ell = \langle a_{\ell m}^{*} a_{\ell m} \rangle$$

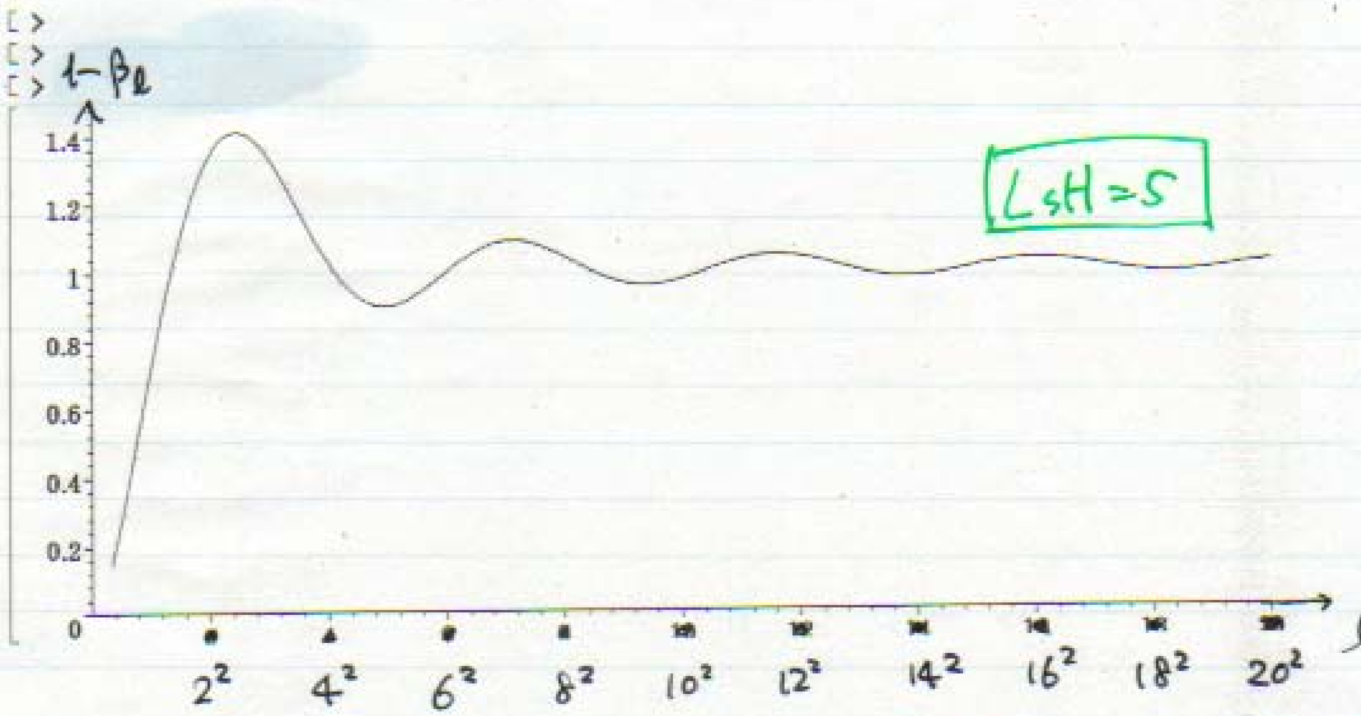
$$= C_\ell^{(0)} (1-\beta_\ell)$$

$$C_\ell^{(0)} \propto \frac{\Gamma(\ell + \frac{n-1}{2})}{\Gamma(\ell + \frac{5-n}{2})} \quad \left(\xrightarrow{n\to 1} \frac{1}{\ell(\ell+1)} \right)$$

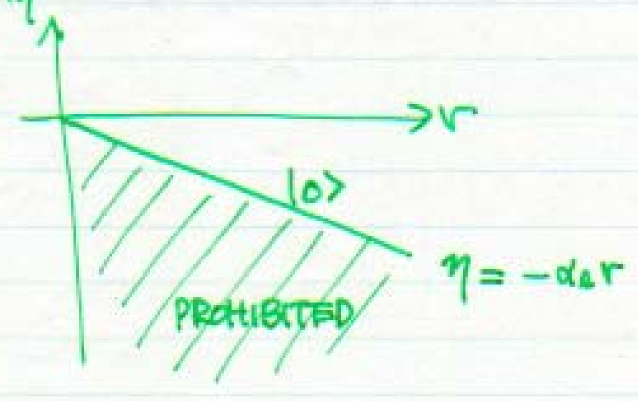
$$\beta_\ell = \frac{4}{\sqrt{\pi}} \frac{\Gamma(\frac{4-n}{2}) \Gamma(\ell + \frac{5-n}{2})}{\Gamma(\frac{3-n}{2}) \Gamma(\ell + \frac{n-1}{2})} \cdot \int_0^{1/\alpha_\ell} dx\, x^{n-2} \left(j_\ell(x) \right)^2$$

$$\left(\alpha_\ell = \frac{1}{L_sH} \sqrt{\frac{4\pi}{\ell+1}} \right)$$





(本当の $\ell(\ell+1)C_\ell$ は、これに $\left\{\begin{array}{l}\text{acoustic peak}\\ \text{Silk damping}\end{array}\right\}$ を合わせたもの)



§4 Holographic interpretation of the large-scale damping

The above analysis was derived from the assumption that the only $\Omega = (\theta, \varphi)$ are NC.

$\Downarrow$

Questions

(1) What will happen if one introduces NC to other coordinates (η, r)?

(2) Does the large-scale damping hold for any cutoff theories?

Partial answer

(1) Some work shows that there is no sharp damping if one puts NC only on (η, r). [Tsujikawa-Maartens-Brandenberger, Huang-Li]

(2) NO

— QFTs with a simple cutoff do not have the critical mode A_c.

§5 Conclusion and outlook

- what we have shown are :
  - — Sharp damping for small l ($l<10$) could be understood as a remnant of the noncommutativity (holography) in the very early universe.
    (large-scale damping)
  - — To fit the result to the observational data, the Hubble length $1/H$ should be of the same order to the NC scale L_s.
    (cf. [M.F.-Kawai-Ninomiya])
    (NB L_s can be much larger than $L_p = 1/\sqrt{G_N}$)
  - — The damping generically holds for any Gaussian fluctuations in the exponentially expanding universe.

    $\Downarrow$

    The same damping pattern is expected to be observed in tensor fluctuations (graviton) if the scenario is correct.

- Future direction
  - — Ansatz 2) seems too strong.

    ⇛ we need to quantize the NC inflaton field with the BC taken into account properly.

    ⇒ Sharper damping is expected.
  - — Only $\Omega = (\theta, \varphi)$ was made noncommutative.

    ⇛ Can we introduce a noncommutativity into space-time in such a way that the cosmological principle manifestly holds (i.e. ISO(3)-invariant way)?

    (or in such a way that the holography is manifest)